Candidate Index Number

Anglo - Chinese School (Independent)

FINAL EXAMINATIONS 2012
YEAR 3 INTEGRATED PROGRAMME
CORE MATHEMATICS
PAPER 1

TUESDAY **2ND OCTOBER 2012** **1 h 30 min**

Additional Material
Graph Paper (1 sheet)

INSTRUCTIONS TO CANDIDATES

- Write your index number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.

For Examiner's Use

This paper consists of 14 printed pages.

[Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

*Answer **all** the questions in the spaces provided.*

1 *[Maximum mark: 6]*

(a) Given that $x = 3\sqrt{\frac{5-y}{y+2}}$, express y in terms of x.

[3 marks]

(b) Simplify $\frac{y^{4n} - x^{4n}}{x^2 y^{2n} - x^{2n+2}}$.

[3 marks]

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2 *[Maximum mark: 4]*

Given that $-5 < x < 2$, $1 \leq y \leq 6$ and x and y are integers, find

(a) the least possible value of $x + y^3$.

[2 marks]

(b) the least possible value of $\frac{y}{x}$.

[2 marks]

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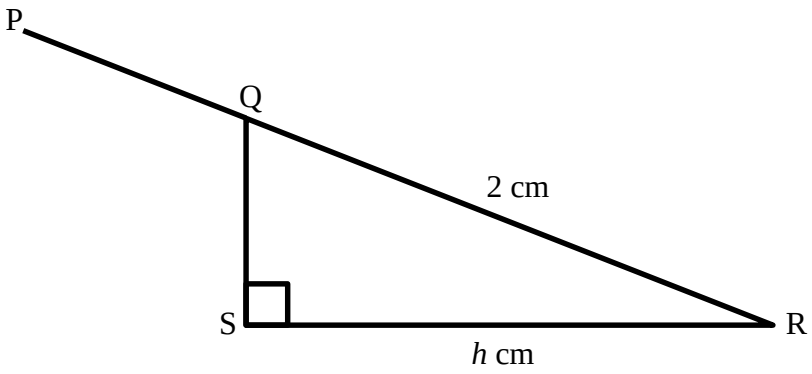
3 *[Maximum mark: 4]*

In the diagram, PQR is a straight line, QR = 2 cm and RS = h cm.



Find the following in terms of h.

(a) $\sin \angle SRQ$,

[2 marks]

(b) $\tan \angle PQS$.

[2 marks]

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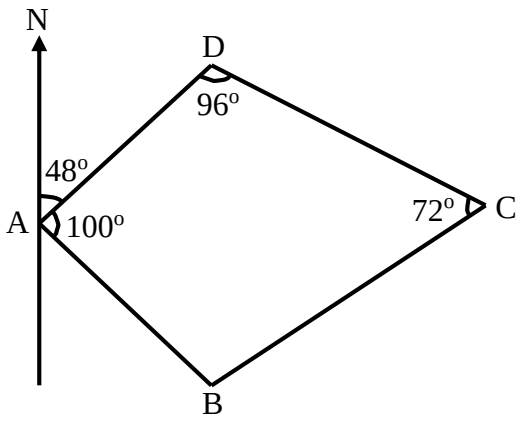
4 *[Maximum mark: 9]*

(a) A, B and C are points on level ground, with A due north of C. $\angle BAC = 45^o$, $\angle BCA = 60^o$ and AB = 48 m. Calculate the length of BC, leaving your answer in the form $a\sqrt{b}$ where *a*, *b* and *c* are constants. [$\sin 120^o = \frac{\sqrt{3}}{2}$, $\cos 120^o = -\frac{1}{2}$, $\tan 120^o = \sqrt{3}$, $\sin 45^o = \frac{1}{\sqrt{2}}$, $\cos 45^o = \frac{1}{\sqrt{2}}$, $\tan 45^o = 1$]

[3 marks]

(b) In the diagram, A, B, C and D are four towns.



Find the bearing of

(i) A from D,
(ii) D from C,
(iii) A from B.

[6 marks]

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5 *[Maximum mark: 5]*

Given that $\cos A = -\frac{1}{3}$ and $90^o \leq A \leq 180^o$, find the value of

(a) $\tan A$,

[2 marks]

(b) $\frac{3\cos A - \tan A}{2 - 3\sin A}$, leaving your answer in the form $a + b\sqrt{2}$ where *a* and *b* are constants.

[3 marks]

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6 *[Maximum mark: 8]*

Joel and Gabriel started together on a 7 km race at the same constant speed of y km/h. After 1 km, Joel increased his speed by 1 km/h and ran the remaining 6 km at the new speed. Gabriel walked at a constant speed of y km/h all the way.

(a) Write down the time, in terms of y, Joel took to complete the 7 km race.

[1 mark]

(b) Given that Joel finished the race 12 minutes earlier than Gabriel, form an equation in y and show that it reduces to $y^2 + y - 30 = 0$.

[3 marks]

(c) Solve the equation $y^2 + y - 30 = 0$.

[2 marks]

(d) Calculate Joel's average speed, in km/h, for the whole journey.

[2 marks]

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7 *[Maximum mark: 5]*

(a) Express $y = 3x^2 - 12x + 16$ in the form $y = a(x-h)^2 + k$.

[2 marks]

(b) Hence, sketch the curve $y = 3x^2 - 12x + 16$, indicating clearly, the points of intersection with the axes (if any) and the turning point.

[3 marks]

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8 *[Maximum mark: 7]*

(a) Solve the equation

$$64^{x^2} = (0.25)(16^x)^2$$

[3 marks]

(b) Solve the simultaneous equations

$$5^x = 125(5^{y-1})$$

$$\frac{1}{\log_3 2} + \log_2(2x + y) = 2 + 2\log_4 6$$

[4 marks]

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9 *[Maximum mark: 8]*

(a) Find the possible values of k for which the line $2y = x + 2k$ is a tangent to the curve $x^2 + y^2 = 8k$.

[4 marks]

(b) Find the range of values of k for which the expression $x^2 + k^2x^2 - 20kx + 90$ is positive for all real values of x.

[4 marks]

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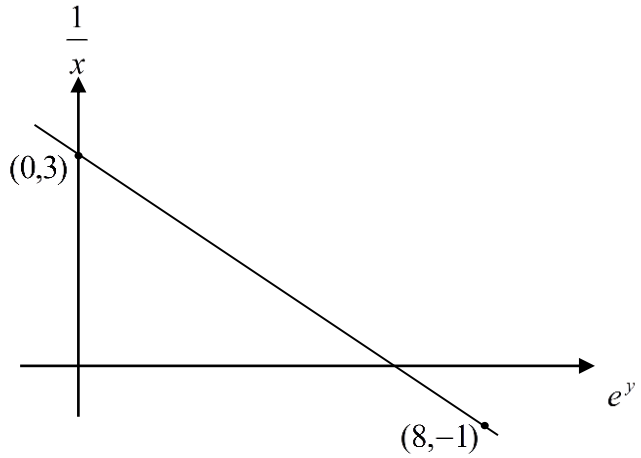
10 *[Maximum mark: 4]*

The diagram below shows the straight line obtained by plotting $\frac{1}{x}$ against e^y. Express y in terms of x.



[4 marks]

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11 *[Maximum mark: 9]*

(a) A line L passes through the point P (-2, 2) and has gradient 1. Find the equation of L.

[2 marks]

(b) The point Q ($3a$, a) lies on L. Find the value of a.

[1 mark]

(c) R is the point (0, -8). Prove that ΔPQR is a right-angled triangle.

[4 marks]

(d) Find the area of ΔPQR and hence, the shortest distance of Q from PR.

[2 marks]

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12 *[Maximum mark: 4]*

Given that $\lg 3 = p$ and $\lg 5 = q$, express the following in terms of p and q.

(a) $\lg \sqrt{\frac{243}{5}}$

[2 marks]

(b) $\lg 0.54$

[2 marks]

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13 *[Maximum mark: 7]*

Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = x + \frac{8}{x} - 5$. The corresponding values of x and y are given in the table below.

x	1	2	3	4	5	6
y	4.00	1.00	0.67	1	1.60	2.33

(a) Using a scale of 2 cm to represent 1 unit on each axis, draw the graph of $y = x + \frac{8}{x} - 5$ where $1 \le x \le 6$.

[4 marks]

(b) By drawing a suitable straight line graph on the same axes, solve the equation $2x + \frac{8}{x} - 9 = 0$.

[3 marks]

********** END OF PAPER 1 **********

Answers:

1a	$y = \frac{45 - 2x^2}{x^2 + 9}$
1b	$\frac{y^{2n} + x^{2n}}{x^2}$
2a	-3
2b	-6
3a	$\sin SRQ = \frac{\sqrt{4 - h^2}}{2}$
3b	$\tan SQP = -\frac{h}{\sqrt{4 - h^2}}$
4a	$B = 16\sqrt{6}$
4bi	228
4bii	312
4biii	328
5a	$\tan A = -\sqrt{8} = -2\sqrt{2}$
5b	$-\frac{3}{2} - \frac{1}{2}\sqrt{2}$
6a	$\frac{1}{y} + \frac{6}{y + 1}$
6c	5
6d	$5\frac{5}{6}$
7a	$y = 3(x - 2)^2 + 4$
8a	$x = \frac{1}{3}$ or $x = 1$
8b	$x = \frac{10}{3}$
9a	$k = 0$ or $k = 10$
9b	$-3 < k < 3$
10	$y = \ln(6 - \frac{2}{x})$
11a	$y = x + 4$
11b	$a = -2$
11d	$\frac{12}{13}\sqrt{26}$
12a	$\frac{1}{2}[5p - q]$
12b	$3p - q - 1$
13