RVHS H2 Mathematics Remedial Programme

Topic: Probability

Basic Mastery Questions

1. **MI Prelim 9758/2021/Q6(i),(ii),(iii)**

The events A and B are such that $\mathrm{P}(A)=0.6$, $\mathrm{P}(A\cup B)=0.8$ and $\mathrm{P}(A\cap B')=0.55$.

(i) Find the probability that B occurs. [1]

(ii) Find the probability that neither A nor B occurs. [1]

A third event C is such that B and C are independent and $\mathrm{P}(C)=0.6$.

(iii) Find $\mathrm{P}(B'\cap C)$. [2]

Answer: (i) 0.25 (ii) 0.2 (iii) 0.45

(i)	$\mathrm{P}(B)=\mathrm{P}(A\cup B)-\mathrm{P}(A\cap B')$ $=0.8-0.55$ $=0.25$
(ii)	Probability that neither A nor B occurs $=\mathrm{P}(A'\cap B')$ $=1-\mathrm{P}(A\cup B)$ $=1-0.8$ $=0.2$
(iii)	**Method 1** $\mathrm{P}(B'\cap C)$ $=\mathrm{P}(B')\times\mathrm{P}(C)$ $\because$ B and C are independent, B' and C are independent, $=(1-0.25)(0.6)$ $=0.45$ **Method 2**

	$\mathrm{P}(B' \cap C)$ $= \mathrm{P}(C) - \mathrm{P}(B \cap C)$ $= \mathrm{P}(C) - \mathrm{P}(B)\mathrm{P}(C)$ $\because B$ and C are independent $= 0.6 - (0.25)(0.6)$ $= 0.45$

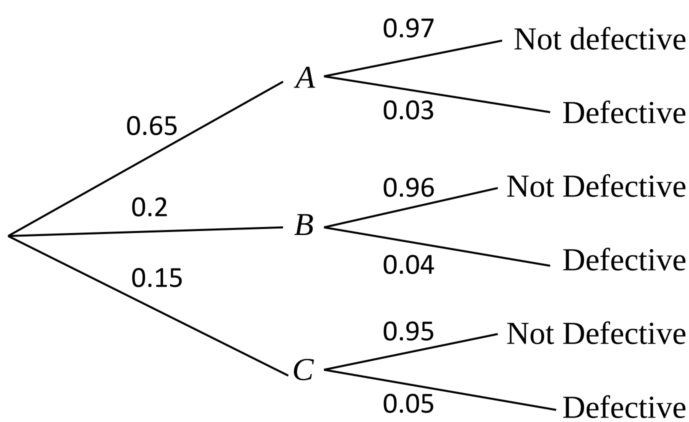
2. **RVHS Prelim 9758/2021/Q8(i),(ii)**

A manufacturer produces 3 types of spray bottles: Type A, Type B and Type C. 65% of the sprayers manufactured are Type A and 20% are Type B.

3% of Type A sprayers, 4% of Type B sprayers and 5% of Type C sprayers have manufacturing defects.

(i) A sprayer is chosen at random. Construct a probability tree to show the above information. [2]
(ii) Find the probability that out of 2 randomly selected sprayers, exactly one of them has manufacturing defects. [3]

Answer: (ii) 0.06755

(i)	
(ii)	$\mathrm{P}(\text{a sprayer has manufacturing defect})$ $= 0.65 \times 0.03 + 0.2 \times 0.04 + 0.15 \times 0.05$ $= 0.035$ $\mathrm{P}(\text{1 out of 2 sprayers has manufacturing defect})$ $= \binom{2}{1} \times 0.035 \times (1 - 0.035)$ $= 0.06755$

3. **CJC Prelim 9758/2020/02/Q6(a)**
A bag contains 4 red counters and 6 blue counters. 4 counters are drawn from the bag at random, without replacement.

Calculate the probability that:
(i) all the counters drawn are blue,
(ii) at least 3 blue counters are drawn,
(iii) at least 1 counter of each colour is drawn,
(iv) at least 3 blue counters are drawn, given that at least 1 of each colour is drawn.

Answer: (i) $\frac{1}{14}$ (ii) $\frac{19}{42}$ (iii) $\frac{97}{105}$ (iv) $\frac{40}{97}$

(i) P (all counters blue) $=\frac{{}^{6}C_{4}}{{}^{10}C_{4}}=\frac{1}{14}$

(ii) <u>Case 1</u> : P(3 blue,1 red) $=\frac{{}^{6}C_{3}\times{}^{4}C_{1}}{{}^{10}C_{4}}=\frac{8}{21}$

<u>Case 2</u> : P (all blue) $=\frac{1}{14}$

P(at least 3 blue)
= P(3 blue, 1 red) + P(all blue)
$=\frac{8}{21}+\frac{1}{14}=\frac{19}{42}$

(iii) P (all counters red) $=\frac{{}^{4}C_{4}}{{}^{10}C_{4}}=\frac{1}{210}$

P(at least 1 counter of each colour is drawn)
= 1 – P(all blue) – P(all red)
$=1-\frac{1}{14}-\frac{1}{210}=\frac{97}{105}$

(iv) P(at least 3 blue | at least 1 of each colour)

$$=\frac{\text{P(at least 3 blue }\cap\text{ at least 1 of each colour)}}{\text{P(at least 1 of each colour)}}$$

$$=\frac{\text{P(3 blue,1 red)}}{\text{P(at least 1 of each colour)}}$$

$$=\frac{\frac{8}{21}}{\frac{97}{105}}=\frac{40}{97}$$

4. **HCI Prelim 9758/2020/02/Q7(a)**

A team of 15 students was selected for an outdoor education trip. One student volunteered to be the trip leader while another volunteered as the assistant trip leader. They decided to have some ice-breaker games, where all 15 students sat in a circle.

Find the probability that both leaders were not seated together. [2]

Answer: 0.857

Required probability

$$=\frac{\frac{13!}{13}\times {}^{13}C_2\times 2!}{\frac{15!}{15}}$$

$$=\frac{6}{7} \text{ or } 0.857 \text{ (3 s.f.)}$$

Alternative

Total numbers of ways = $(15-1)!=14!$

Number of ways leaders seated together

$=(14-1)!\times 2=13!\times 2$

Probability = $1-\frac{13!\times 2}{14!}=1-\frac{1}{7}=\frac{6}{7}$

5. **JPJC Prelim 9758/2020/02/Q6(i)**

The eleven letters in the word CORONAVIRUS are rearranged to form 'words' which may not make sense. Find the probability that the two 'R's are together; [3]

Answer: 0.182

No. of ways where the two 'R's are together = $\frac{10!}{2!}=1814400$
Required probability = $\frac{1814400}{\frac{11!}{2!2!}}=\frac{1814400}{9979200}=\frac{2}{11}$ or 0.182

Standard Questions

1. **RVHS Prelim 9758/2020/02/Q6**

A set of 36 cards is made up of cards chosen from a few packs of ordinary playing cards. The breakdown of the number of cards in each suit and denomination is given in the following table.

Denomination \ Suit	Spade	Heart	Diamond	Club
Ace	1	2	2	1
King	2	2	2	3
Queen	2	4	4	3
Jack	1	3	2	2

For example, there are 2 Aces of Diamond and 3 Jacks of Heart in the set of 36 cards.

(i) This set of 36 cards is put into a bag and 1 card is selected at random.

(a) Find the probability that the card is either an Ace or a Jack but not a Club. [1]

(b) Find the probability that the card is neither a Jack nor a Heart. [1]

(ii) The cards are all placed back into the bag and 2 cards are selected at random. Find the probability that both cards are Aces given that neither of the cards is Spade. [2]

Answer: (i)(a) $\frac{11}{36}$ **(i)(b)** $\frac{5}{9}$ **(ii)** $\frac{2}{87}$

(i)(a) $\frac{5+6}{36}=\frac{11}{36}$
(i)(b) $\frac{5+8+7}{36}=\frac{5}{9}$
(ii)(a) P(both are Aces | neither of the cards is Spade) $=\frac{\text{P(both Aces \& not Spade)}}{\text{P(not Spade)}}=\frac{\binom{5}{2}}{\binom{30}{2}}=\frac{2}{87}$

2. **DHS Prelim 9758/2022/02/Q7(a),(b)**

Commodity X is traded four times a week from Monday to Thursday. The unit price of X can only rise or fall on any day. If the unit price of X rises on a day, there is a probability of 0.6 that it will rise on the next trading day. If the unit price of X falls on a day, there is a probability of 0.15 that it will rise on the next trading day.

In a particular week, the unit price of X rises on Monday and the events A and B are defined as follows.

A: the unit price of X falls on Tuesday.
B: the unit price of X rises on Thursday.

(a) Find

(i) $P(A \cap B)$, [2]

(ii) P(B), [2]

(iii) P(B|A). [2]

(b) State, with a reason, whether A and B are independent. [1] [3]

Answer: (a)(i) 0.087 (ii) 0.339 (iii) 0.2175 (B) not independent

7(a)(i)	$P(A \cap B)$ $= P(\text{fall, rise, rise}) + P(\text{fall, fall, rise})$ $= (0.4 \times 0.15 \times 0.6) + (0.4 \times 0.85 \times 0.15)$ $= 0.087$
(ii)	$P(B)$ $= P(A \cap B) + P(A' \cap B)$ $= 0.087 + P(\text{rise, rise, rise}) + P(\text{rise, fall, rise})$ $= 0.087 + (0.6 \times 0.6 \times 0.6) + (0.6 \times 0.4 \times 0.15)$ $= 0.339$
(iii)	$P(B \mid A)$ $= \dfrac{P(B \cap A)}{P(A)}$ $= \dfrac{0.087}{0.4}$ $= 0.2175$
(b)	Since $P(B \mid A) = 0.2175 \neq 0.339 = P(B)$, A and B are not independent.