Marking Scheme for 2024 4E5N A.Math Prelim Paper 2

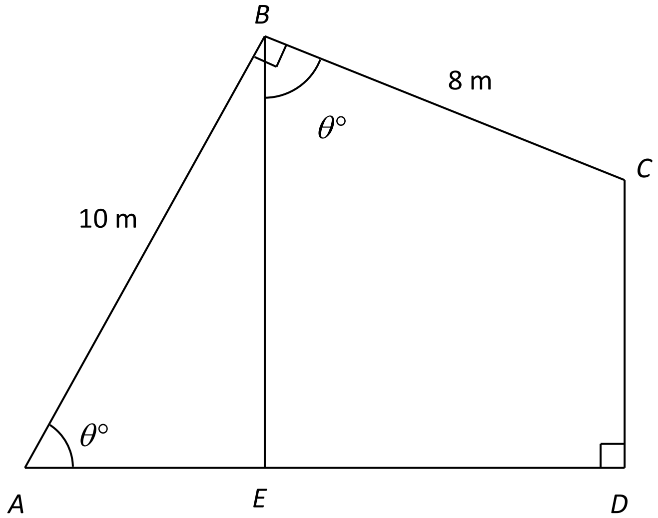
[√ means follow through] **Total Marks : 90 Setter:Mdm Ho**

Qn				
1	(a)	Let $f(x) = 2x^3 + 3x^2 - 8x + 3$. By the Remainder Theorem, $f(-1) = p$ $p = 2(-1)^3 + 3(-1)^2 - 8(-1) + 3$ $p = -2 + 3 + 8 + 3$ $p = 12$	M1 A1	[2]
	(b)	$(2x-1)$ is a factor of $f(x)$ [or (x+3) or (x-1) is a factor] $2x^3 + 3x^2 - 8x + 3 = 0$ $(2x-1)(x^2 + 2x - 3) = 0$ $(2x-1)(x+3)(x-1) = 0$ $2x - 1 = 0$ xo $x + 3 = 0$ or $x - 1 = 0$ $2x = 1$ $x = -3$ $x = 1$ $x = \frac{1}{2}$ $x = -3, \frac{1}{2}$ or 1	M1 M1 [another quad expression] A1 [3 ans]	[3]
	(c)	$2(x-1)^3 + 3(x-1)^2 - 8x + 11 = 0$ $2(x-1)^3 + 3(x-1)^2 - 8(x-1) + 3 = 0$ Let $y = x - 1$. $2y^3 + 3y^2 - 8y + 3 = 0$ $y = -3, \frac{1}{2}$ or 1 $x - 1 = -3, \frac{1}{2}$ or 1 $x = -2, \frac{3}{2}$ or 2 Hence, $x = -2, \frac{3}{2}$ or 2.	M1 A1	[2]
2	(a)	$120 000	B1	[1]

	(b)	(i) $\text{Value after 1 year} = 120000e^{-12a}$ $90000 = 120000e^{-12a}$ $e^{-12a} = 0.75$ $-12a = \ln 0.75$ $a = \dfrac{\ln 0.75}{-12} = 0.0239735$ $= 0.02397 \textit{shown}$	 M1 M1 must show 0.0239735	[2]
		(ii) $70000 = 120000e^{-at}$ $\dfrac{7}{12} = e^{-at}$ $-0.02397t = \ln\dfrac{7}{12}$ $t = 22.49$ $\approx 23 \text{ months}$	 M1 M1 A1	[3]
		(iii) 5 years = 60 months $\text{Value after 5 years} = 120000e^{-60(0.02397)}$ $= \$28482.55$ Yes, since car dealer is paying more ($29000). or No, there is not much difference, so I would rather use the car.	 M1 A1	[2]
3	**(a)**	$\cos 75° = \cos(45° + 30°)$ $= \cos 45° \cos 30° - \sin 45° \sin 30°$ $= \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right) - \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{1}{2}\right)$ $= \dfrac{\sqrt{6} - \sqrt{2}}{4}$ (Shown)	M1[use 45+30] M1 [any one -surd form]] M1 [simplify to desired ans]	[3]
	(b)	$\text{LHS} = \dfrac{\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}}{\dfrac{1}{\cos x} + \dfrac{1}{\sin x}}$ $= \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x} \times \dfrac{\sin x \cos x}{\cos x + \sin x}$ $= \dfrac{1}{\cos x + \sin x}$ $= \text{RHS}$	M1 use identities M1 [same denominator] M1 simplify	[6]

		$\dfrac{1}{\cos x+\sin x}=\dfrac{3}{4\cos x-3\sin x}$ $3\cos x+3\sin x=4\cos x-3\sin x$ $6\sin x=\cos x$ $\tan x=\dfrac{1}{6}$ $\alpha=0.165148677$ $x=0.165,\ 3.31$	M1 cross multiply M1 [change to tan] A1	
4	**(a)**	$AB=\sqrt{196-x^2},\ BC=2x$ $A=\dfrac{1}{2}(x+2x)\left(\sqrt{196-x^2}\right)$ $=\dfrac{3x}{2}\left(\sqrt{196-x^2}\right)$	B1 [$AB=\sqrt{196-x^2}$] M1 formula with h=3x	[2]
	(b)	$\dfrac{dA}{dx}=\dfrac{3x}{2}\left(-\dfrac{2x}{2\sqrt{196-x^2}}\right)+\dfrac{3\sqrt{196-x^2}}{2}$ $=\dfrac{3}{2}\left(-\dfrac{x^2}{\sqrt{196-x^2}}+\sqrt{196-x^2}\right)$ $=\dfrac{3}{2}\left(\dfrac{196-x^2-x^2}{\sqrt{196-x^2}}\right)$ $=\dfrac{3(98-x^2)}{\sqrt{196-x^2}}$ $\dfrac{dA}{dx}=0$ $\dfrac{3(98-x^2)}{\sqrt{196-x^2}}=0$ $x=\sqrt{98}$ $x\approx 9.90$ $\dfrac{d^2A}{dx_2}=\dfrac{3\sqrt{196-x^2}(-2x)-3(98-x^2)(\frac{1}{2})(196-x^2)^{-\frac{1}{2}}(-2x)}{196-x^2}$ $\dfrac{d^2A}{dx^2}<0$, *A* is max. [or Using first derivative test]	M1 M1 A1 M1 A1	[5]

	(c)	$A=\frac{3\sqrt{98}}{2}\left(\sqrt{196-98}\right)$ $=147\text{ cm}^2$	M1 A1	[2]
5	**(a)**	$2^{3+2x}+2^{5+x}=2^x+4$ $2^3\times\left(2^x\right)^2+2^5\times 2^x=2^x+4$ $8\left(2^x\right)^2+32\left(2^x\right)=2^x+4$ Let $y=2^x$. $8y^2+32y=y+4$ $8y^2+31y-4=0$ $(8y-1)(y+4)=0$ $8y-1=0$ or $y+4=0$ $8y=1$ $\quad y=-4$ $y=\frac{1}{8}$ $\quad 2^x=-4$ $2^x=\frac{1}{8}$ $2^x=2^{-3}$ $x=-3$ Since $2^x>0$, $x=-3$.	M1 [$8\left(2^x\right)^2$ seen] M1 [$32\left(2^x\right)$ seen] M1 factorise A1 for both y A1	[5]
	(b)	$\log_a\sqrt{ab^3}=\frac{\log_2\sqrt{ab^3}}{\log_2 a}$ $=\frac{\frac{1}{2}[\log_2(ab^3)]}{\log_2 a}$ $=\frac{\log_2(a)+3\log_2(b)}{2(\log_2 a)}$ $=\frac{x+3y}{2x}$	M1 [change base 2] M1 [$\log_2(a)+3\log_2(b)$] A1	[3]

	(c)	$\log_5 50 + 4\log_{25} y = \log_5(2y+4) + 2$ $\log_5 50 + 4\dfrac{\log_5 y}{\log_5 25} = \log_5(2y+4) + \log_5 5^2$ $\log_5 50 + 2\log_5 y = \log_5 25(2y+4)$ $\log_5 50y^2 = \log_5 25(2y+4)$ $50y^2 = 50y + 100$ $y^2 - y - 2 = 0$ $(y-2)(y+1) = 0$ $y = 2$ or -1 (reject)	M1[$\dfrac{\log_5 y}{\log_5 25}$] M1[2= $\log_5 5^2$] M1 use product law A1	[4]
6	**(a)**	 $AE = 10\cos\theta$ $\sin\theta = \dfrac{ED}{8}$ $ED = 8\sin\theta$ $AD = 10\cos\theta + 8\sin\theta$ (shown)	Use trigo ratio M1 M1	[2]
	(b)	$10\cos\theta + 8\sin\theta = R\cos(\theta - \alpha)$ $R = \sqrt{8^2 + 10^2}$ $= \sqrt{164}$ or 12.8 $\alpha = \tan^{-1}\left(\dfrac{8}{10}\right)$ $= 38.66$ $\approx 38.7^o$ $10\cos\theta + 8\sin\theta = \sqrt{164}\cos(\theta - 38.7)$	M1 M1 A1	[3]

	(c)	$\sqrt{164}\cos(\theta-38.7)=12$ $\cos(\theta-38.7)=\dfrac{12}{\sqrt{164}}$ basic $\angle = 20.4$ $\theta-38.7=20.4,\ -20.4$ $\theta=59.1,\ 18.3$ (1 decimal place)	M1 M1 A1 both	[3]
7	**(a)**	Show that $\dfrac{d}{dx}\{x(3x-1)^{\frac{5}{3}}\}=(8x-1)(3x-1)^{\frac{2}{3}}$. $\dfrac{d}{dx}\{x(3x-1)^{\frac{5}{3}}\}$ $=x\left(\dfrac{5}{3}\right)(3x-1)^{2/3}(3)+(3x-1)^{5/3}$ $=5x(3x-1)^{2/3}+(3x-1)^{5/3}$ $=(3x-1)^{2/3}[5x+3x-1]$ $=(3x-1)^{2/3}[8x-1]$	B1 for $x\left(\dfrac{5}{3}\right)(3x-1)^{2/3}$ seen B1 for (3) seen B1 for $(3x-1)^{5/3}$ seen M1 for factorization A1 for showing clear working leading to the desired expression	[5]
	(b)	$\int(8x-1)(3x-1)^{\frac{2}{3}}\,dx=x(3x-1)^{\frac{5}{3}}+c$ $\int 8x(3x-1)^{\frac{2}{3}}\,dx-\int (3x-1)^{\frac{2}{3}}\,dx=x(3x-1)^{\frac{5}{3}}+c$ $\int 8x(3x-1)^{\frac{2}{3}}\,dx=x(3x-1)^{\frac{5}{3}}+\left(\dfrac{(3x-1)^{\frac{5}{3}}}{\frac{5}{3}(3)}\right)+c$ $\int 8x(3x-1)^{\frac{2}{3}}\,dx=x(3x-1)^{\frac{5}{3}}+\left(\dfrac{(3x-1)^{\frac{5}{3}}}{5}\right)+c$ $\int x(3x-1)^{\frac{2}{3}}\,dx=\dfrac{1}{8}x(3x-1)^{\frac{5}{3}}+\dfrac{1}{40}(3x-1)^{\frac{5}{3}}+c$ $=\left(\dfrac{1}{8}x+\dfrac{1}{40}\right)(3x-1)^{\frac{5}{3}}+c$	M1 M1 M1 $\left(\dfrac{(3x-1)^{\frac{5}{3}}}{\frac{5}{3}(3)}\right)$ A1 [1/8 seen] A1 [1/40 seen]	[5]

8	**(a)**	(i) From the diagram, by similar triangles, $\frac{r}{h}=\frac{2}{4}$ $4r=2h$ $r=\frac{h}{2}$ (shown) (ii) From (a)(i), $V=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$ $=\frac{1}{3}\pi\left(\frac{h^2}{4}\right)h$ $=\frac{\pi}{12}h^3$ (shown)	M1 M1 M1 M1	[2] [2]
	(b)	$V=\frac{\pi}{12}h^3$ $\frac{dV}{dh}=\frac{\pi}{4}h^2$ Given: $\frac{dV}{dt}=-2\text{m}^3/\text{min}$ When $h=2$, $\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$ $-2=\frac{\pi}{4}(2)^2\times\frac{dh}{dt}$ $-2=\pi\times\frac{dh}{dt}$ $\frac{dh}{dt}=\frac{-2}{\pi}$ $=-0.637\text{m/min}$ (correct to 3 sig. fig.)	M1 B1 M1 M1 A1	[5]
9	**(a)**	$x^2+y^2+2x-6y-6=0$ Radius $=\sqrt{1^2+(-3)^2-(-6)}$ $=\sqrt{16}$ $=4\text{ cm}$ $\therefore$ Centre is $(-1,3)$	M1A1 M1A1 [or A2]	[4]

	(b)	$(-1)^2 + k^2 + 2(-1) - 6k - 6 = 0$ $k^2 - 6k - 7 = 0$ $(k-7)(k+1) = 0$ $k = 7$ or $k = -1$ (NA)	M1 M1 A1	[3]
	(c)	(i) Coordinates of *centre* C_2 = (-1, -3). (ii) $\sqrt{(5+1)^2 + (3+0)^2}$ $= \sqrt{45} = 6.71 > 4 = radius$ P(5,0) lies outside the circle C_2.	B1 M1 A1	[1] [2]
10	**(a)**	$2\sin(2x+\pi) - 1 = 0$ $\sin(2x+\pi) = \frac{1}{2}$ $\alpha = \sin^{-1}\frac{1}{2}$ $= \frac{\pi}{6}$ $2x + \pi = \frac{\pi}{6},\quad \pi - \frac{\pi}{6},\quad \frac{\pi}{6} + 2\pi,\quad \pi - \frac{\pi}{6} + 2\pi$ $x = -\frac{5\pi}{12},\quad -\frac{\pi}{12},\quad \frac{7\pi}{12},\quad \frac{11\pi}{12}$ x coordinate of $A = \frac{7\pi}{12}$	M1 M1 A1	[3]
	(b)	x coordinate of $B = \frac{11\pi}{12}$	B1	[1]
	(c)	Shaded area $= -\int_0^{\frac{7\pi}{12}} 2\sin(2x+\pi) - 1\,\mathrm{d}x + \int_{\frac{7\pi}{12}}^{\frac{11\pi}{12}} 2\sin(2x+\pi) - 1\,\mathrm{d}x$ $= -\left[-\cos(2x+\pi) - x\right]_0^{\frac{7\pi}{12}} + \left[-\cos(2x+\pi) - x\right]_{\frac{7\pi}{12}}^{\frac{11\pi}{12}}$ $= -\left[-2.69862 - 1\right] + \left[-2.01377 - (-2.69862)\right]$ $= 3.69862 + 0.684853$ $= 4.38 \text{ unit}^2$	M1 M1 M1 A1	[4]

Total = 90 m