Name: ______________________ Class: ________

For Marker's Use
90

南僑中學

NAN CHIAU HIGH SCHOOL

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PRELIMINARY EXAMINATION 2023
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS 4049/02
Paper 2 23 August 2023, Wednesday

Candidates answer on the Question Paper. **2 hours 15 minutes**

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + \binom{n}{r} a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

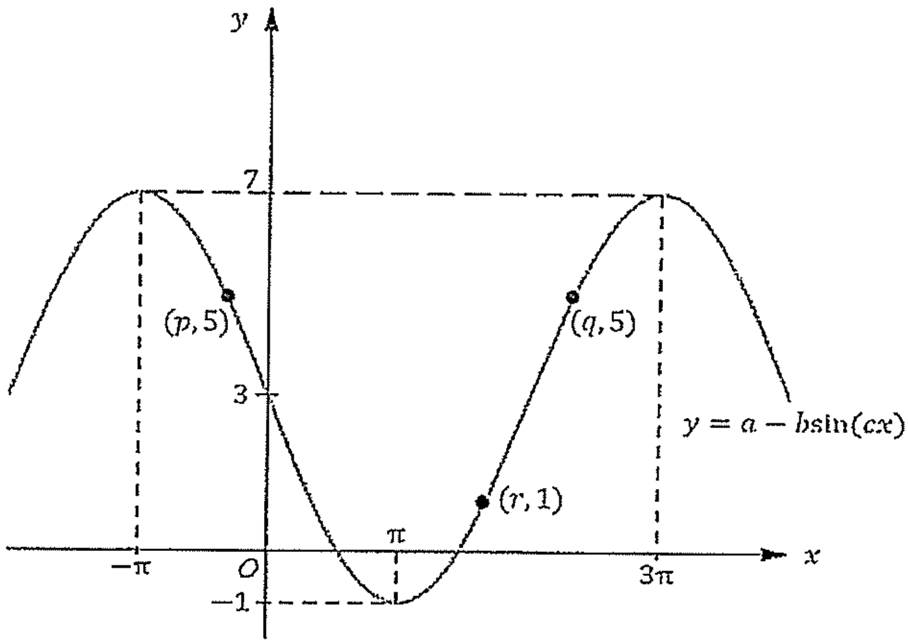
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1



The diagram above shows part of the curve $y = a - b\sin(cx)$, where a, b and c are constants.

(a) Write down the values of a, b and c. [3]

The curve passes through the points $(p, 5)$, $(q, 5)$ and $(r, 1)$, where p, q and r are constants.

(b) Find an equation, in terms of π, connecting

(i) p and q, [1]

(ii) q and r. [1]

2 (a) Explain with the aid of a sketch why $\int_0^a \sqrt{a^2 - x^2}\ \mathrm{d}x = \frac{\pi a^2}{4}$, for $a > 0$. [2]

(b) Hence, find, in terms of a and π,

(i) $\int_a^0 \sqrt{4(a^2 - x^2)}\ \mathrm{d}x$, [2]

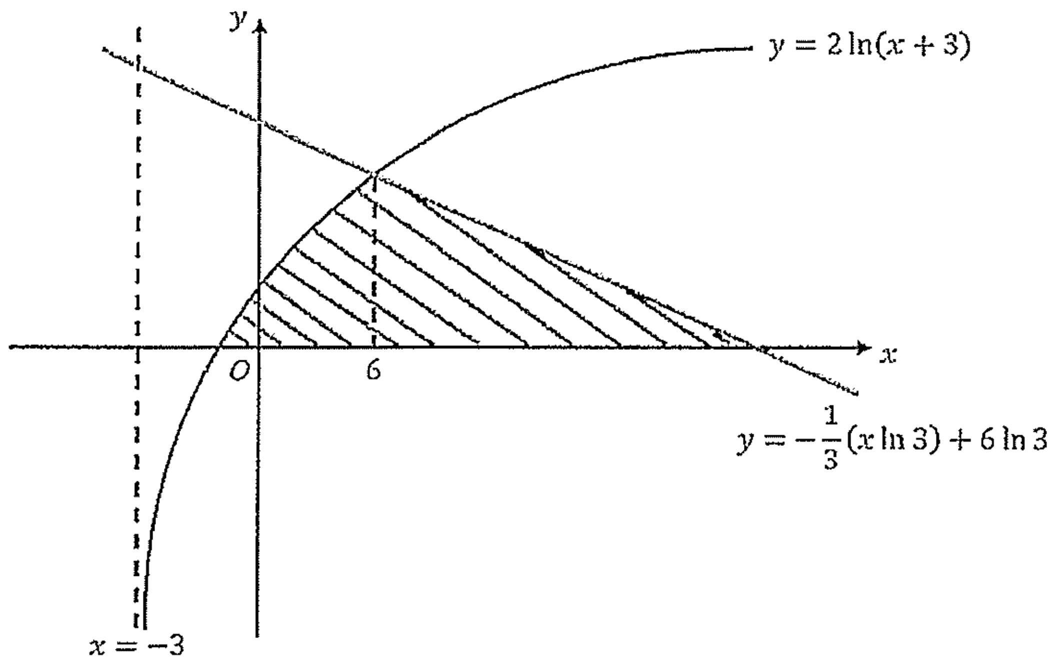
(ii) $\int_{-a}^a \sqrt{a^2 - x^2}\ \mathrm{d}x$. [2]

3 Air is being pumped into a spherical elastic ball at a constant rate of 10 cm^3 per second. It is assumed that the ball maintains a spherical shape throughout the process.

(a) Find the rate of increase of the radius of the ball at the instant when the radius is 5 cm. [The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.] [3]

(b) Find the rate of increase of the surface area of the ball at the instant when the radius of the ball is increasing at the rate of $\frac{5}{72\pi}$ cm per second.
[The surface area of a sphere of radius r is $4\pi r^2$.] [5]

4



The diagram shows part of the curve $y = 2\ln(x+3)$ and a line $y = -\frac{1}{3}(x \ln 3) + 6\ln 3$. The curve and the line intersect at $x = 6$. Find the exact area of the shaded region bounded by the curve, the line and the x–axis. Express your answer in the form $a\ln 3 + b$, where a and b are integers to be determined. [9]

Continuation of working space for question 4.

5 On 01 January 2010, there were 1500 predators of a particular species in a habitat. Ecologists believe that the population of the predator, P in thousands, can be modelled by the formula $P = N - 3.5(e^{kt})$, where N and k are positive constants and t is the time in years after 01 January 2010.

(a) Show that $N = 5$. [1]

(b) Comment on the feasibility of the ecologist's model for the population of the predator in the long run. [2]

(c) The population of the prey, Q in thousands, can be modelled by the formula $Q = 8e^{-t}$, where t is the time in years after 01 January 2010. The population of predator and prey first became equal on 01 January 2020. Find the value of k. [3]

(d) A mathematician believes that the population of the predator, P in thousands, should be modelled by the formula $P = 5 - 3.5(e^{-2t})$, where t is the time in years after 01 January 2010, instead. Determine the year and month in which the population of the predator first doubles the population of the prey under this model. [4]

6 The polynomial $f(x)$ is given by $f(x) = 2x^3 - 6x^2 + Ax + B$, where A and B are constants. Find the values of A and B such that

(a) $x^2 - 9$ is a factor, [4]

(b) $x^2 + 9$ is a factor, [4]

(c) the curve $y = \mathrm{f}(x)$ cuts the x-axis at $x = -1$ and just touches the x-axis at $x = 2$. [3]

7 (a) (i) By considering the general term in the binomial expansion of $\left(\frac{1}{2}x^2 + \frac{\sqrt{2}}{x^2}\right)^8$, explain why there are no terms with odd powers of x. [3]

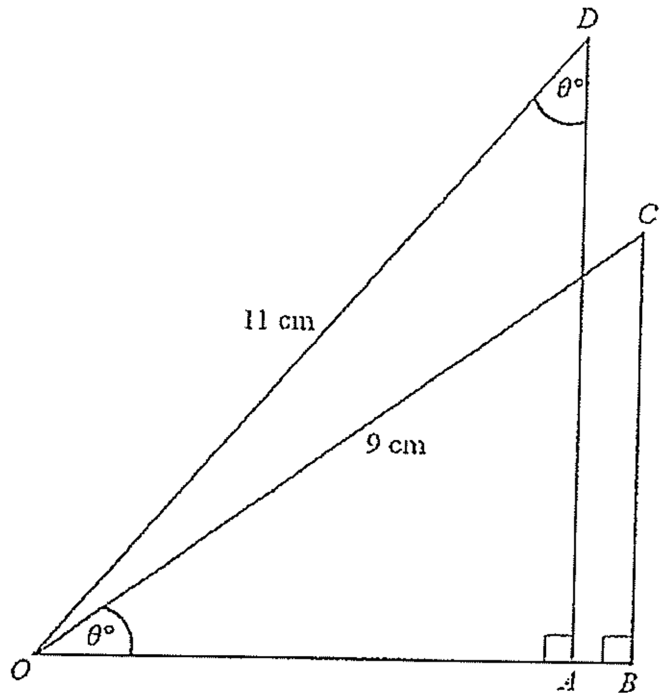
(ii) Find the term independent of x in the expansion of $\left(\frac{1}{2}x^2 + \frac{\sqrt{2}}{x^2}\right)^8 (3x - 7)^2$. [3]

(b) In the binomial expansion of $\left(1+3\sqrt{x}\right)^n$, the coefficient of $x\sqrt{x}$ is 7 times the coefficient of x.

(i) Show that $3\binom{n}{3} = 7\binom{n}{2}$. [3]

(ii) Hence, find the value of n. [2]

8



In the diagram, triangles OBC and OAD are right-angled triangles such that $OC = 9$ cm and $OD = 11$ cm. Angles BOC and ODA are each equal to $\theta°$, where $0° < \theta < 90°$.

(a) Express AB in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(b) Find the minimum value of AB and the corresponding value of θ. [3]

(c) Find the range of values of θ for which A is between O and B. [3]

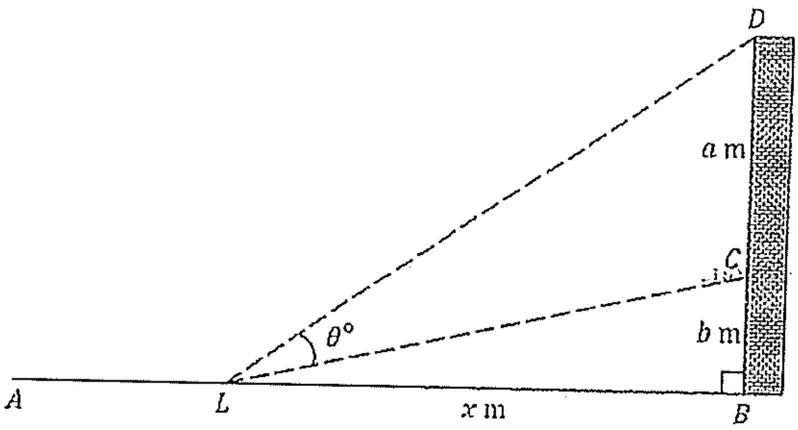
9 The equation of a circle is $x^2 + y^2 - 10x - 4y + 19 = 0$.

(a) Point P lies on the circle, and the tangent to the circle at point P has a gradient of -3. Find the possible coordinates of point P. [5]

(b) Another two tangents to the circle intersect at the origin. Find the gradients of these two tangents.

[5]

10



In the diagram, A and B are two fixed points on a horizontal ground and a projector is positioned on the ground at L which is x m away from B. The projector casts a beam of light on a screen CD, of fixed height a m. C is the bottom of the screen, where $BC = b$ m. Angle CLD is $\theta°$. Assume that the thickness of the screen is negligible.

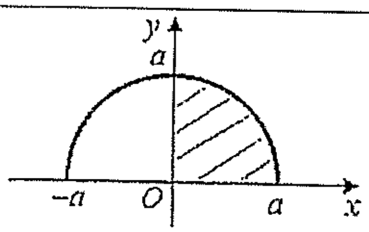
(a) Express $\tan(\angle DLB)$ and $\tan(\angle CLB)$ in terms of a, b or/and x. Hence, show that $\tan\theta = \frac{ax}{x^2+ab+b^2}$. [4]

(b) Given that x can vary, find, in terms of a and b, the value of x for which $\tan\theta$ is stationary. [4]

(c) Given that $a = 10$ and $b = 3$, find the value of θ which gives the stationary value of $\tan\theta$ found in part (b). [2]

End of Paper

Answer Key

1(a)	$a=3,\ b=4,\ c=\frac{1}{2}$
1(b)(i)	$p+q=2\pi$
1(b)(ii)	$q+r=4\pi$
2(a)	 $\int_0^a \sqrt{a^2-x^2}\ \mathrm{d}x$ $= Area\ of\ quarter\ circle$ $=\frac{\pi a^2}{4}$
2(b)(i)	$-\frac{\pi a^2}{2}$
2(b)(ii)	$\frac{\pi a^2}{2}$
3(a)	$\frac{dr}{dt}=\frac{1}{10\pi}$ cm/s Accept: 0.0318 cm/s (3 s.f.)
3(b)	$\frac{dS}{dt}=\frac{10}{3}$ cm^2/s *Accept*: 3.33 cm^2/s (3 s.f.)
4	$(60\ln 3-16)$ units^2
5(a)	$N=5$ *(shown)*
5(b)	$P=5-3.5(e^{kt})$ *As* $t\to\infty$, $3.5(e^{kt})\to\infty$ for $k>0$ $\therefore P\to-\infty$ Since the number of predator become <u>negative</u> in the long run, the ecologist's model <u>may not be feasible</u> in the long run.
5(c)	0.0357 (3 s. f.)
5(d)	Year 2011, March

6(a)	$B = 54,\ A = -18$
6(b)	$B = -54,\ A = 18$
6(c)	$B = 8,\ A = 0$
7(a)(i)	Since $16 - 4r = 4(4 - r)$ is a multiple of 4 / even for all real values of $r, 0 \leq r \leq 8$, there are no terms with odd powers of x.
7(a)(ii)	857.5
7(b)(i)	Shown Question
7(b)(ii)	$n = 9$
8(a)	$\therefore AB = 14.2\cos(\theta + 50.7°)$
8(b)	minimum value of $AB = 0$ when $\theta + 50.7° = 90°$ $\theta = 39.3°$ (1 d.p.)
8(c)	$\therefore 0° < \theta < 39.3°$
9(a)	$\therefore P(2,1)$ or $P(8,3)$
9(b)	1.59 or -0.252 (3 s.f.)
10(a)	$\tan(\angle DLB) = \dfrac{a+b}{x}$ $\tan(\angle CLB) = \dfrac{b}{x}$
10(b)	$x = \sqrt{ab + b^2}$ $(x > 0)$
10(c)	38.7° (1 d.p.)