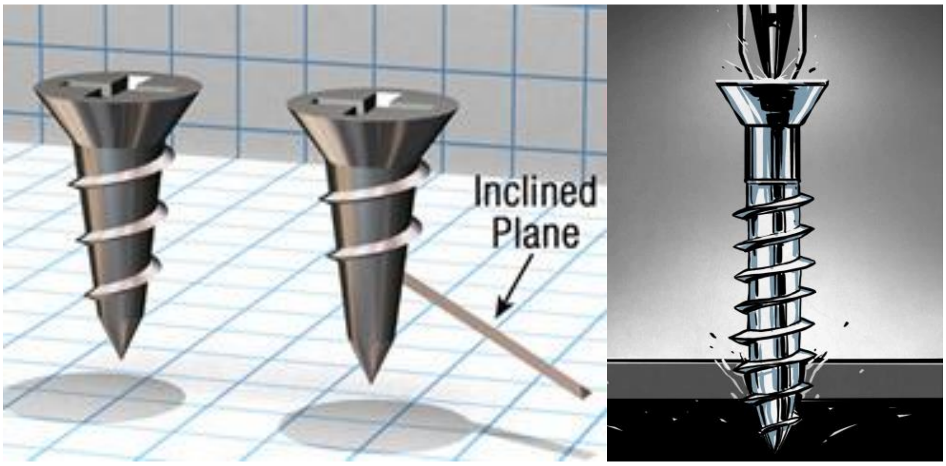
H2 Topic 04 – Forces



A screw *is a long inclined plane wrapped around an axis. A force (that is lower in magnitude) is exerted over a long distance along the "ramp" – it takes less effort to rotate the screw but you will need to make many rotations to secure the screw which will then provide a large retention force.*

Content

- Types of force
- Centre of gravity
- Turning effects of forces
- Equilibrium of forces
- Upthrust

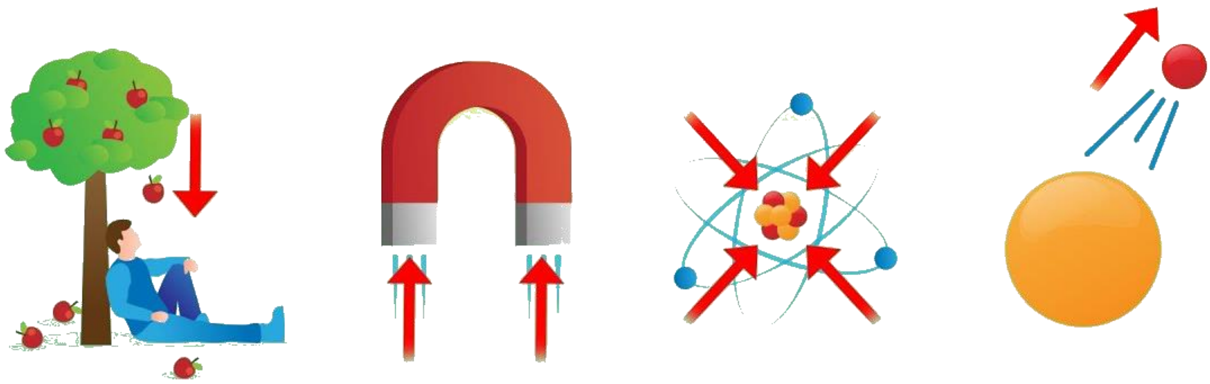
Learning Outcomes

Candidates should be able to:

(a) show recall and apply Hooke's law ($F = kx$, where k is the force constant) to new situations or to solve related problems
(b) describe the forces on mass, charge and current in gravitational, electric and magnetic fields as appropriate
(c) show a qualitative understanding of normal contact forces, frictional forces and viscous forces including air resistance (No treatment of the coefficients of friction and viscosity is required)
(d) show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity
(e) define and apply the moment of a force and the torque of a couple
(f) show an understanding that a couple is a pair of forces which tends to produce rotation only
(g) apply the principle of moments to new situations or to solve related problems
(h) show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium.
(i) use a vector triangle to represent forces in equilibrium
(j) derive, from definitions of pressure and density, the equation $p = \rho gh$
(k) solve problems using the equation $p = \rho gh$
(l) show an understanding of the origin of the force of upthrust acting on a body in a fluid
(m) state that upthrust is equal in magnitude and opposite in direction to the weight of the fluid displaced by a submerged or floating object
(n) calculate the upthrust in terms of the weight of the displaced fluid
(o) recall and apply the principle that, for an object floating in equilibrium, the upthrust is equal in magnitude and opposite in direction to the weight of the object to new situations or to solve related problems

4.1 Introduction

Physics tries to understand all the different types of forces in the Universes using the fewest number of basic laws. We currently categorise all forces in terms of 4 fundamental interactions.



The 4 Fundamental Forces. *(from left to right) Gravity, electromagnetism, strong (nuclear) force and weak (nuclear) force.*

force	effects	range
gravitational force	acts on all masses	infinite
electromagnetic force	acts on electric charges	infinite
strong nuclear force	acts on protons and neutrons holds them together in an atomic nucleus	10^{-15} m
weak nuclear force	acts on elementary particles results in radioactive decay	10^{-17} m

Force is a vector quantity. It has both magnitude and direction.

In recognition of Isaac Newton's work on classical mechanics, specifically Newton's 2nd Law of motion (see topic on Dynamics), the SI unit of force is the newton (N).

1 N is the magnitude of a force that acts on a mass of 1 kg resulting in an acceleration of 1 m s^{-2} in the direction of the force.

4.2 Field of Force

A **field of force** is

a region of space
where a particle experiences a force.

Forces can act on particles (such as masses and electric charges) without the particle *touching* other particle(s). Some resources refer to these as "force at a distance". For A-Levels, we refer to a *field of force.*

If asked to consider the *types* of force more specifically, we should specify the nature of the force when discussing the field of force.

field of force	specific definition
gravitational field (see topic on gravitational field)	a region of space in which a **gravitational** force acts on a **mass**
electric field (see topic on electric field)	a region of space in which an **electric** force acts on a **stationary charge**
magnetic field (see topic on electricity and magnetism)	a region of space in which a **magnetic** force acts on a **current-carrying conductor**, **moving charge**, or a **permanent magnet**,

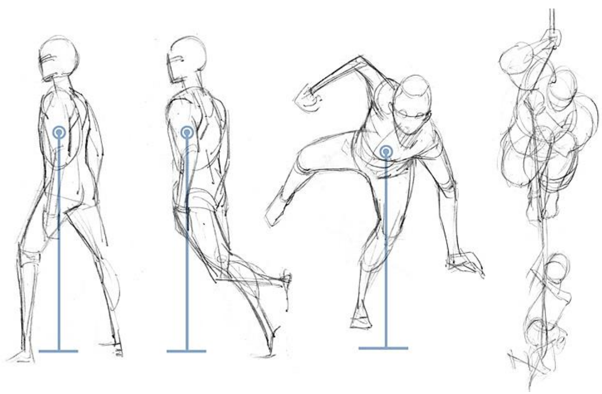
In our everyday lives, we are familiar with Earth's gravitational field. We have weight because of the gravitational force that is acting on *every* part of our body ranging from the head to the toes.

It will be simpler to picture the overall effect of gravity acting at a single point – the centre of gravity.

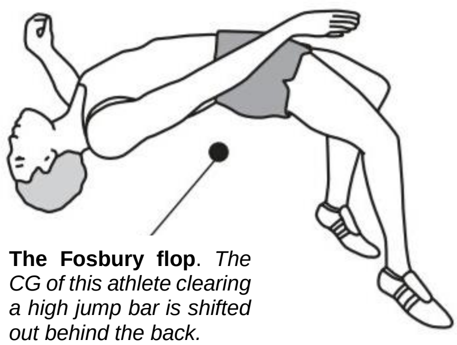
The **centre of gravity** is

the point from where all the weight of a body seems to act

The centre of gravity (CG) of an object changes with the shape. For a person standing upright, the CG is somewhere in the middle of the body, approximately behind the belly button. The CG will shift as the posture changes.



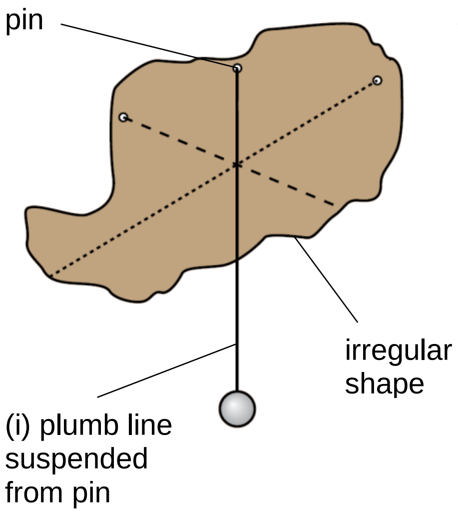
Art Meets Science. *Animators and game designers work with the concept of CG to convey body language or create realistic animations.*



The Fosbury flop. *The CG of this athlete clearing a high jump bar is shifted out behind the back.*

Example 1

Describe how to find the centre of gravity of a thin sheet of irregularly-shaped cardboard.



Solution

(i) Suspend a mass on a string from a pin to form a plumb line.

(ii) Suspend cardboard freely through a hole near the edge.

(iii) Draw a line along the vertical string on the cardboard.

(iv) Repeat steps (ii) and (iii) for one or two more different holes.

(v) Centre of gravity is at point of intersection of lines.

4.3 Translational Equilibrium

A body is in **translational equilibrium** when there is no resultant force.

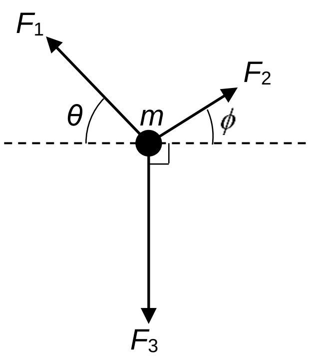
When there is no resultant force on a body, there is no linear acceleration of the body's centre of gravity. 2 possible scenarios are:

- body is stationary
- body moves with constant velocity

Example 2

3 forces F_1, F_2 and F_3 are parallel to the plane of paper and acts on mass *m* shown below.

If *m* is in translational equilibrium, state an equation expressing the magnitude of F_3 in terms of F_1.



(not to scale)

Solution

no resultant force vertically:

$$F_1 \sin\theta + F_2 \sin\phi = F_3$$

no resultant force horizontally:

$$F_1 \cos\theta = F_2 \cos\phi$$

since $F_1 \dfrac{\cos\theta}{\cos\phi} = F_2$

$$F_3 = F_1 \sin\theta + F_1 \left(\frac{\cos\theta}{\cos\phi}\right) \sin\phi$$

$$F_3 = F_1 (\sin\theta + \cos\theta \tan\phi)$$

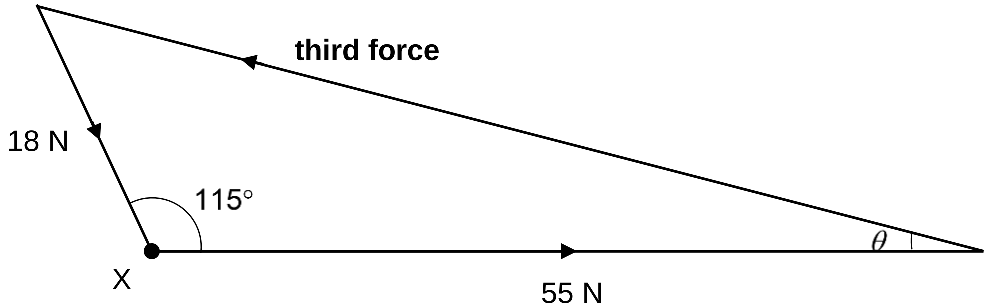
Note: We can *resolve* vector quantities along 2 perpendicular axes and perform vector addition.

Example 3

An object X rests on a smooth horizontal surface. Two horizontal forces act on X as shown below.

(a) By means of a scale diagram, determine a third horizontal force that allows X to be stationary.

(b) Using the *resolution* of forces, verify that your answer to part (a) is correct.



(to scale)

Solution

(a)

scale is 1 N : 0.2 cm

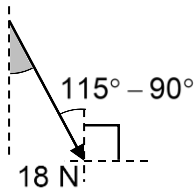
magnitude: 64.7 N

direction: 15° as shown in the diagram

(b) Let the 3rd force be F_3

no resultant force vertically:

$$(18)\cos(115^\circ - 90^\circ) = F_3 \sin\theta \qquad (1)$$



no resultant force horizontally:

$$(18)\sin(115^\circ - 90^\circ) + 55 = F_3 \cos\theta \qquad (2)$$

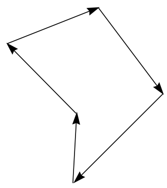
(1) ÷ (2)

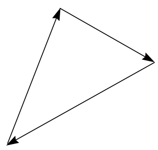
$$\tan\theta = \frac{\cos 25^\circ}{\sin 25^\circ} \quad \Rightarrow \quad \theta = 14.6^\circ \approx 15^\circ$$

Sub θ = 14.6° into (1), F_3 = 64.7 N

Note:

1. For vector quantities, answer includes both ***magnitude*** and ***direction***.
2. The forces acting on an object in translational equilibrium form a closed polygon – the net effect of the vectors is zero as the vector sum of the forces "close upon themselves".

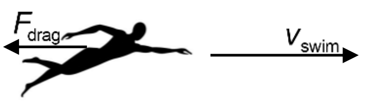




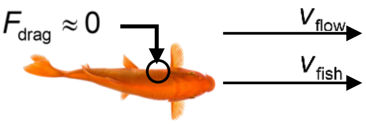
4.4 Commonly-Encountered Forces

4.4.1 Viscous Forces

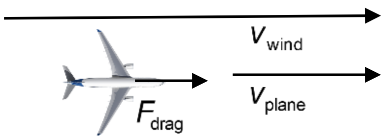
Viscous forces act when there is relative motion between a body and the fluid (either a gas or liquid) surrounding the body. It is known as a *dissipative* force – some energy is converted and lost as heat to the surroundings when a body experiences viscous force. It acts along (parallel to) the surfaces.



In still water, the drag force acting on a swimmer opposes direction of swim.



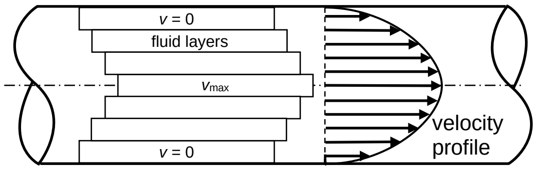
There is virtually no water resistance along the skin-water interface if a fish moves at the same speed as the flow of water. There is <u>*no*</u> *relative motion between fish and water.*



If wind velocity is greater than a plane and is along same direction, it is known as "tail wind" and cuts travel time. The viscous force is in same direction as the plane – forwards!

Note: *Viscous forces only manifest when there is* ***relative motion.***

For a body moving in a fluid, the magnitude of viscous force(s) depends on
- shape of the body (whether the shape is streamlined or not)
- size of the body
- texture of the body's surface
- viscosity of fluid



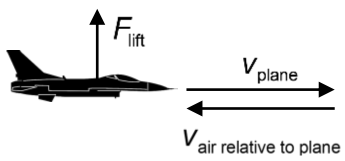
Fluid flow in a pipe. *Under smooth (laminar) flow conditions in a pipe with circular cross-section, the centre fluid layer has the greatest speed. There are infinitely many fluid layers and each layer exert viscous force on adjacent layers, resulting in the velocity profile.*

If all of the above conditions are kept constant, then the magnitude of viscous force on a body increases with the relative speed of the body with respect to the fluid.

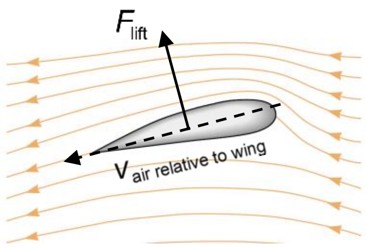
We can also consider different *layers* of fluid flowing past each other and exerting viscous force mutually. In reality when fluid flows through a pipe, the fluid layer immediately in contact with the (stationary) inner pipe walls have zero velocity, and causes drag on the adjacent layer.

4.4.2 Lift

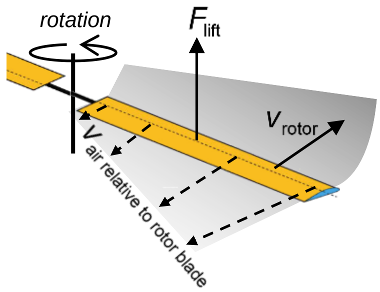
Lift is a force acting on a body (e.g. aeroplane wing or helicopter rotor) when there is relative motion between body and fluid. Unlike viscous forces which act along (parallel to) surfaces, lift acts *perpendicular* to direction of relative flow of surrounding fluid. Hence, for an aeroplane, lift acts perpendicular to the plane's wings.



In still air, the relative flow of air has the same magnitude as the plane's speed but in opposite direction.



When a plane and/or its wings tilt up, the direction of lift changes as well.



The lift generated by a helicopter rotor blade varies – less lift nearer to the rotation axis and more lift "outside".

Lift *acts at right angles to the relative flow of surrounding fluid.*

4.4.3 Normal Contact Force

"Normal" here refers to the direction - normal contact force is exerted *perpendicular* to surfaces that are physically touching.

In each of the following scenarios, a mass experiences a normal contact force N. In a labelled diagram, forces are represented by arrows:
- length of arrow indicate magnitude of force relative to other forces in the same diagram
- direction of arrow indicate the line of action of the force
- starting point depicts where the force is applied to

We then practise drawing the free body diagram (FBD) which often uses the CG to simplify depiction of forces. A FBD shows all external forces acting on the body or system. All external forces acting on the body should be depicted using arrows.

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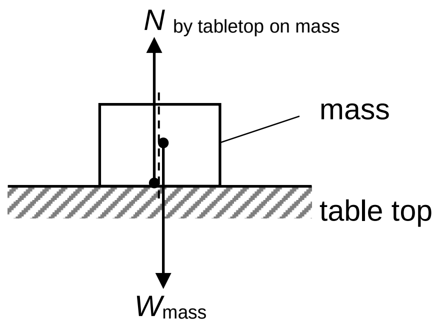
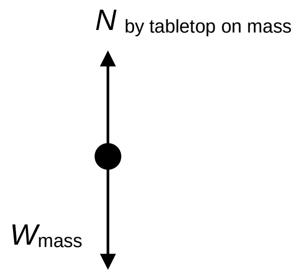
<tr><th>labelled force diagram</th><th>free body diagram of mass</th></tr>
<tr><td>
</td><td>
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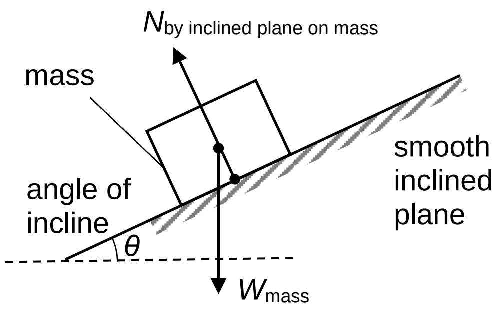
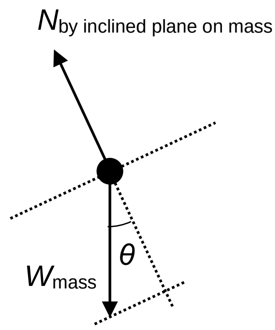
<tr><td>
</td><td>
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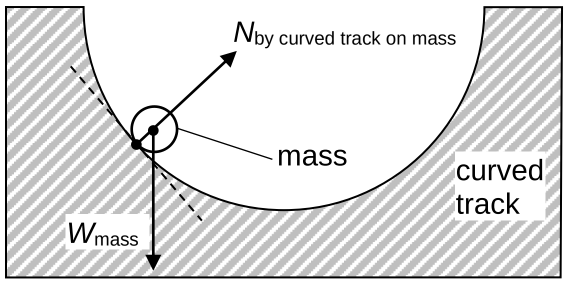
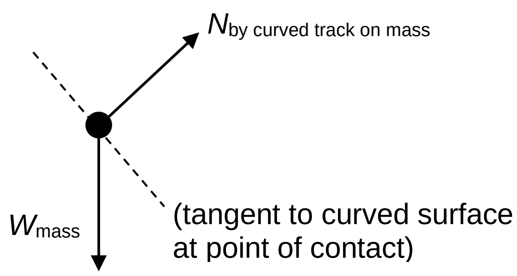
<tr><td>
</td><td>
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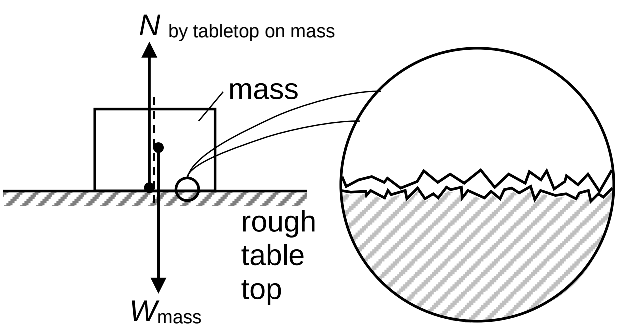
4.4.4 Friction

Friction is a force that opposes relative motion between surfaces in contact. It can also act to oppose *impending* relative motion of surfaces. Friction acts along (i.e. "parallel to") the surfaces.

The magnitude of friction depends on

- magnitude of normal contact force*
- material / type of surfaces in contact



Pictorial representation of non-ideal surfaces in contact. *If the mass moves along the table top, the "peaks" must skip along the surfaces and some "peaks" will be broken off. Therefore, a force is required to set the mass into motion and to keeping it moving. Even between "perfectly flat" real surfaces, there are attractive forces between molecules of the 2 surfaces. Such adhesion depends on the type of material.*

Example 4

(a) A mass m rests stationary on a rough floor. What is the magnitude of friction acting on it?

(b) A force of 10 N acts on the mass of 3.0 kg to the right but it remains at rest.

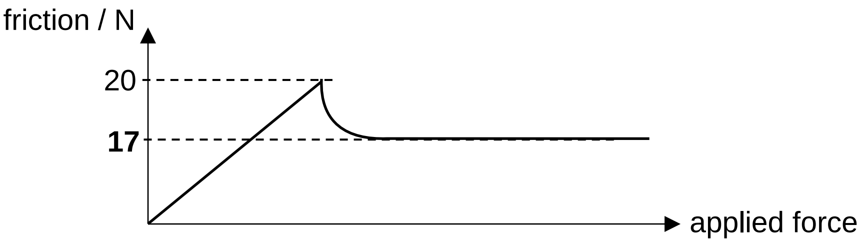
(i) State and explain the magnitude and direction of friction acting on the mass.

(ii) State and explain the magnitude and direction of *contact* force (or *reaction* force) acting on the mass.

(c) The magnitude of friction acting on the mass varies with the magnitude of applied external force. The variation of friction acting on the mass with applied external force is shown. Using the diagram, state the magnitude of friction

(i) when the mass just starts to slide,

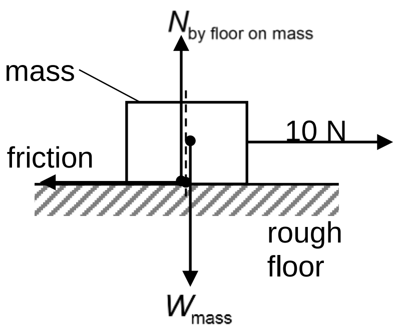
(ii) when the mass is moving at constant speed.

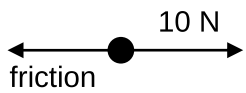


Solution

(a) There is no friction. The mass is at rest so no resultant force horizontally.

(b)



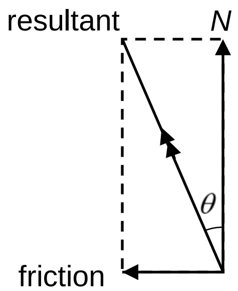


(i) Friction is 10 N to the left. The mass is at rest so no resultant force horizontally.

(ii) Normal contact force $N = W_{mass} = mg$

$$\text{Magnitude of contact force} = \sqrt{N^2 + \mathit{friction}^2} = \sqrt{\left(3(9.81)\right)^2 + 10^2} = 31.1\ \text{N}$$

$$\tan\theta = \frac{10}{3g} \Rightarrow \qquad \theta = \tan^{-1}\left(\frac{10}{3(9.81)}\right) = 18.8^\circ$$



Contact force is **[magnitude]** 31.1 N

[direction] acting 18.8° left of normal to surface

(c) For objects at rest, the magnitude of friction will match externally-applied force(s) parallel to surfaces. The friction when mass starts to slide is (i) 20 N and (ii) 17 N when moving.

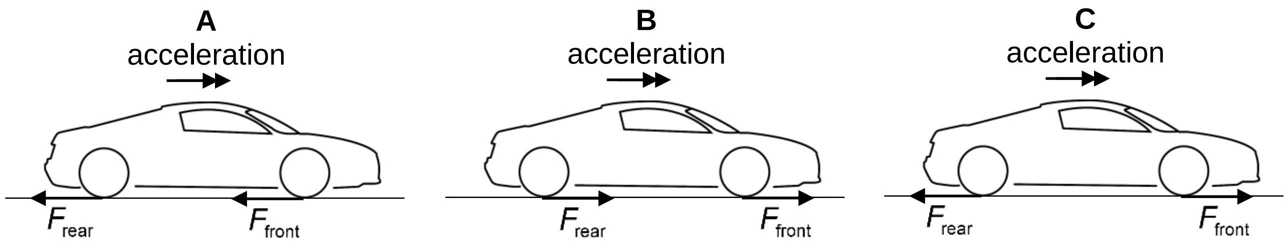
Notes: (b) Reminder to answer both *magnitude* and *direction* for vector quantities.

(b)(ii) Vector sum of all forces at contact surfaces is the resultant **contact / reaction force**.

Example 5

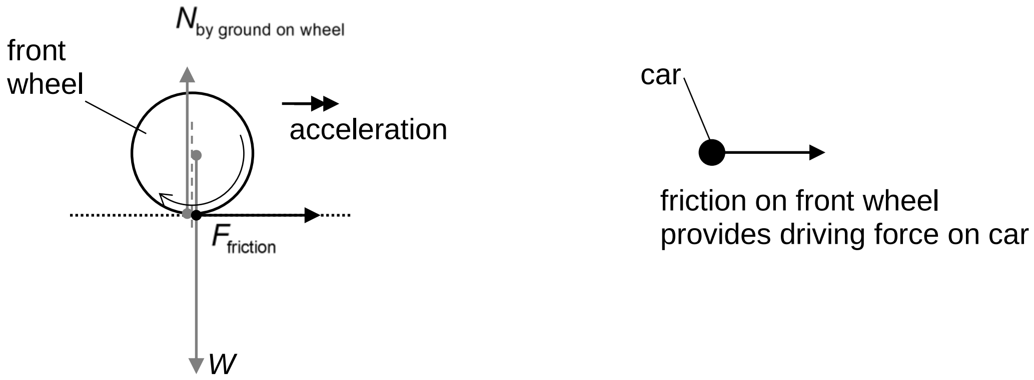
A car is accelerating along a level road. Only the front wheels of the car are engine-driven.

Which diagram shows the direction (only) of friction F_{front} on the front wheels and the direction of friction F_{rear} on the rear wheels?

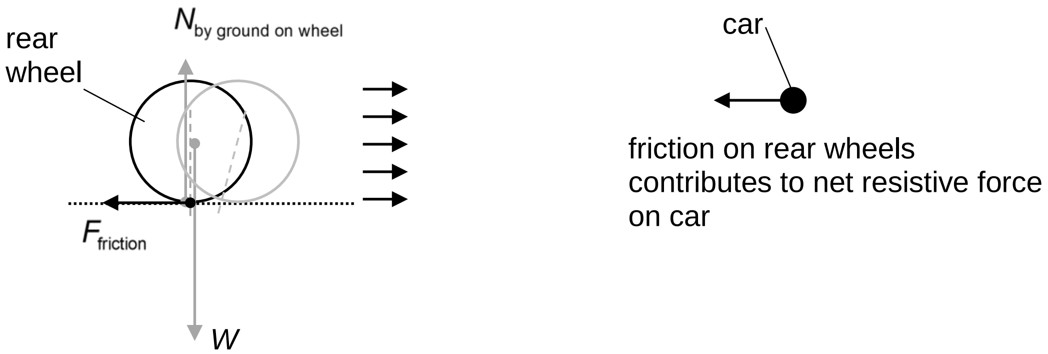


Solution: C

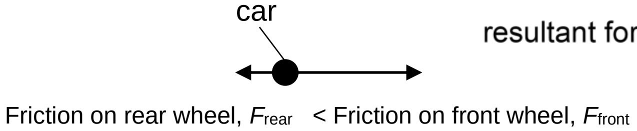
Part of the front wheel in contact with the ground moves left. Friction opposes relative motion:



Rear wheels are being "moved" to the right – the surface of the wheel in contact with the ground is similarly being moved to the right. Friction opposes relative motion so acts to the left.



Considering the car *as one system*, the friction on front wheels must have bigger magnitude compared to those acting on the rear wheels to have a net forward force



resultant force, $F_{drive} = m_{car}a$

$= F_{front} - F_{rear}$

Notes: Car has no vertical resultant force so

$$W_{car} = N_{by\ ground\ on\ front\ wheels} + N_{by\ ground\ on\ rear\ wheels}$$

in terms of magnitude (the vertical arrows are not to scale but drawn to different lengths)

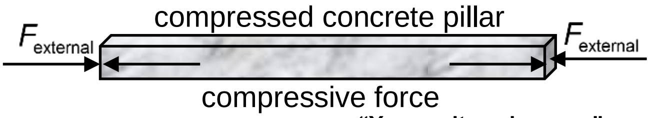
*(Example 5 is a 2020 GCE O level Physics Question)

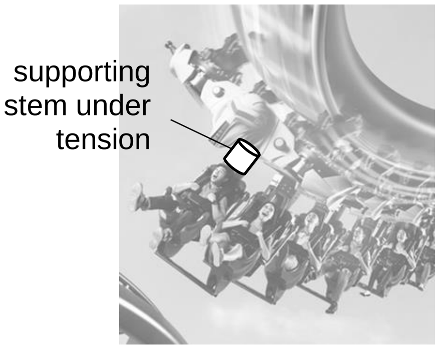
4.4.5 Tension and Compression

Tension is a force along the length of a body. We often work with tension that acts *parallel* to a flexible medium such as a rope, cable or a string.

When a body is stretched along its axis by equal and opposite externally-applied forces, we can consider tension to be the internal forces that try to return the body to its original length.

Compression is the opposite of tension. We can consider compression to be the internal forces that try to return the body to its original length when the body is being pushed inwards along the axis.

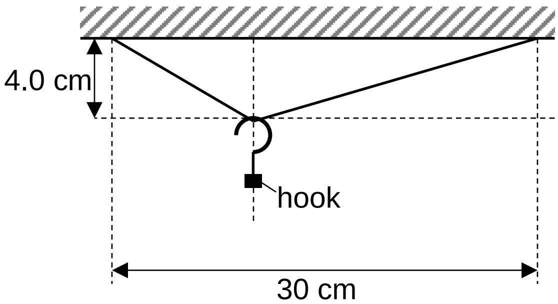




"You can't push a rope"
Flexible medium such as ropes and strings cannot have compressive forces. They can have tension which pulls inwards against the direction of externally applied force(s).

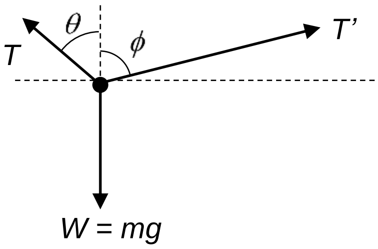
However, rigid structures can have both compression and tension forces (not at the same time). The supporting stem highlighted here is under tension.

Example 6
A hook of mass 50 g is hung on a light string as shown. There is no friction between the hook and the string. Find the magnitude of tension in the string.



(not to scale)

Solution



no resultant force along horizontal so

$$T \sin\theta = T' \sin\phi$$

Since the string is light and there is no friction, the tension along the string is the same:

$$T = T' \rightarrow \theta = \phi$$

Thus, hook must be hung from middle of string:

$$\tan\theta = \frac{15}{4}$$

no resultant force along vertical so

$T \cos\theta + T \cos\theta = mg$

$$T = \frac{mg}{2\cos\theta} = \frac{(50\times10^{-3})(9.81)}{2\cos\left(\tan^{-1}\left(\frac{15}{4}\right)\right)} = 0.95 \text{ N}$$

Notes: Tension has the same magnitude throughout a *light* ("massless") string and acts along the length of the string.

4.4.5.1 Hooke's Law

Springs can be stretched or compressed to provide a force that is governed by Hooke's Law.

Hooke's Law states that

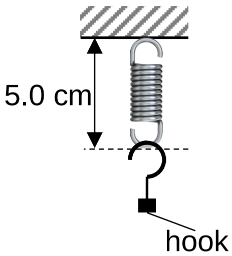
the change in length of a material is directly proportional to the force applied on it, provided that the limit of proportionality is not exceeded.

$$F = kx$$

F : is the force applied (N)
k : is the proportionality constant (N m^{-1})
x : is the change in length (m)

Example 7

A hook of mass 80 g is being hung onto a light spring of natural length 5.0 cm as shown. The spring has an elastic spring constant of 20 N m^{-1}. Find the final length of the spring at equilibrium.



(not to scale)

Solution

By Hooke's Law,

$$F = kx$$

$$x = \frac{F}{k} = \frac{mg}{k}$$

$$= \frac{(80 \times 10^{-3})(9.81)}{20} = 0.0392 \text{ m}$$

$$\text{final length} = L_0 + x = 5.0 + 3.92 = 8.92 \text{ cm}$$

4.4.5.2 Elastic Deformation

Depending on its original state, a spring is capable of providing tension, compressive or both forces, *elastically*.

In Physics, *elastic* means the ability to return to the original value *before* and *after* a change has taken place.

This is a "closed coiled helical spring". It cannot be compressed any further to result in a change in length.

This is an "open coiled helical spring". It can be compressed to result in a change in length with the effect of providing an "outward" force.

When not specified, *assume that a spring (mentioned as part of an exam question) to be capable of either being stretched or compressed to provide an elastic force.*

For springs, *elastic* refers to the ability of a spring to return to its natural length after the externally-applied force stops acting. The *elastic* limit is the point beyond which the spring does not return to its original length when the force is removed.

In Dynamics, an *elastic* collision refers to one where the sum of kinetic energies of the bodies colliding remains the same *before and after the* collision.

Example 8

The variation in length x with force F for a spring is shown below.

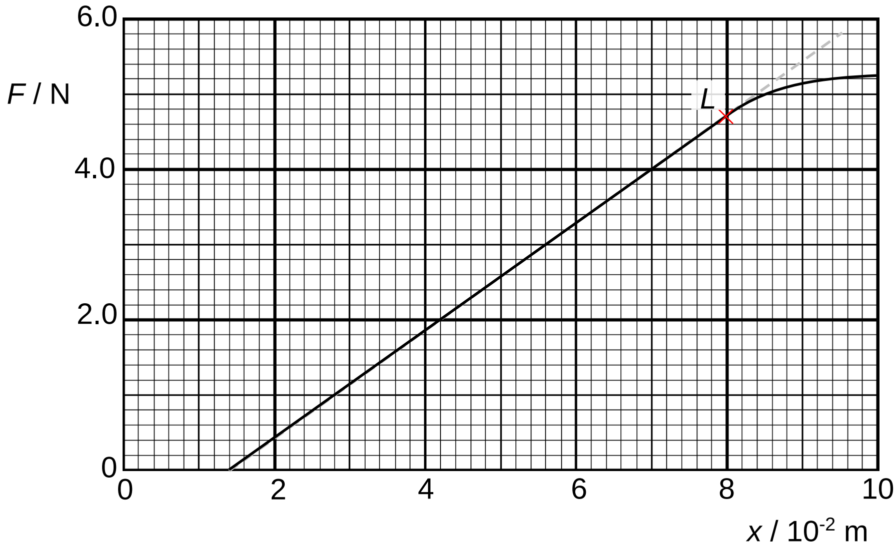
(a) Identify the *proportionality* limit of the spring by marking the point as L.

(b) Describe the meaning of *proportionality* limit.

(c) Another identical spring has not been stretched beyond the limit of proportionality. Determine

  (i) the natural length of the spring, and

  (ii) the spring constant k for the spring.



Solution

(b) The point beyond which
the force applied is no longer proportional to the extension of the spring

(c) (i) Natural length is when no force is exerted on spring, $F = 0$.
$x_0 = 1.4$ cm = 1.4×10^{-2} m.

(ii) By Hooke's Law,

$$F = k\left(x - x_0\right)$$
$$4.7 = k\left(8 - 1.4\right) \times 10^{-2}$$
$$k = 71.2 \text{ N m}^{-1}$$

Notes: Graphs should be read to half-smallest square accuracy. For (c)(ii), do **take note of the unit conversion**. Such mistakes are very common especially when working under time constraints.

4.4.6 Fluids

4.4.6.1 Pressure

Pressure is defined as the force per unit area (normal to force).	$p = \dfrac{F_{\perp}}{A}$	Pressure is a scalar quantity. Some commonly seen units include N m^{-2}, Pa, atm, bar, torr, mm Hg.

Example 9

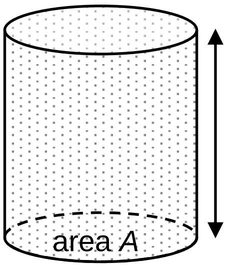
Derive, from the definitions of pressure and density, the equation

$$p = \rho g h$$

where p is the pressure exerted by the liquid on the base of the container and g is the acceleration of free fall.

Solution

Consider a cylindrical column of liquid with circular cross-sectional area A:



$$\text{density} = \frac{\text{mass of fluid}}{\text{volume}}$$

$$\rho = \frac{m}{Ah}$$

$$\frac{m}{A} = \rho h$$

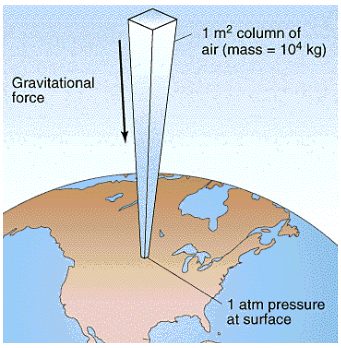
pressure is force acting normally per unit area of a surface

$$p = \frac{F}{A}$$

$$= \frac{W}{A} = \left(\frac{m}{A}\right) g$$
$$= \rho h g$$

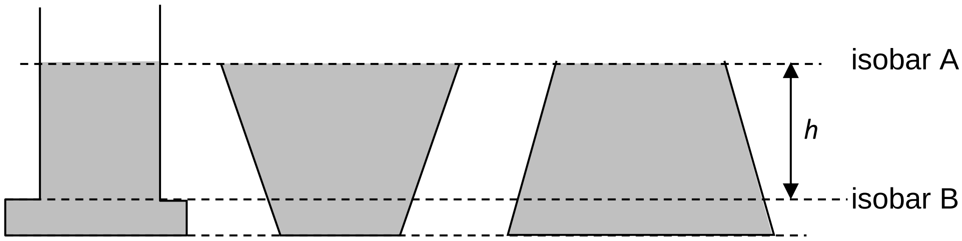
Notes: $\rho g h$ is the hydrostatic pressure of that particular fluid only. If this is a cylinder of water near Earth's surface, the total pressure acting on area A includes the column of water and column of atmosphere above it

$$p_{\text{total}} \simeq \rho_{\text{water}} h_{\text{water}} g + (1\text{ atm})$$



4.4.6.2 Upthrust

Fluid pressure does not depend on shape (of container), total fluid mass or surface area of fluid:

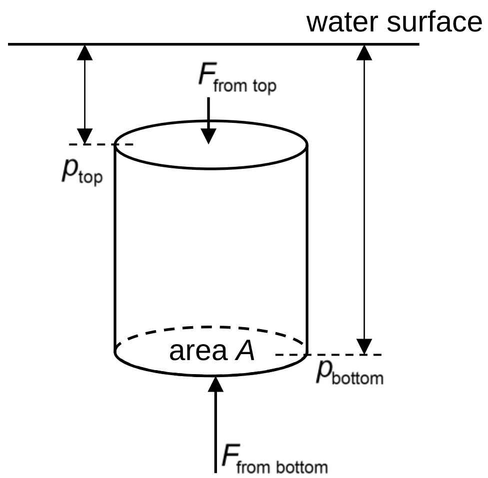


Isobars. *Each of the dotted lines is an isobar for the fluid. The fluid pressure along each dotted line is the same and is dependent on height h. ("Isobar" : Line of equal pressure)*

Because pressure increases with depth, the pressure at isobar B is greater than the pressure at isobar A. If an object is submerged into the fluid to the extent the volume spans both isobars, the object encounters a difference in the pressure exerted normally across its cross-sectional areas.

Example 10

A cylinder is totally submerged into water. An upthrust acts on it. **Explain the origin of the upthrust.**



Solution

[recall] Pressure exerted by water increases with depth

[apply] Pressure on bottom surface area of cylinder greater than pressure on top surface area of cylinder

[magnitude] Upwards force on bottom surface larger than downwards force on top surface

[direction] Net upwards force is upthrust

Note: Pressure acts equally in **all** directions in a fluid.

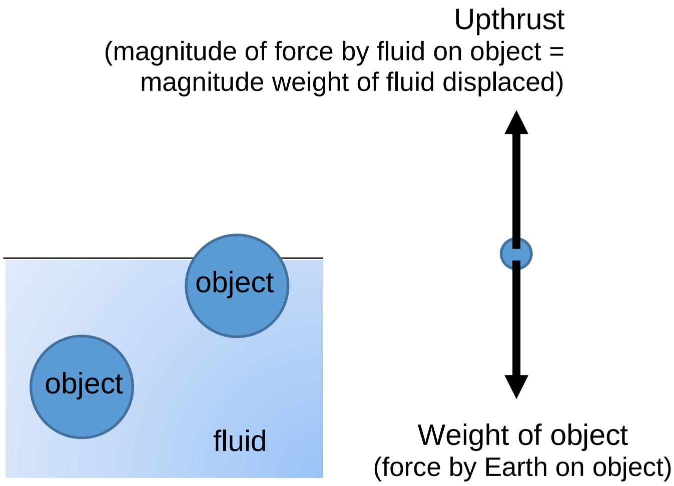
Mathematically,

$$\begin{aligned}
U_{\text{on object by fluid}} &= F_{bottom} - F_{top} \\
&= \left(P_{bottom} - P_{top}\right)A \\
&= \left(h_{bottom}\rho_{fluid}g - h_{top}\rho_{fluid}g\right)A \\
&= \left(h_{bottom} - h_{top}\right)\rho_{fluid}gA \\
&= V_{displaced\ fluid}\,\rho_{fluid}\,g
\end{aligned}$$

Hence $\boxed{U_{\text{on object by fluid}} = \text{ weight of fluid displaced}}$

A free-body diagram helps us identify the forces acting on an object and form the force equation.

In the simplest case of a **floating or submerged object** that is in equilibrium, upthrust is equal in magnitude and opposite in direction to the weight of fluid displaced by that object.



For a floating/submerged object,

upthrust = weight of object

$$U_{\text{on object by fluid}} = W_{\text{object}}$$

$$V_{\text{displaced fluid}} \rho_{\text{fluid}} g = V_{\text{object}} \rho_{\text{object}} g$$

Note: When the object is floating such that some part of the object is above the fluid, volume of displaced fluid $V_{\text{displaced fluid}}$ is smaller than the volume of object V_{object}.

When the object **sinks,** upthrust is smaller in magnitude than the weight of the object. Using the relevant formula for upthrust and weight of object (as well as the fact that the volume of displaced fluid would be the same as the volume of the object when the object is fully submerged in the fluid), we arrive at a conclusion that you learnt in primary school – an object with higher density than the fluid sinks!

upthrust < weight of object

$$U_{\text{on object by fluid}} < W_{\text{object}}$$

$$V_{\text{displaced fluid}} \rho_{\text{fluid}} g < V_{\text{object}} \rho_{\text{object}} g$$

$$\rho_{\text{fluid}} < \rho_{\text{object}}$$

Example 11

The average human body occupies a volume of 7.4×10^{-2} m^3. Find the magnitude of upthrust acting on the body immersed in

(a) (i) air of density 1.3 kg m^{-3}, and

(ii) sea water of density 1030 kg m^{-3}.

(b) Hence, explain why upthrust due to air is usually ignored when body mass is taken at health check-ups.

Solution

(a) Assuming full submersion,

$$\begin{aligned} U_{\text{due to air}} &= \rho_{\text{air}} V_{\text{body}} g \\ &= (1.3)\left(7.4\times10^{-2}\right)(9.81) \\ &= 0.944 \text{ N} \end{aligned}$$

$$\begin{aligned} U_{\text{due to sea water}} &= \rho_{\text{sea water}} V_{\text{body}} g \\ &= (1030)\left(7.4\times10^{-2}\right)(9.81) \\ &= 748 \text{ N} \end{aligned}$$

(b) Majority of human body is water. Using sea water density as estimate:

$$\begin{aligned} \rho_{\text{body}} &\approx \frac{m_{\text{body}}}{V_{\text{body}}} \\ m_{\text{body}} &\approx \rho_{\text{sea water}} V_{\text{body}} \end{aligned}$$

$$\begin{aligned} \frac{U_{\text{due to air}}}{W_{\text{body}}}\times100\% &= \frac{\rho_{\text{air}} V_{\text{body}} g}{\rho_{\text{sea water}} V_{\text{body}} g}\times100\% \\ &\approx \frac{\rho_{\text{air}}}{\rho_{\text{sea water}}}\times100\% = \frac{1.3}{1030}\times100\% \\ &= 0.13\% \end{aligned}$$

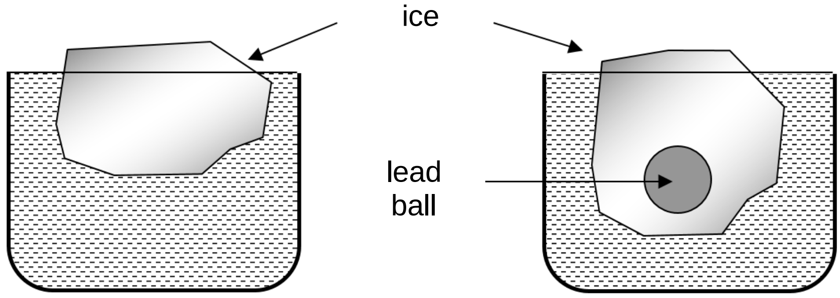
The upthrust due to air is 3 orders of magnitude less than the weight and is insignificant.

Notes:

- When "hence" is used in the question, answers to previous parts need to be referenced.
- When comparing figures, it is good practice to think in terms of ratios such as percentages.

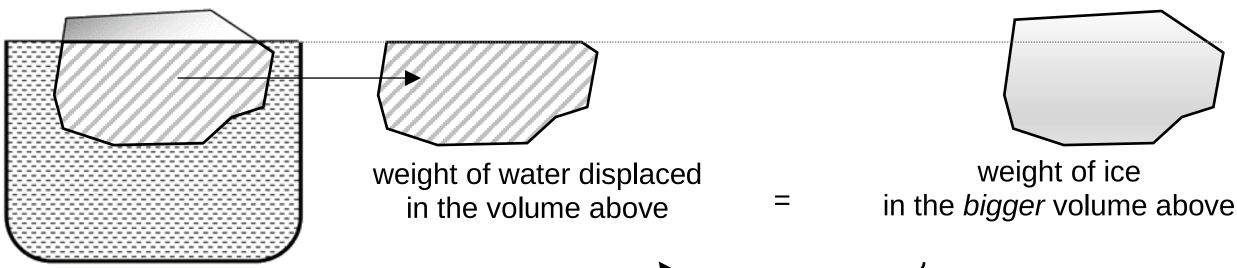
Example 12

Two beakers of water are shown below. Both beakers are filled to the brim with cold water. A solid ice block floats in beaker A while an ice block encasing a lead ball floats in beaker B.
State and explain the change (if any) to the water level in both beakers when all of the ice melts.



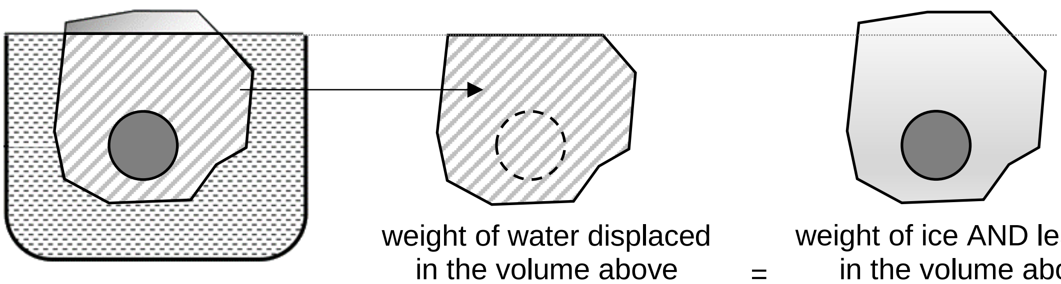
beaker A beaker B

Solution



weight of water displaced in the volume above = weight of ice in the *bigger* volume above

no change in water level; the ice melts into the volume of displaced water



weight of water displaced in the volume above = weight of ice AND lead ball in the volume above

From results of beaker A, pure ice will not cause change to water level when it melts completely.
For lead ball to float, the upthrust on the ball has same magnitude as its weight

$$U_{\text{on ball}} = \rho_{\text{water}} V_{\text{water displaced for ball}} g$$
$$W_{\text{ball}} = \rho_{\text{water}} V_{\text{water displaced for ball}} g$$
$$m_{\text{ball}} = \rho_{\text{water}} V_{\text{water displaced for ball}}$$
$$\rho_{\text{ball}} V_{\text{ball}} = \rho_{\text{water}} V_{\text{water displaced for ball}}$$

$$\frac{\rho_{\text{ball}}}{\rho_{\text{water}}} = \frac{V_{\text{water displaced for ball}}}{V_{\text{ball}}}$$

Since lead is denser than water,

a volume of water that is greater than the ball's volume is displaced to float the ball.

When the ball no longer floats (because ice has melted)

there is less water being displaced so water level falls

A ship made of steel $\left(\rho_{\text{steel}} \gg \rho_{\text{water}}\right)$ can float because its internal hollow volume displaces a large mass of water. Sufficient upthrust is generated to keep the ship afloat.

Example 13

A boat floats on fresh water and displaces 35.6 kN of water.

Find the volume of sea water displaced if the boat floats on sea water of density 1030 kg m^{-3}.

Solution

Boat floats so upthrust on boat is
equal in magnitude and
opposite in direction to the weight of boat.

$$W_{\text{sea water displaced}} = W_{\text{boat}} = W_{\text{fresh water displaced}}$$

$$W_{\text{sea water displaced}} = 35.6 \text{ kN}$$

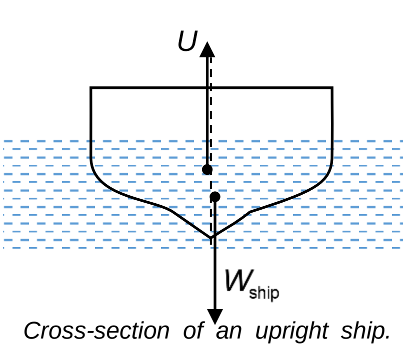
Let density of sea water = ρ_s and
volume of sea water displaced $= V_s$

$$U = \rho_s V_s g = W_{\text{boat}}$$

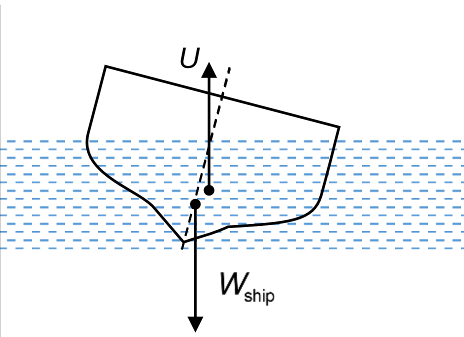
$$V_s = \frac{W_{\text{boat}}}{\rho_s g} = \frac{35.6 \times 10^3}{(1030)(9.81)} = 3.52 \text{ m}^3$$

Notes: Sea water is denser, so *less* of the boat needs to be submerged to displace sufficient fluid.

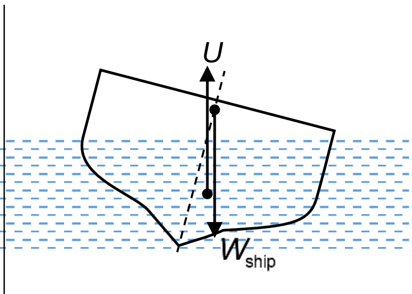
Similar to the concept of CG, all of upthrust can be regarded as acting upwards through the centre of mass of the displaced fluid. This is referred to as the centre of buoyancy.



Cross-section of an upright ship. Magnitude of upthrust equals magnitude of weight. More massive cargo are loaded near bottom.



*If the ship tilts, the upthrust and weight forms a **restoring couple** which returns the ship back upright.*



If the ship is top-heavy, the resulting couple from a tilt can act to rotate the ship in the direction of the tilt, perpetuating an unstable equilibrium.

A labelled force diagram sometimes helps us visualize better. *In the above, knowing the relative positions from which forces act helps predict a cargo ship's stability. It takes deliberate practice to better gauge when to sketch labelled force diagrams versus FBDs.*

4.5 Labelled Force Diagram

During problem-solving, it is often useful to draw the labelled force diagrams or free body diagrams of the appropriate body or system.

A *free-body diagram* of a body (or system) shows all external forces acting on the body (or system) acting through the CG.

A labelled force diagram can additionally show the point(s) of exertion and line(s) of action on simple representation(s) of the body (or system). When considered together with approximate representations of the magnitudes and direction of forces, a labelled force diagram helps much in determining equilibrium and stability of a body or system.

In drawing a labelled force diagram, one should

1. identify the body (or system) that is being considered and draw a simple sketch (e.g. boxes, blocks etc) representing the body (or system).
2. mark and label on the sketch from (1), all external forces acting on the body (or system), paying attention to the point of exertion and line of action of each force.
3. determine the magnitude of the forces (if necessary) that can be directly calculated via a formula.
4. Identify 2 suitable perpendicular axes for analysis and resolve all forces along these 2 perpendicular axes.

A good understanding of Newton's 3rd Law of Motion will help us decide whether a force is to be included in a labeled force diagram or not.

4.5.1 Newton's Third Law

> **Newton's Third Law of Motion** states that
>
> when body A exerts a force on body B,
> body B exerts on body A
>
> a force of the same type
> that is equal in magnitude and
> opposite in direction.

One implication of this law is that the free-body diagram of body A (or that of body B) should not have both "action-reaction" forces drawn on it.

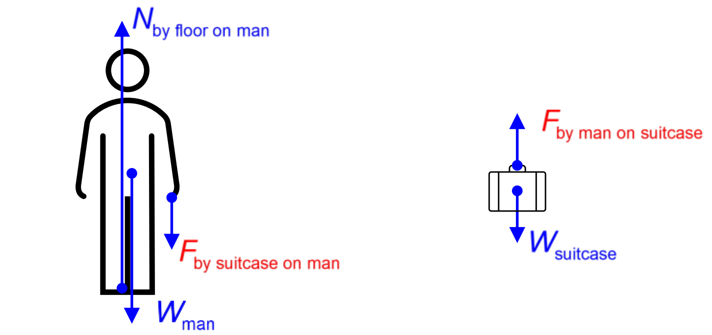
Note that:

1. Newton's Third Law of Motion involves two different bodies; the 'action and reaction' pair of forces as stated within the law acts on **separate bodies** (if the 'action' acts on body A, then the 'reaction' must act on body B).
2. The pair of forces is equal in magnitude and opposite in direction; they must also be of the same type/nature. In other words, if the force that A exerts on B is a gravitational force, then the equal and opposite force exerted by B on A is also a gravitational force.

Study the forces in the following scenarios for each of the underlined bodies. Identify which are action-reaction pairs (“Newton’s Third Law pairs”):

Scenario 1: A man standing stationary on the floor holding a suitcase.

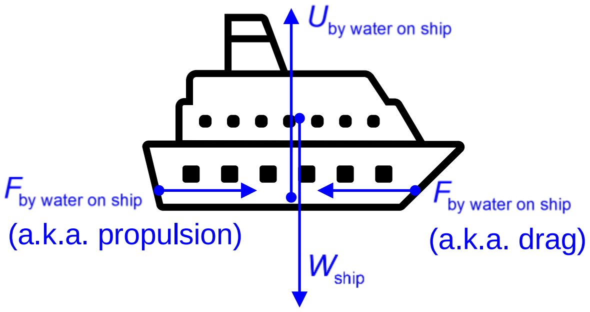
labelled force diagrams:



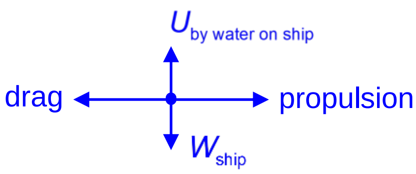
free body diagrams:

Scenario 2: A ship moving through the sea at constant velocity to the right.

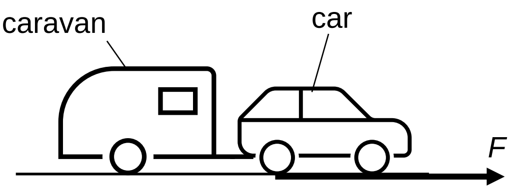
labelled force diagram:



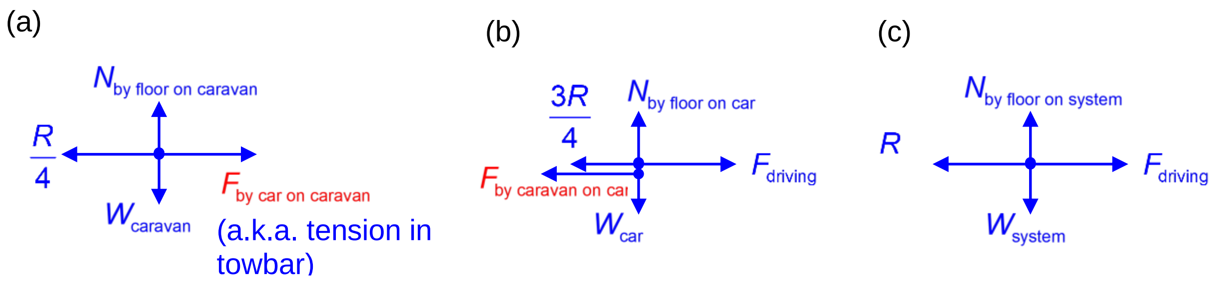
free body diagram:



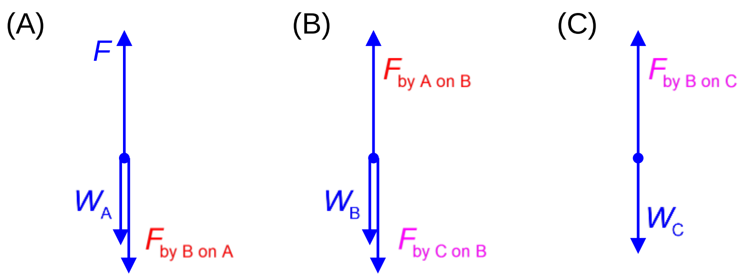
Scenario 3: A car tows a caravan. The engine of the car provides a driving force *F*. The total resistance to motion has a constant value of *R*. A quarter of this resistance acts on the caravan.

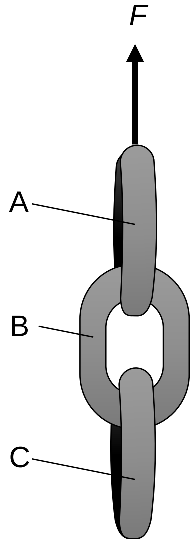


The free body diagrams of the **(a)** caravan, **(b)** car and **(c)** two vehicles as a single body as shown.

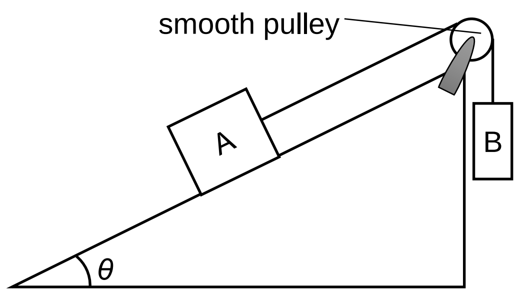


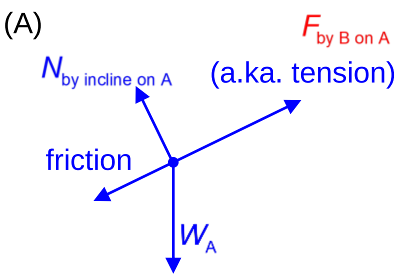
Scenario 4: A chain has 3 identical links A, B and C. The chain is accelerated upwards by force *F*. The free body diagrams for each of the chain links A, B and C are shown

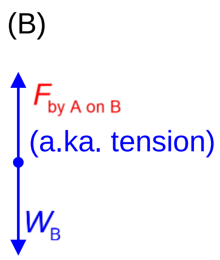




Scenario 5: Mass A slides up a rough incline with constant speed. The free body diagrams of mass A and mass B are shown.







4.6 Rotation

4.6.1 Rotational Equilibrium

A body is in **rotational equilibrium** when there is

[magnitude] no resultant torque
[direction] about any point.

Principle of moments states that for a body in rotational equilibrium,

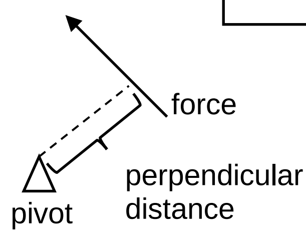
sum of clockwise moments
about <u>any</u> point
is equal to
sum of anti-clockwise moments
about <u>the same</u> point

4.6.2 Moment of a Force

Moment of a force <u>about a point</u> is the

product of the force and its perpendicular distance to the pivot.

moment $= F \times d$



4.6.3 Torque of a Couple

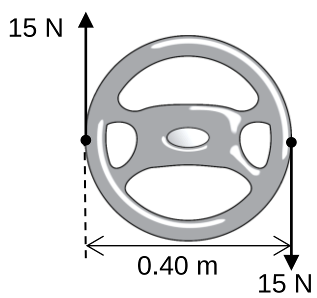
A couple is *a pair of forces* acting on the same body (or system) that
(i) are equal in magnitude;
(ii) are opposite in direction; and
(iii) the lines of action do <u>not</u> coincide.

Torque of a couple is the

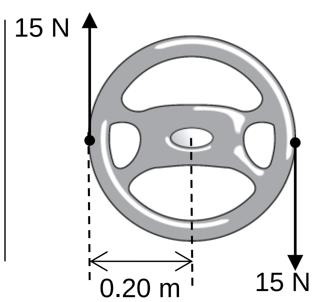
product of one of the forces and the perpendicular distance between the forces

Because the forces above give no resultant force, couples tend to produce only rotation:

torque = $F \times d_{\perp}$



torque of couple
$= F \times d_{\perp}$
$= (15)(0.40)$
$= 6.0$ N m



sum of
clockwise moments
$= F \times d + F \times d$
$= 2(F \times d)$
$= 2(15)(0.20)$
$= 6.0$ N m

A couple is a special case of moments. *The above diagram shows the forces needed to turn a car's steering wheel. The two forces give no resultant force so the wheel will not accelerate up, down or sideways. However, the wheel is not in equilibrium as the pair of forces will cause it to rotate. In finding the torque, you can treat the forces as a couple (left), or as 2 clockwise moments about the centre of the steering wheel.(right).*

4.7 Equilibrium

A body is in **equilibrium** when

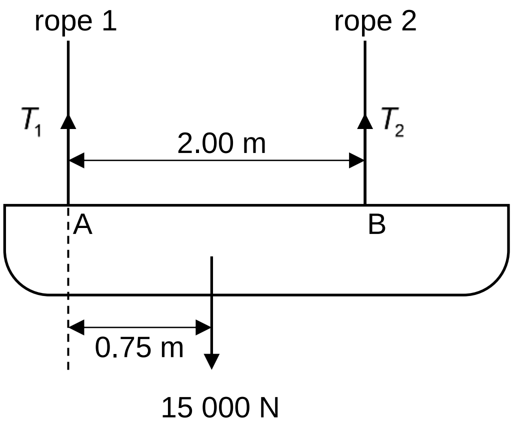
there is no resultant force and
there is no resultant torque about any point on it.

Sometimes this state of equilibrium is referred to as "dynamic equilibrium" or "total mechanical equilibrium".

Basically it is when a body has reached both translational and rotational equilibrium.

Example 14

A boat of weight 15 000 N is being lifted by two ropes attached at points A and B. It remains horizontal and travels up vertically at constant speed. Find the tensions T_1 and T_2 in rope 1 and 2 respectively.



(not to scale)

Solution

Boat is in rotational equilibrium.
By Principle of Moments, taking moments about A,
sum of clockwise moments
= sum of anticlockwise moments

$$(15\ 000)(0.75) = (T_2)(2.00)$$
$$T_2 = \frac{(15\ 000)(0.75)}{2.00}$$
$$= 5625\text{ N}$$

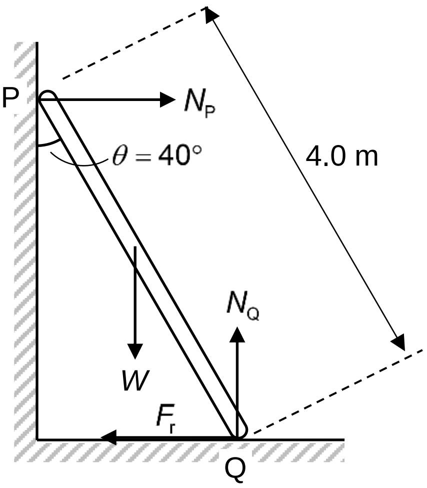
Boat is in translational equilibrium.
No resultant force vertically.

$$T_1 + T_2 = 15\ 000\text{ N}$$
$$T_1 = 15\ 000 - T_2 = 15\ 000 - 5625$$
$$= 9375\text{ N}$$

Notes: Note the presentation expected for questions involving rotational equilibrium: the principle of moments, the pivot, and the relationship relating to sum of moments.
Be reminded that a body under translational equilibrium does not have linear *acceleration* so it can either (i) have constant velocity or (ii) be stationary.

Example 15

A 4.0 m long ladder of uniform density and mass 6.0 kg leans $40°$ against a smooth vertical wall at P. Its lower end contacts the floor at Q. Find the force exerted by ladder on floor at Q, $F_{\text{on floor}}$.



(not to scale)

Solution

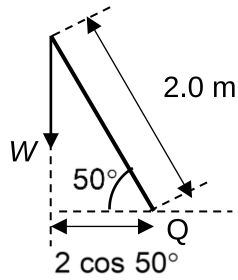
Vertical wall is smooth so only normal contact force N_P acts on ladder to the right at P.

Ladder is in translational equilibrium. No resultant force horizontally: there must be friction F_r acting on ladder to the left at Q:

$$|N_P| = |F_r|$$

No resultant force vertically:

$$|N_Q| = |W|$$



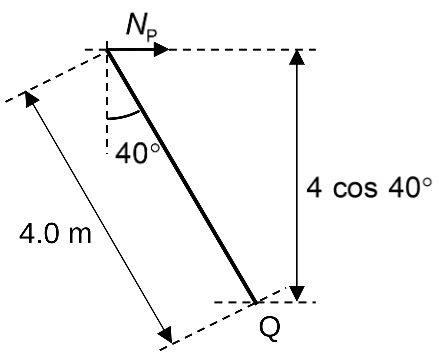
Ladder is in rotational equilibrium.
By Principle of Moments, taking moments about Q,
sum of clockwise moments
= sum of anticlockwise moments

$$(W)(2\cos 50°) = (N_P)(4\cos 40°)$$

$$N_P = \frac{(W)(2\cos 50°)}{(4\cos 40°)} = \frac{(mg)(2\cos 50°)}{(4\cos 40°)}$$

$$= \frac{(6.0)(9.81)(2\cos 50°)}{(4\cos 40°)}$$

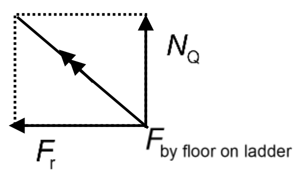
$$= 24.69 \text{ N}$$



[magnitude] $|F_{\text{on floor}}| = \sqrt{|W|^2 + |F_r|^2} = \sqrt{|mg|^2 + |N_P|^2}$

$$= \sqrt{((6.0)(9.81))^2 + (24.69)^2}$$

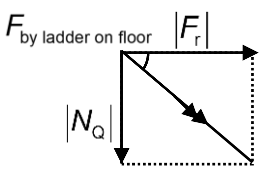
$$= 63.8 \text{ N } (3 \text{ s.f.})$$



[direction] angle below horizon $= \tan^{-1}\frac{|N_Q|}{|F_r|} = \tan^{-1}\left|\frac{mg}{N_P}\right|$

$$= \tan^{-1}\left|\frac{(6.0)(9.81)}{24.69}\right|$$

$$= 67.2°$$



Notes: The force by ladder <u>on floor</u> is the reaction force by floor <u>on ladder</u>.

Example 16

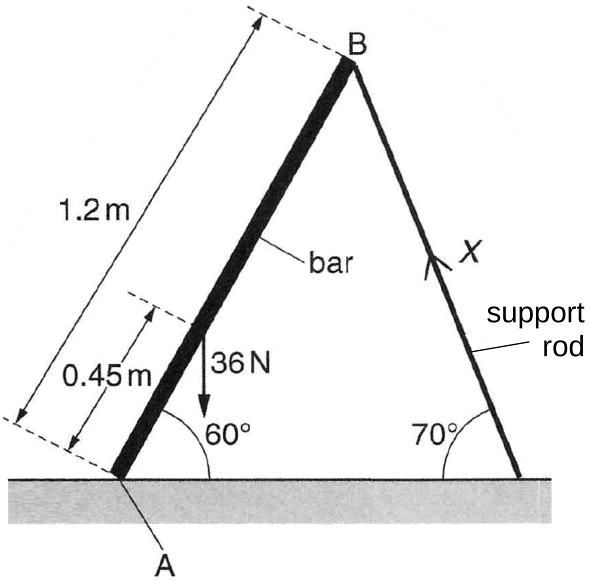
A non-uniform bar AB of weight 36 N makes an angle of 60° with a horizontal surface.

(a) Show that the magnitude of force X which supports the bar is 8.8 N.

(b) (i) Explain why a force *F* must act at A to keep the bar in equilibrium

(ii) Find the magnitude of *F*.

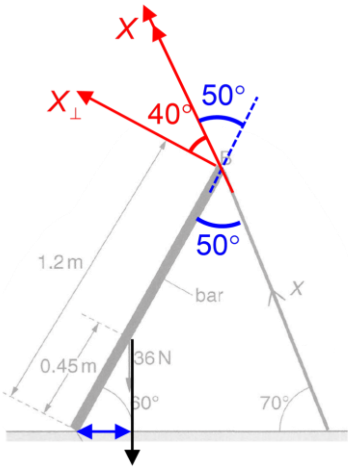
(iii) Draw an arrow to show the approximate direction of *F*.



(not to scale)

Solution

(a)



By Principle of moments, take moments about A,
sum of clockwise moments =
sum of anti-clockwise moments

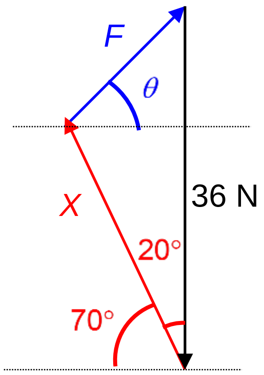
$$(36)\left[0.45\cos(60^\circ)\right] = \left[X\cos(40^\circ)\right](1.2)$$
$$X = 8.8 \text{ N}$$

(b)(i) Since bar is in translational equilibrium, there is no resultant force along horizontal so there needs to be a force acting right at A to balance the left component of X.

Bar is in rotational equilibrium.
By principle of moments and taking moments about B,
weight gives anti-clockwise moment so there needs to be an upwards force at A to give clockwise moment

Notes: For (b)(i), you need to address both conditions of equilibrium to gain full credit.

Example 16 (continued)

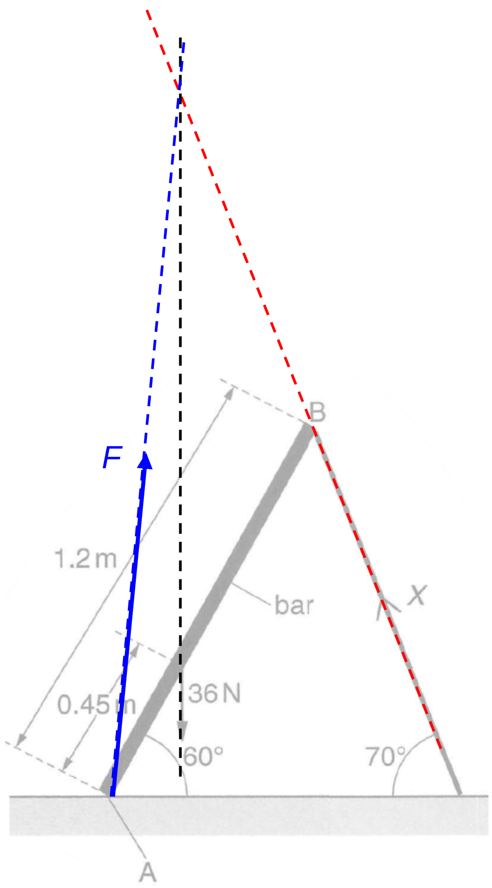


Solution

(b)(ii)

By cosine rule,

$$F^2 = X^2 + 36^2 - 2(36)(X)\cos 20^\circ$$
$$F = \sqrt{8.8^2 + 36^2 - 2(36)(8.8)\cos 20^\circ}$$
$$= 27.9 \text{ N}$$



(b)(iii)

For a body that is in equilbrium,
when <u>none</u> of the forces acting on the body are parallel all the lines of action from the forces will meet at a common point.

i.e the three forces must be **CONCURRENT.**

(*4 or more non parallel forces need not be concurrent for rotational equilibrium to take place.*

Why?

A body in equilibrium under the action of more than three forces may be reduced to a three-force member by a combination of two or more of the known forces.)

Notes: (b)(iii) is a special geometric result, often useful in answering MCQs.

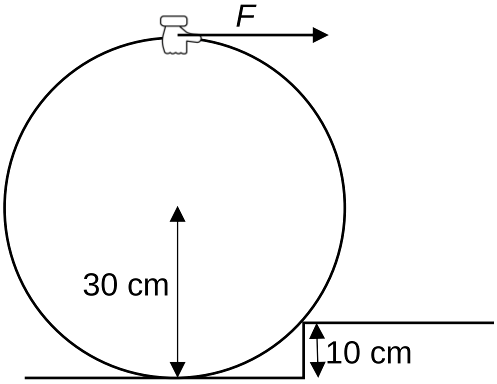
We can understand using rotational equilibrium: the sum of moments about **any** point (which can be outside the body) must sum up to zero torque.

The common point where the lines of action meet results in zero perpendicular distance to the point for each of the forces:

$$\text{torque} = F(0) + W(0) + X(0) = 0$$

Example 17

A wheel is being rolled up a kerb by a forward force F at the top as shown. The weight of 1400 N passes through the centre of wheel, the wheel has a radius of 30 cm and the kerb is 10 cm high. Determine (i) F and (ii) the reaction force on the wheel by the kerb **just** as wheel mounts the kerb.

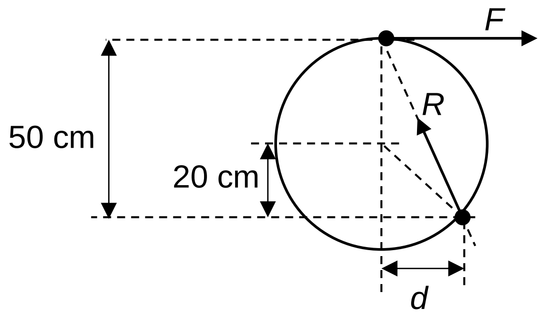


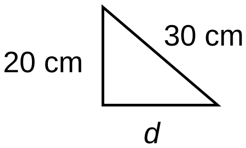
(not to scale)

Solution

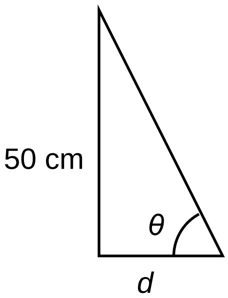
Let reaction force be R

To just mount kerb, the wheel loses contact with the lower ground so no normal contact force there.





$d = \sqrt{30^2 - 20^2}$



$$\tan\theta = \frac{50}{d} = \frac{50}{\sqrt{30^2 - 20^2}}$$

$$\theta = \tan^{-1}\left(\frac{50}{\sqrt{30^2 - 20^2}}\right)$$

$$= 65.9^\circ$$

[direction] R acts 24.1° left of vertical

By principle of moments, take moments about pivot, sum of clockwise moments = sum of anti-clockwise moments

$$F(50) = Wd$$

$$F = \frac{(1400)\sqrt{30^2 - 20^2}}{50}$$

$$= 626 \text{ N to the right}$$

wheel in translational equilibrium
no resultant force vertically:

$$R\sin\theta = W$$

[magnitude] $$R = \frac{1400}{\sin 65.9^\circ}$$

$$= 1530 \text{ N}$$

Notes: The geometric result in Example 16(b)(iii) was used – can you spot it?

4.8 Ending Notes

Force is one of the fundamental building blocks on which the rest of the A-Level Physics syllabus will build on. You should master the skills necessary to solve these problems soonest possible.

In general, if you ever find yourself uncertain on how to approach answering any question, a good "fall-back strategy" is to consider the following matrix:

	magnitude	**direction**
force	"which force?" "how strong?" "any "cancellation?" "what is the resultant?"	"where is the force directed to?" "is the force being displaced?"
energy	"what type of energy?" "how much energy?"	"being converted from what type to what type?" "being converted from where to where?" "is there energy loss / dissipation?"

You can use the space below to do up your own mind-map to summarise this topic.