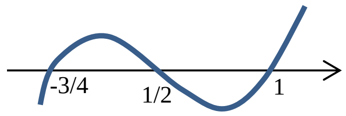
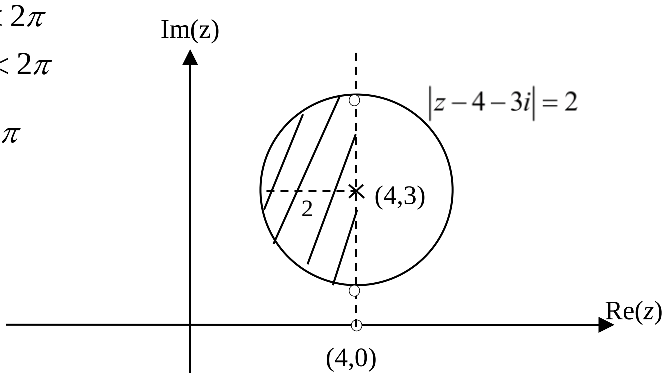
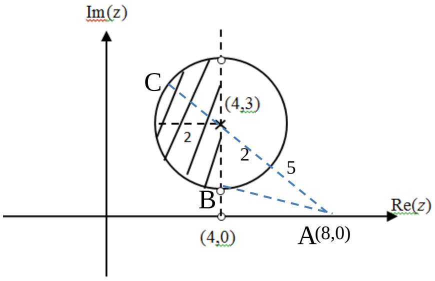
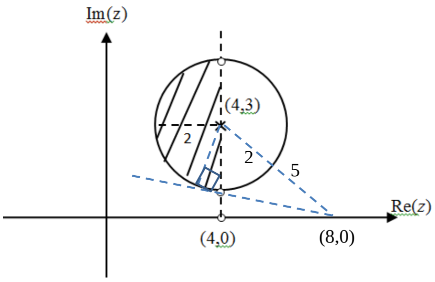
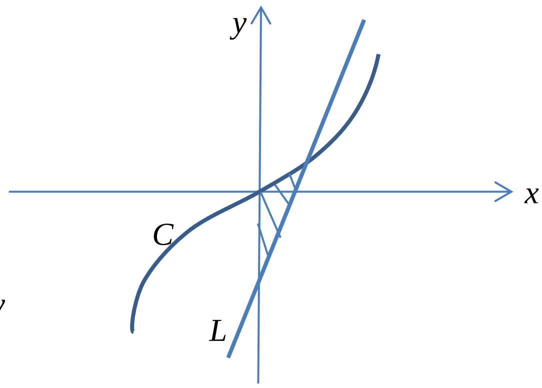
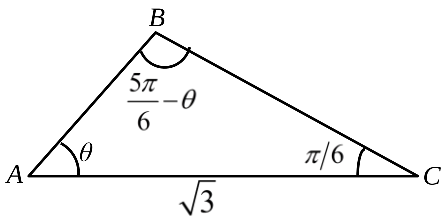
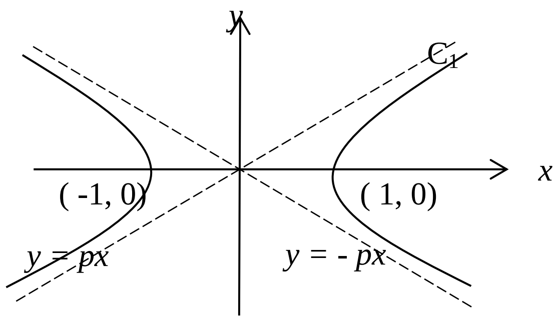
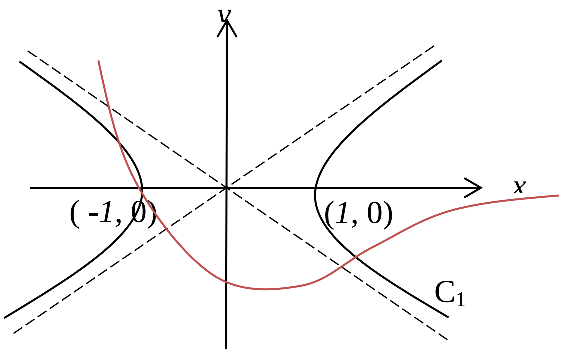
Anderson Junior College
Preliminary Examination 2015
H2 Mathematics Paper 1 (9740/01)

Qn	Solution
1	$\dfrac{x+2}{2x-1}<2x+1$ $\dfrac{x+2-(2x+1)(2x-1)}{2x-1}<0$ $\dfrac{-4x^2+x+3}{2x-1}<0$ $(2x-1)\left(4x^2-x-3\right)>0$ $(2x-1)(4x+3)(x-1)>0$ $-\dfrac{3}{4}<x<\dfrac{1}{2}$ or $x>1$ (ans)  $\dfrac{2x^2+1}{2-x^2}<\dfrac{2+x^2}{x^2}$ $\dfrac{2+\dfrac{1}{x^2}}{\dfrac{2}{x^2}-1}<\dfrac{2}{x^2}+1$ $\Rightarrow -\dfrac{3}{4}<\dfrac{1}{x^2}<\dfrac{1}{2}$ or $\dfrac{1}{x^2}>1$ $\Rightarrow 0<\dfrac{1}{x^2}<\dfrac{1}{2}$ or $\dfrac{1}{x^2}>1$ $\Rightarrow x^2>2$ or $x^2<1$ $\Rightarrow x>\sqrt{2}$ **or** $x<-\sqrt{2}$ or $-1<x<1,\ x\neq 0$
2	$\dfrac{dy}{dx}=\dfrac{x+5}{y^2}\Rightarrow y^2\dfrac{dy}{dx}=x+5$ $2y\left(\dfrac{dy}{dx}\right)^2+y^2\dfrac{d^2y}{dx^2}=1$ $2\left[\left(\dfrac{dy}{dx}\right)^3+2y\dfrac{dy}{dx}\cdot\dfrac{d^2y}{dx^2}\right]+2y\dfrac{dy}{dx}\cdot\dfrac{d^2y}{dx^2}+y^2\dfrac{d^3y}{dx^3}=0$ $\Rightarrow 2\left(\dfrac{dy}{dx}\right)^3+6y\dfrac{dy}{dx}\cdot\dfrac{d^2y}{dx^2}+y^2\dfrac{d^3y}{dx^3}=0$ When $x=0$, $y=5\Rightarrow\dfrac{dy}{dx}=\dfrac{1}{5}$ $10\left(\dfrac{1}{25}\right)+25\dfrac{d^2y}{dx^2}=1\Rightarrow\dfrac{d^2y}{dx^2}=\dfrac{3}{125}$ $2\left(\dfrac{1}{5}\right)^3+30\left(\dfrac{1}{5}\right)\left(\dfrac{3}{125}\right)+25\dfrac{d^3y}{dx^3}=0\Rightarrow\dfrac{d^3y}{dx^3}=-\dfrac{4}{625}$ $\therefore\ y=5+\dfrac{1}{5}x+\dfrac{3}{125(2!)}x^2-\dfrac{4}{625(3!)}x^3+\ldots$ $=5+\dfrac{1}{5}x+\dfrac{3}{250}x^2-\dfrac{2}{1875}x^3+\ldots$

3(i)	$\pi < \arg(z-4)^2 < 2\pi$ $\pi < 2\arg(z-4) < 2\pi$ $\frac{\pi}{2} < \arg(z-4) < \pi$ 
3(ii)	 From diagram, $AB < \lvert z-8\rvert \le AC$ $\sqrt{1^2+4^2} < \lvert z-8\rvert \le \sqrt{3^2+4^2}+2$ $\sqrt{17} < \lvert z-8\rvert \le 7$
3(iii)	 maximum $\arg(z-8)$ $= \pi - \tan^{-1}\frac{3}{4} + \sin^{-1}\frac{2}{5}$ $= 2.9096$ $= 2.91\text{rad}$ (3sf)
4	C: $y = 2\sin^{-1}x$ L: $y = \frac{8\pi}{3}x - \pi$ C & L intersect at $\left(\frac{1}{2}, \frac{\pi}{3}\right)$ And y-intercept of L is -π.  Volume obtained when S is rotated 2π radians about the y-axis $= \frac{1}{3}\pi\left(\frac{1}{2}\right)^2\left(\frac{\pi}{3}\right) + \pi\left(\frac{1}{2}\right)^2\left(\frac{\pi}{3}\right) - \pi\int_0^{\frac{\pi}{3}}\left[\sin\left(\frac{y}{2}\right)\right]^2 dy$ $= \frac{\pi}{12}\left(\frac{4\pi}{3}\right) - \pi\int_0^{\frac{\pi}{3}}\frac{1-\cos y}{2}\,dy$

	$= \frac{\pi^2}{9} - \frac{\pi}{2}\left[y - \sin y\right]_0^{\frac{\pi}{3}}$ $= \frac{\pi^2}{9} - \frac{\pi}{2}\left[\frac{\pi}{3} - \sin\frac{\pi}{3}\right]$ $= \frac{\pi^2}{9} - \frac{\pi^2}{6} + \frac{\pi}{2}\left[\frac{\sqrt{3}}{2}\right]$ $= \frac{\pi\sqrt{3}}{4} - \frac{\pi^2}{18}$
5	By sine rule,  $\frac{AB}{\sin\frac{\pi}{6}} = \frac{\sqrt{3}}{\sin(\frac{5\pi}{6}-\theta)}$ $AB = \frac{\frac{1}{2}\sqrt{3}}{\sin\frac{5\pi}{6}\cos\theta - \cos\frac{5\pi}{6}\sin\theta}$ $= \frac{\frac{1}{2}\sqrt{3}}{\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta}$ $\approx \frac{2\sqrt{3}}{2\left(1-\frac{1}{2}\theta^2\right)+2\sqrt{3}(\theta)}$ since θ is small $= \frac{2\sqrt{3}}{2+2\sqrt{3}\theta-\theta^2}$ (shown) Applying binomial expansion, $AB \approx \sqrt{3}\left[1+\left(\sqrt{3}\theta-\frac{1}{2}\theta^2\right)\right]^{-1}$ $\approx \sqrt{3}\left[1-\left(\sqrt{3}\theta-\frac{1}{2}\theta^2\right)+\left(\sqrt{3}\theta-\frac{1}{2}\theta^2\right)^2\right]$ $\approx \sqrt{3}\left[1-\sqrt{3}\theta+\frac{1}{2}\theta^2+3\theta^2\right]$ $= \sqrt{3}-3\theta+\frac{7\sqrt{3}}{2}\theta^2$ $\left(a=\sqrt{3},\quad b=-3,\quad c=\frac{7\sqrt{3}}{2}\right)$
6(i)	Given that $\sin x > \frac{2x}{\pi}$ $\Rightarrow$ $e^{\sin x} > e^{\frac{2x}{\pi}}$ since $y=e^x$ is increasing $\Rightarrow$ $0 < \frac{1}{e^{\sin x}} < \frac{1}{e^{\frac{2x}{\pi}}}$ $\Rightarrow$ $\int_0^{\frac{\pi}{2}} e^{-\sin x}dx < \int_0^{\frac{\pi}{2}} e^{-\frac{2x}{\pi}}dx$
6(ii)	$\int_{\frac{\pi}{2}}^{\pi} e^{-\sin x}dx = \int_{\frac{\pi}{2}}^{0} e^{-\sin(\pi-u)}(-du)$ Let $u=\pi-x \Rightarrow \frac{du}{dx}=-1$ = $\int_0^{\frac{\pi}{2}} e^{-\sin(u)}du$ since sin (π-u) = sin u

6(iii)	$\int_0^{\pi} e^{-\sin x}dx = \int_0^{\frac{\pi}{2}} e^{-\sin x}dx + \int_{\frac{\pi}{2}}^{\pi} e^{-\sin x}dx$ $= 2\int_0^{\pi/2} e^{-\sin x}dx$ from the result in (ii) $< 2\int_0^{\frac{\pi}{2}} e^{-\frac{2x}{\pi}}dx$ from the result in (i) $= 2\left[\frac{-\pi}{2}e^{-\frac{2x}{\pi}}\right]_0^{\frac{\pi}{2}}$ $= -\pi [e^{-1} - e^{0}]$ $= \frac{\pi(e-1)}{e}$
7(i)	$p^2x^2 - y^2 = p^2$ where $p > 1$ $x^2 - \frac{y^2}{p^2} = 1$ $x \to \infty, \quad \frac{y}{p} \to \pm x, \quad$ ie. $y \to \pm px$ 
7(ii)	The transformation is that of a translation of 2 units in the direction of the positive *x*-axis. The equation of C_2: $p^2(x-2)^2 - y^2 = p^2$ Sub (4,3) into C_2: $p^2(4-2)^2 - 3^2 = p^2$ $4p^2 - 9 = p^2$ $3p^2 = 9$ $p^2 = 3$ $p = \sqrt{3}$ (rej $-\sqrt{3}$ $\because$ $p > 1$)
7(iii)	 No. of roots = no. of intersection points between both graphs = 3
8(i)	Let P_n be the proposition: $\sum_{r=2}^{n} \frac{2}{(r+3)(r+5)} = \frac{11}{30} - \frac{2n+9}{(n+4)(n+5)}$, $n \in \mathbf{Z}^+$, $n \geq 2$. When n = 2, LHS = $\frac{2}{(5)(7)} = \frac{2}{35}$,

	RHS = $\frac{11}{30}-\frac{2(2)+9}{(6)(7)}=\frac{2}{35}$. Since LHS = RHS, P_2 is true. Assume P_k is true for some $k\in\mathbf{Z}^+$, $k\geq 2$ i.e. $\sum_{r=2}^{k}\frac{2}{(r+3)(r+5)}=\frac{11}{30}-\frac{2k+9}{(k+4)(k+5)}$. Need to show that P_{k+1} is also true. i.e. $\sum_{r=2}^{k+1}\frac{2}{(r+3)(r+5)}=\frac{11}{30}-\frac{2(k+1)+9}{(k+1+4)(k+1+5)}=\frac{11}{30}-\frac{2k+11}{(k+5)(k+6)}$. LHS of $P_{k+1}=\sum_{r=2}^{k+1}\frac{2}{(r+3)(r+5)}$ $=\sum_{r=2}^{k}\frac{2}{(r+3)(r+5)}+\frac{2}{(k+4)(k+6)}$ $=\left[\frac{11}{30}-\frac{2k+9}{(k+4)(k+5)}\right]+\frac{2}{(k+4)(k+6)}$ $=\frac{11}{30}-\frac{(2k+9)(k+6)-2(k+5)}{(k+4)(k+5)(k+6)}$ $=\frac{11}{30}-\frac{2k^2+21k+54-2k-10}{(k+4)(k+5)(k+6)}$ $=\frac{11}{30}-\frac{2k^2+19k+44}{(k+4)(k+5)(k+6)}$ $=\frac{11}{30}-\frac{(k+4)(2k+11)}{(k+4)(k+5)(k+6)}$ $=\frac{11}{30}-\frac{2k+11}{(k+5)(k+6)}$ Since P_2 is true, and P_k is true $\Rightarrow P_{k+1}$ is true , by mathematical induction, P_n is true for all $n\in\mathbf{Z}^+$, $n\geq 2$.
8(ii)	$\sum_{r=4}^{n+4}\frac{2}{r(r+2)}=\sum_{r=1}^{n+1}\frac{2}{(r+3)(r+5)}$ $=\sum_{r=2}^{n+1}\frac{2}{(r+3)(r+5)}+\frac{2}{(4)(6)}$ $=\frac{11}{30}-\frac{2n+11}{(n+5)(n+6)}+\frac{2}{24}$ $=\frac{9}{20}-\frac{2n+11}{(n+5)(n+6)}$
8(iii)	$\sum_{r=4}^{n+4}\frac{1}{(r+1)^2}=\frac{1}{2}\sum_{r=4}^{n+4}\frac{2}{(r+1)^2}$ $<\frac{1}{2}\sum_{r=4}^{n+4}\frac{2}{r(r+2)}$ (Since $(r+1)^2=r^2+2r+1>r^2+2r=r(r+2)$)

	$=\frac{1}{2}\left[\frac{9}{20}-\frac{2n+11}{(n+5)(n+6)}\right]$ $<\frac{9}{40}$ (since $\frac{2n+11}{(n+5)(n+6)}>0$ for all $n\in\mathbb{Z}^+$)
9(i)	$\frac{d}{dx}\left(\frac{1}{\sqrt{x^2-3}}\right)=\frac{d}{dx}\left((x^2-3)^{-\frac{1}{2}}\right)=\left(-\frac{1}{2}\right)(x^2-3)^{-\frac{3}{2}}(2x)=\frac{-x}{(x^2-3)^{\frac{3}{2}}}$
9(ii)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1}\frac{1}{x}\right)=\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\left(-\frac{1}{x^2}\right)$ $=\frac{1}{\sqrt{\frac{x^2-1}{x^2}}}\left(-\frac{1}{x^2}\right)=\frac{x}{\sqrt{x^2-1}}\left(-\frac{1}{x^2}\right)$ ($\sqrt{x^2}=x\ \ as\ \ x>1$) $=\ -\frac{1}{x\sqrt{x^2-1}}$
9(iii)	Area $=\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} y\,\mathrm{d}x$ $=\int_{2}^{\frac{2}{\sqrt{3}}}\ln t\,\frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t$ $=\int_{2}^{2/\sqrt{3}}\ln(t)\,\frac{-t}{(t^2-1)^{3/2}}\mathrm{d}t$ from(i) $=\int_{\frac{2}{\sqrt{3}}}^{2}\ln t.\frac{t}{(t^2-1)^{\frac{3}{2}}}\,\mathrm{d}t$ $(as\ \ \int_a^b f(x)dx=-\int_b^a f(x)dx)$ $=\left[\left(\frac{-1}{\sqrt{t^2-1}}\right)\ln t\right]_{t=\frac{2}{\sqrt{3}}}^{t=2}-\int_{\frac{2}{\sqrt{3}}}^{2}\frac{-1}{\sqrt{t^2-1}}\left(\frac{1}{t}\right)\mathrm{d}t$ $=\left[\left(\frac{-\ln 2}{\sqrt{3}}\right)-\left(\frac{-\ln\frac{2}{\sqrt{3}}}{\sqrt{\frac{1}{3}}}\right)\right]+\int_{\frac{2}{\sqrt{3}}}^{2}\frac{1}{t\sqrt{t^2-1}}\mathrm{d}t$ $=\left[\frac{-\ln 2}{\sqrt{3}}+\sqrt{3}\ln\frac{2}{\sqrt{3}}\right]-\left[\sin^{-1}\frac{1}{t}\right]_{\frac{2}{\sqrt{3}}}^{2}$ (by part (ii)) $=-\frac{1}{\sqrt{3}}ln2+\sqrt{3}ln2-\sqrt{3}\ln(\sqrt{3})-[sin^{-1}\left(\frac{1}{2}\right)-sin^{-1}\left(\frac{\sqrt{3}}{2}\right)]$ $=(\sqrt{3}-\frac{1}{\sqrt{3}})ln2-\frac{\sqrt{3}}{2}ln3-(\frac{\pi}{6}-\frac{\pi}{3})$ $=\frac{2\sqrt{3}}{3}ln2-\frac{\sqrt{3}}{2}ln3+\frac{\pi}{6}$
10(i)	Let $y=\mathrm{f}(x)\Rightarrow y=\frac{1}{2-x^2}$ $\Rightarrow x^2=2-\frac{1}{y}$

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$\Rightarrow x = -\sqrt{2-\frac{1}{y}}$ since $x \le 0$

Therefore $f^{-1} : x \to -\sqrt{2-\frac{1}{x}}$, $x \in (-\infty, 0) \cup \left[\frac{1}{2}, \infty\right)$

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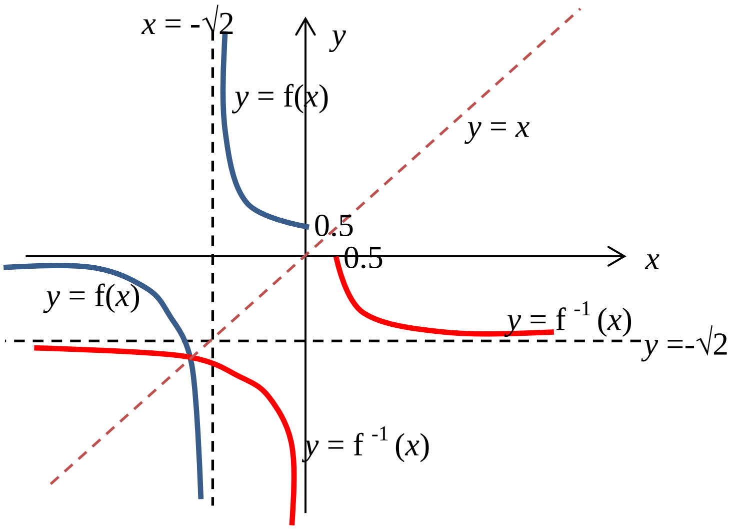

$f(x) = f^{-1}(x)$

$\Rightarrow f(x) = x$

$\Rightarrow \frac{1}{2-x^2} = x$

$\Rightarrow x^3 - 2x + 1 = 0$

$\Rightarrow (x-1)(x^2 + x - 1) = 0$

$\Rightarrow x = 1,\ x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

$f(x) \le f^{-1}(x)$

$\Rightarrow \frac{-1-\sqrt{5}}{2} \le x < -\sqrt{2}$

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<tr><td>10(iii)</td><td>

For $f^{-1}g$ to exist,

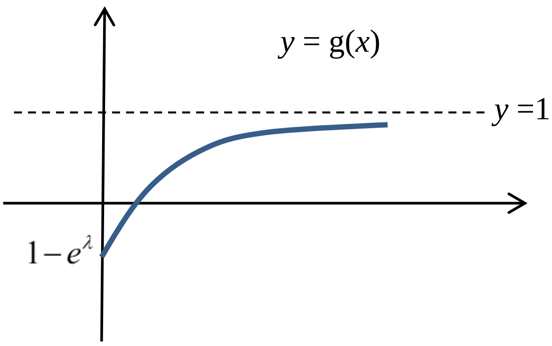
range of g $\subseteq$ Domain of f^{-1}

$\Rightarrow [1-e^{\lambda}, 1) \subseteq \left[\frac{1}{2}, \infty\right)$

$\Rightarrow 1 - e^{\lambda} \ge \frac{1}{2}$

$\Rightarrow \lambda \le \ln\frac{1}{2}$

Greatest value of $\lambda = \ln\frac{1}{2}$



$[0, \infty) \xrightarrow{g} \left[\frac{1}{2}, 1\right) \xrightarrow{f^{-1}} (-1, 0]$.

Range of $f^{-1}g = = (-1, 0]$

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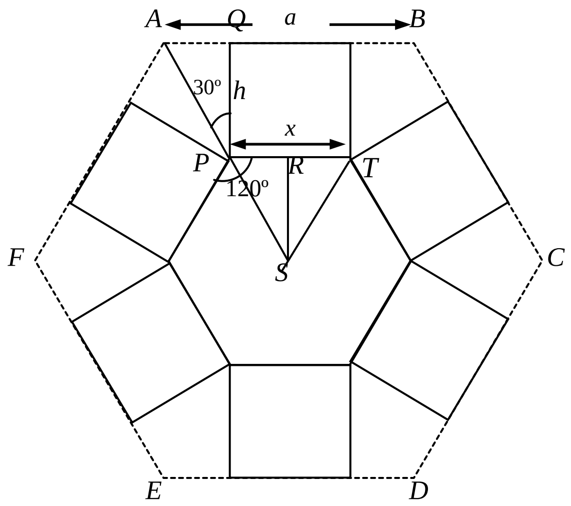
11(i)

i) From triangle *APQ*,



$$\tan 30° = \frac{\frac{a-x}{2}}{h} \Rightarrow \frac{1}{\sqrt{3}} = \frac{a-x}{2h}$$

$$\Rightarrow x = a - \frac{2}{\sqrt{3}}h$$

$$\begin{aligned}
\text{Volume, } V &= \text{base area} \times \text{height} \\
&= 6(\text{area of □PST}) \times h \\
&= 6\left(\frac{1}{2}x^2 \sin 60^o\right) \times h \\
&= 3h\left(a - \frac{2}{\sqrt{3}}h\right)^2 \\
&= 3h\left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{\sqrt{3}}{2}a - h\right)^2 \\
&= 2\sqrt{3}\, h\left(\frac{\sqrt{3}}{2}a - h\right)^2 \quad \text{(shown)}
\end{aligned}$$

<u>Alternative method: find the height *RS* of triangle *PST*</u>

From triangle *PSR*,

$$\tan 30° = \frac{\frac{x}{2}}{RS} \Rightarrow \frac{1}{\sqrt{3}} = \frac{\frac{x}{2}}{RS}$$

$$\Rightarrow RS = \frac{\sqrt{3}}{2}\left(a - \frac{2}{\sqrt{3}}h\right)$$

$$\begin{aligned}
\text{Volume, } V &= \text{base area} \times \text{height} \\
&= 6\left[\frac{1}{2}(x)RS\right] \times h \\
&= 3h\left(a - \frac{2}{\sqrt{3}}h\right)\left[\frac{\sqrt{3}}{2}\left(a - \frac{2}{\sqrt{3}}h\right)\right] \\
&= \frac{3\sqrt{3}}{2}h\left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{\sqrt{3}}{2}a - h\right)^2 = 2\sqrt{3}\, h\left(\frac{\sqrt{3}}{2}a - h\right)^2 \quad \text{(shown)}
\end{aligned}$$

11(ii)

$$V = 2\sqrt{3}\, h\left(\frac{\sqrt{3}}{2}a - h\right)^2$$

$$\frac{dV}{dh} = 2\sqrt{3}\left[2\left(\frac{\sqrt{3}}{2}a - h\right)(-1)h + \left(\frac{\sqrt{3}}{2}a - h\right)^2\right]$$

	$= 2\sqrt{3}\left(\frac{\sqrt{3}}{2}a-h\right)\left[-2h+\frac{\sqrt{3}}{2}a-h\right]$ $= 2\sqrt{3}\left(\frac{\sqrt{3}}{2}a-h\right)\left(\frac{\sqrt{3}}{2}a-3h\right)$ For stationary value of V, $\frac{\mathrm{d}V}{\mathrm{d}h}=0.$ $\Rightarrow h=\frac{\sqrt{3}}{2}a \quad \text{or} \quad h=\frac{\sqrt{3}}{6}a$ When $h=\frac{\sqrt{3}}{2}a,$ base area of the box is zero (or the volume is zero). Hence this value of h does not give a maximum volume of the box. (OR: show that $\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=6a>0$ for this value of h) To check for maximum at $h=\frac{\sqrt{3}}{6}a,$ *1st Derivative Test* <table><tr><th>h</th><th>$(\frac{\sqrt{3}}{6}a)^-$</th><th>$\frac{\sqrt{3}}{6}h$</th><th>$(\frac{\sqrt{3}}{6}h)^+$</th></tr><tr><td>$\frac{dV}{dh}$</td><td>>0</td><td>0</td><td><0</td></tr></table>When $h=\left(\frac{\sqrt{3}}{6}a\right)^-$, $\left(\frac{\sqrt{3}}{2}a-h\right)>0$ and $\left(\frac{\sqrt{3}}{2}a-3h\right)>0 \Rightarrow \frac{dV}{dh}>0$ When $h=\left(\frac{\sqrt{3}}{6}a\right)^+$, $\left(\frac{\sqrt{3}}{2}a-h\right)>0$ and $\left(\frac{\sqrt{3}}{2}a-3h\right)<0 \Rightarrow \frac{dV}{dh}<0$ *<u>2nd Derivative Test</u>* $\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=2\sqrt{3}\left[-\left(\frac{\sqrt{3}}{2}a-3h\right)-3\left(\frac{\sqrt{3}}{2}a-h\right)\right]=12\sqrt{3}h-12a$ When $h=\frac{\sqrt{3}}{6}a,$ $\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=-6a<0$. So V is maximum when $h=\frac{\sqrt{3}}{6}a.$
12(i)	Line l : $\mathbf{r} = \begin{pmatrix}0\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}0\\6\\1\end{pmatrix}, \lambda \in \mathbb{R}$ Let θ be the angle between the line l and the plane p_1.

	$\sin\theta = \left\lvert \dfrac{\begin{pmatrix}0\\6\\1\end{pmatrix}\bullet\begin{pmatrix}2\\-1\\1\end{pmatrix}}{\sqrt{6^2+1^2}\sqrt{2^2+1^2+1^2}} \right\rvert = \left\lvert \dfrac{-6+1}{\sqrt{37}\sqrt{6}} \right\rvert \Rightarrow \theta = 19.6^\circ$
12(ii)	Let M be the point of $PQ = \left(0,3,\frac{3}{2}\right)$. R lies on x-y plane $\Rightarrow R = (a,b,0)$. Thus $\overrightarrow{MR} = \begin{pmatrix}a\\b-3\\-\frac{3}{2}\end{pmatrix}$ $\overrightarrow{MR} \perp$ line l $\Rightarrow \begin{pmatrix}a\\b-3\\-\frac{3}{2}\end{pmatrix}\bullet\begin{pmatrix}0\\6\\1\end{pmatrix} = 0 \Rightarrow 6b-18-\frac{3}{2}=0 \Rightarrow b=\frac{13}{4}$ Length of MR = 2 $\Rightarrow \sqrt{a^2+(b-3)^2+\left(-\frac{3}{2}\right)^2} = 2$ $\Rightarrow a^2+\left(\frac{13}{4}-3\right)^2+\left(-\frac{3}{2}\right)^2 = 4 \quad \Rightarrow a = \pm\frac{3\sqrt{3}}{4}$ $\therefore R = \left(\frac{3\sqrt{3}}{4},\frac{13}{4},0\right)$ or $\left(-\frac{3\sqrt{3}}{4},\frac{13}{4},0\right)$
12(iii)	p_1: $\mathbf{r}\bullet\begin{pmatrix}2\\-1\\1\end{pmatrix} = \begin{pmatrix}2\\-1\\1\end{pmatrix}\bullet\begin{pmatrix}0\\6\\2\end{pmatrix} = -5 \Rightarrow 2x-y+z=-4$ p_2: $x+5y-10z=0$ Using GC, line of intersection of the two planes is $\mathbf{r} = \begin{pmatrix}-\frac{10}{11}\\\frac{24}{11}\\0\end{pmatrix}+\beta\begin{pmatrix}5\\21\\11\end{pmatrix}, \beta\in\mathbb{R}$ [OR $\mathbf{r} = \begin{pmatrix}\frac{30}{11}\\\frac{16}{11}\\0\end{pmatrix}+\beta\begin{pmatrix}5\\21\\11\end{pmatrix}, \beta\in\mathbb{R}$] A vector parallel to p_3 is $\begin{pmatrix}5\\21\\11\end{pmatrix}$. Another vector parallel to $p_3 = \begin{pmatrix}-\frac{10}{11}\\\frac{24}{11}\\0\end{pmatrix}-\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}-\frac{21}{11}\\\frac{13}{11}\\-1\end{pmatrix} = -\frac{1}{11}\begin{pmatrix}21\\-13\\11\end{pmatrix}$

	[Note that $(0,6,2)$ lies on p_3 since it lies on both p_1 and p_2 $\Rightarrow$ it lies on p_3. Accept $\begin{pmatrix}-1\\5\\1\end{pmatrix}$ or equivalent as direction vector] Normal to p_3 $= \begin{pmatrix}5\\21\\11\end{pmatrix} \times \begin{pmatrix}-\frac{21}{11}\\ \frac{13}{11}\\ -1\end{pmatrix} = \begin{pmatrix}-34\\-16\\46\end{pmatrix} = -2\begin{pmatrix}17\\8\\-23\end{pmatrix}$ Equation of p_3 : $\mathbf{r} \bullet \begin{pmatrix}17\\8\\-23\end{pmatrix} = \begin{pmatrix}17\\8\\-23\end{pmatrix} \bullet \begin{pmatrix}1\\1\\1\end{pmatrix} \Rightarrow \mathbf{r} \bullet \begin{pmatrix}17\\8\\-23\end{pmatrix} = 2$