Name : SOLUTIONS

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2023
Secondary 4

TUESDAY	**MATHEMATICS**	**4052/01**
15 August 2023	**Paper 1**	2 hours 15 minutes

Candidates answer on the Question Paper.

INSTRUCTIONS TO CANDIDATES

Write your class, name and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For p, use either your calculator value or 3.142, unless the question requires the answer in terms of p.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90**.**

90

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{\text{r}}{100}\right)^{n}$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 It is given that M is a prime number and is a digit of the number 508 M45.
If the number, correct to 3 significant figures, is 509 000, find the maximum value of M.

Answer M = …….7……. [1]

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2 Simplify $3+2(3x-2)^2$.

$3+2(3x-2)^2$
$=3+2(9x^2-12x+4)$
$=3+18x^2-24x+8$
$=18x^2-24x+11$

Answer …….$18x^2-24x+11$……..... [2]

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3 Solve the equation $\left(\frac{1}{7}\right)^{2x}=\sqrt[3]{49}$.

$\left(\frac{1}{7}\right)^{2x}=\sqrt[3]{49}$
$7^{-2x}=\sqrt[3]{7^2}$
$7^{-2x}=7^{\frac{2}{3}}$
$-2x=\frac{2}{3}$
$x=-\frac{1}{3}$

Answer x = ……$-\frac{1}{3}$……. [2]

---

4 (a) Factorise $3y^{2a}+6y^{a}$ completely.

$3y^{2a}+6y^{a}$
$=3y^{a}\left(y^{a}+2\right)$

Answer $\ldots\ldots 3y^{a}\left(y^{a}+2\right)\ldots\ldots\ldots$ [1]

(b) Factorise $20pq-15sq+3sr-4pr$ completely.

$20pq-15sq+3sr-4pr$
$=5q(4p-3s)+r(3s-4p)$
$=5q(4p-3s)-r(4p-3s)$
$=(4p-3s)(5q-r)$

Answer $\ldots\ldots\ldots\ldots(4p-3s)(5q-r)\ldots\ldots\ldots$ [2]

---

5 Express $\dfrac{3n^2-8n-3}{9-n^2}+\dfrac{5n-3}{n+3}$ as a single fraction in its simplest form.

$\dfrac{3n^2-8n-3}{9-n^2}+\dfrac{5n-3}{n+3}$
$=\dfrac{(3n+1)(n-3)}{(3-n)(3+n)}+\dfrac{5n-3}{n+3}$
$=\dfrac{-(3n+1)(3-n)}{(3-n)(3+n)}+\dfrac{5n-3}{n+3}$
$=\dfrac{-3n-1+5n-3}{3+n}$
$=\dfrac{2n-4}{3+n}$

Answer $\ldots\ldots\ldots\dfrac{2n-4}{3+n}\ldots\ldots\ldots$ [3]

---

6 Given that x satisfies the inequality $\frac{1}{2}x-(x-2)\le\frac{3}{4}(2x-3)-5$, find the smallest value of x if x is an integer.

$$\begin{aligned}\frac{1}{2}x-(x-2)&\le\frac{3}{4}(2x-3)-5\\ 2x-4(x-2)&\le 3(2x-3)-20\\ 2x-4x+8&\le 6x-9-20\\ -2x-6x&\le -29-8\\ -8x&\le -37\\ x&\ge 4.625\end{aligned}$$

Answer x =……5……..… [3]

---

7 It is given that $90°<A<180°$ and $\sin A=\frac{7}{25}$.

(a) Find the value of A.

$$\begin{aligned}A&=180°-\sin^{-1}\left(\frac{7}{25}\right)\\ &=180°-16.260...°\\ &=163.739...°\\ &=163.7°\text{ (1dp)}\end{aligned}$$

Answer …… 163.7 …..… [1]

(b) Without the use of the calculator, find the numerical value of $\cos A$.

$$\begin{aligned}x&=\sqrt{25^2-7^2}\\ &=24\\ \cos A&=-\frac{24}{25}\end{aligned}$$

Answer ……$-\frac{24}{25}$……..… [2]

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8 The time, t seconds, taken to download a file from a computer is inversely proportional to the internet connect speed, v Mbps. When the speed is 250 Mbps, the time taken is 4 seconds.

(a) Sketch the graph of t against v in the axes below. [1]

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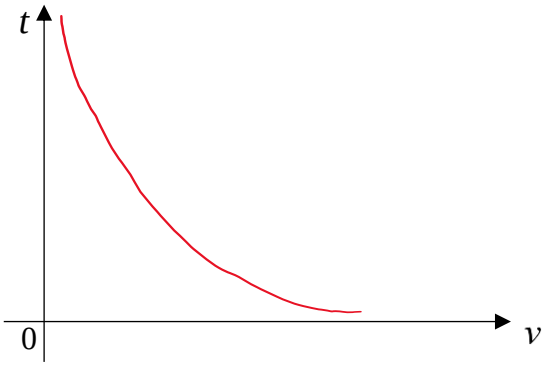


(b) Find the time taken to download a file when the speed is 160 Mbps.

$$t = \frac{k}{v}$$
$$k = 4 \times 250$$
$$= 1000$$
$$t = \frac{1000}{v}$$
$$= \frac{1000}{160}$$
$$= 6.25$$

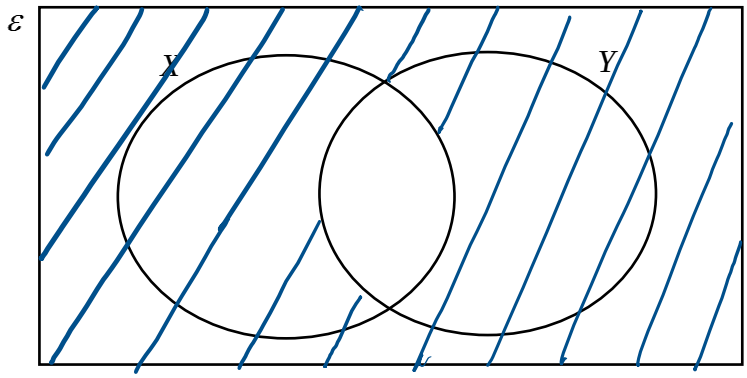
Answer ……6.25……. sec [2]

(c) If the speed is increased by 25%, find the percentage change in the time taken.

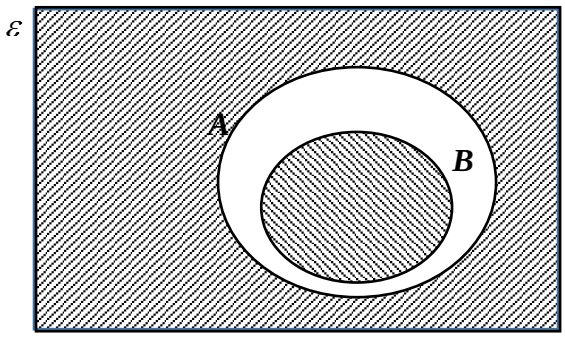
$$t = \frac{k}{v}$$
$$t_1 = \frac{4k}{5v}$$
$$t_1 = \frac{4}{5}t$$
$$\text{Percentage change} = -\frac{1}{5} \times 100\%$$
$$= -20\%$$

Answer ……–20……% [2]

9 **(a)** On the Venn Diagram, shade the the region which represents $X' \cup Y'$. [1]



(b)



From the Venn diagram above,

(i) write, in set notation that illustrates the relationship between A and B.

Answer $B \subset A$ [1]

(ii) write down the set notation that represents the shaded region.

Answer …… $A' \cup B$ ……. [1]

---

10 It is given that $\xi = \{x : x \text{ is an integer and } 0 < x \leq 15\}$,

$P = \{x : x \text{ is a prime number}\}$ and

$Q = \{x : x \text{ is divisible by 3 or 5}\}$.

List the elements in $P' \cap Q$.

$P' = \{1, 4, 6, 8, 9, 10, 12, 14, 15\}$

$Q = \{3, 5, 6, 9, 10, 12, 15\}$

$P' \cap Q = \{6, 9, 10, 12, 15\}$

Answer {……6, 9, 10, 12, 15……} [2]

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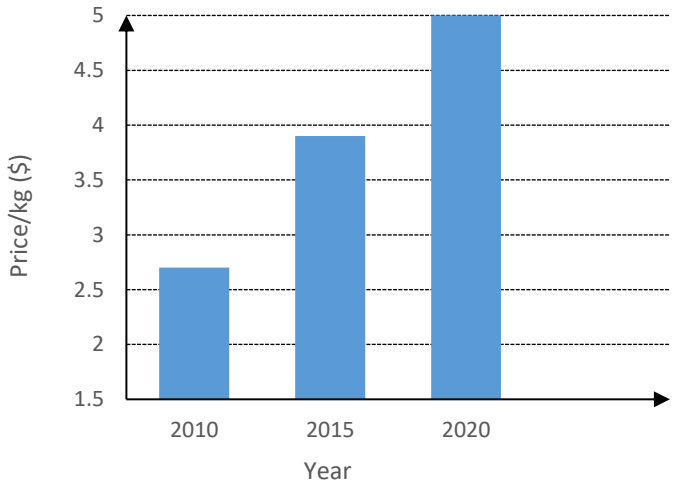
11 The graph shows the price of 1 kg of apples in Country *X* from 2010 – 2020.



(a) State the misleading feature of the graph.

Answer The **vertical axis** does not start at zero [1]

(b) Explain how this feature affects the reader's interpretation of the graph.

Answer

The graph misleads the reader thinking that the price of apples in 2020 is thrice that of 2010's when it is only about twice. [1]

12 Consider the pattern,

$$L_1: 3^2 - 2^2 = 3 + 2 = 5$$
$$L_2: 4^2 - 3^2 = 4 + 3 = 7$$
$$L_3: 5^2 - 4^2 = 5 + 4 = 9$$
$$L_4: 6^2 - 5^2 = 6 + 5 = 11$$
$$\vdots \quad \vdots \quad \vdots$$
$$\vdots \quad \vdots \quad \vdots$$
$$L_n: \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

(i) Write down L_n of the pattern. [1]

$L_n : (n+2)^2 - (n+1)^2 = n+2+n+1 = 2n+3$

(ii) Explain, with calculations, why 1137 is a term of the sequence 5, 7, 9, 11, …. [2]

$2n+3 = 1137$

$n = 567$

***n* is an integer**. Hence 1137 is the 567^{th} term of the sequence.

(iii) Find the exact value of $L_1 + L_2 + L_3 + \ldots + L_{99}$.

$101^2 - 2^2 = 10197$

Answer ……10197……[1]

13 The numbers 720 and 1575, written as the products of their prime factors are $720 = 2^4 \times 3^2 \times 5$ and $1575 = 3^2 \times 5^2 \times 7$.

(i) Find the largest common factor of both 720 and 1575.

HCF = $3^2 \times 5 = 45$

Answer ……45……[1]

(ii) Find the smallest whole number that is divisible by both 720 and 1575.

LCM = $2^4 \times 3^2 \times 5^2 \times 7 = 25200$

Answer ……25200……[1]

(iii) When the product of 720 and 1575 is divided by p, the result is a perfect square.

Find the smallest integer value of p, where $p > 0$.

$$\frac{2^4 \times 3^2 \times 5 \times 3^2 \times 5^2 \times 7}{p} = \frac{2^4 \times 3^4 \times 5^3 \times 7}{p}$$

$\therefore p = 5 \times 7 = 35$

Answer p = ……35…… [1]

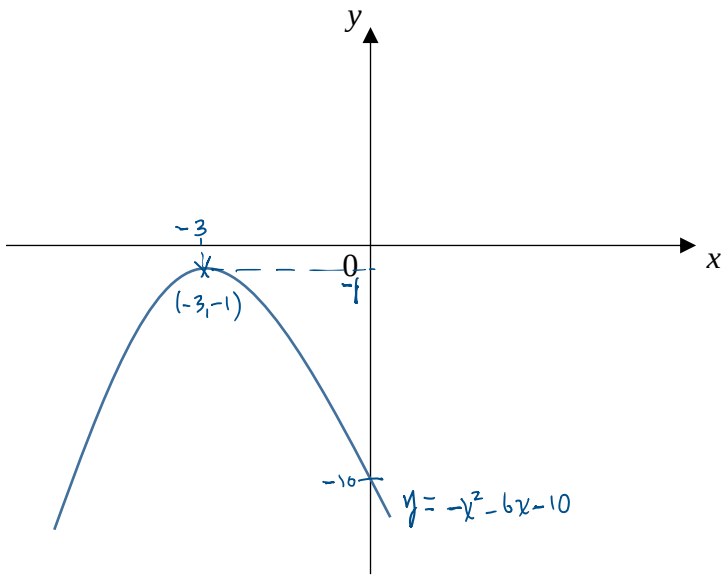
14 **(i)** Express $-x^2-6x-10$ in the form $a(x-p)^2+q$, where a, p and q are integers

$$\begin{aligned}-x^2-6x-10 &= -\left(x^2+6x\right)^2-10\\ &= -\left[(x+3)^2-9\right]-10\\ &= -(x+3)^2+9-10\\ &= -(x+3)^2-1\end{aligned}$$

Answer $\ldots\ldots -(x+3)^2-1\ldots\ldots$ [2]

(ii) Sketch the graph of $y=-x^2-6x-10$ on the axes below.

Indicate clearly the coordinates of the point where the graph crosses the y-axis and the turning point. [1]



(iii) Hence, explain, without solving, why there are no solutions of x when $-x^2-6x=10$.

Answer $-x^2-6x-10=0$

There are no solutions as the curve does not intersect the x-axis as shown above. [1]

15 The following stem-and-leaf diagram shows the duration of each of the 20 calls, in minutes, made by Tammy using her mobile phone on a certain day.

Stem	Leaves
1	3 4 4 5 7 9
2	0 1 2 5 6 6 7 9
3	1
4	4 6
5	1
6	2 *R*

Key: 1 | 3 means 13 minutes

(i) Find the percentage of calls that are shorter than 31 minutes.

Percentage

$= \frac{14}{20} \times 100\%$

$= 70\%$

Answer ……70…% [1]

(ii) Find the median call duration.

median call duration

$= \frac{25 + 26}{2}$

$= 25.5$ min

Answer ……25.5……min [1]

(iii) Given that the range of the duration of the calls made is 52 minutes.

Find the value of *R*.

Answer ……5…… [1]

16 The table below summarises the number of spam calls received by 80 elderly males of a community club in a month.

No. of spam calls	$5 \le n < 10$	$10 \le n < 15$	$15 \le n < 20$	$20 \le n < 25$
Frequency	30	24	15	11

(i) Calculate an estimate for

(a) the mean number of spam calls received,

Answer ……12.9375…… [1]

(b) the standard deviation.

Answer ……5.26 (3sf)…… [1]

(ii) The survey is also conducted for 80 elderly females at the community club. The mean number of spam calls received is 16 calls and the standard deviation is 9 calls.

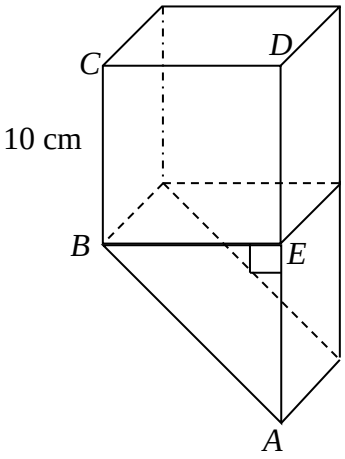
Make two comparisons between the results of the survey conducted for the elderly males and females.

Answer

The elderly **females received more spam calls** in the month as compared to the elderly males as they received a mean number of 16 calls.

However, the results from the elderly **males are more consistent as it has a standard deviation which is about 3.74 calls lower** than that of the elderly females. [2]

17 A container is made up of a cube and a right-angled triangular prism.
Each side of the cube is 10 cm and $EB = EA$.



(a) Find the volume of the container.

Volume of container

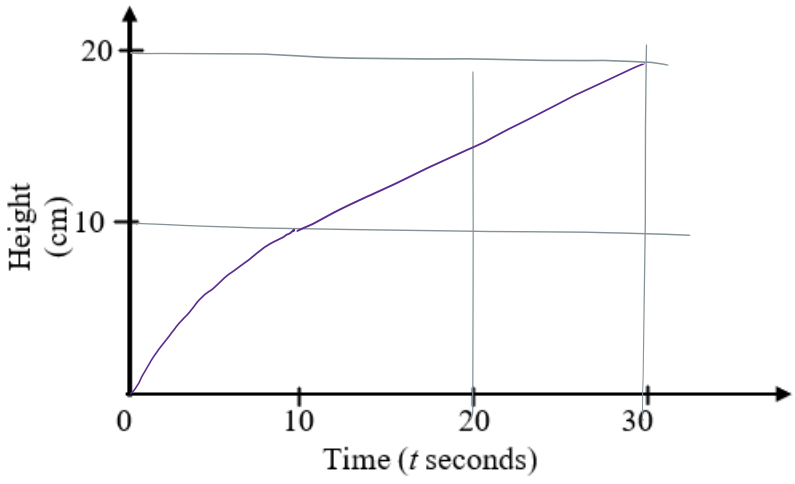
$$= \frac{1}{2}(10+20)(10)\times 10$$

$$= 1500 \text{ cm}^3$$

Answer ……1500…..…cm^3 [2]

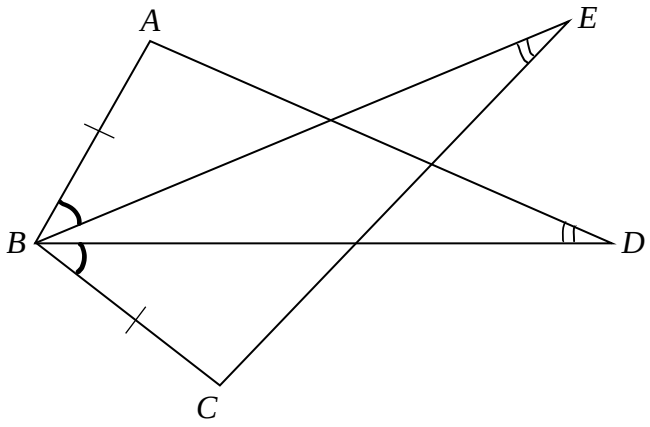
(b) Water is poured from the top at a constant rate and fills the container in 30 seconds.

Sketch the graph showing how the depth of the water varies with time. [1]



18 In the diagram, $BA = BC$, $\angle ABE = \angle DBC$ and $\angle BEC = \angle BDA$.

Prove that the triangles ABD and CBE are congruent.



Answer

$\angle ABE = \angle DBC$ (given)

$\angle ABD = \angle ABE + \angle EBD$

$= \angle CBD + \angle EBD$

$= \angle CBE$

$AB = CB$ (given)

$\angle BEC = \angle BDA$ (given)

$\therefore \Delta ABD \equiv \Delta CBE$ (AAS)

[3]

19 The points *A, B* and *C* are vertices of a triangle. The point *A* is $(2,\ 7)$ and the gradient of the line *AB* is -3.

(a) Find the equation of the line *AB*.

$y-7=-3(x-2)$

$y=-3x+6+7$

$y=-3x+13$

Answer …… $y=-3x+13$……..… [2]

(b) The equation of the line *BC* is $2x + y = 17$.

Find the coordinates of *B*.

$-3x+13=17-2x$

$-x=4$

$x=-4$

$y=17-2(-4)$

$=25$

$B(-4,\ 25)$

Answer ……(-4, 25)……..… [2]

(c) Find the length of *AB*.

$AB=\sqrt{(2+4)^2+(7-25)^2}$

$=18.9736...$

$=19.0$ units (3sf)

Answer ……19.0……units [2]

20 2 regular polygons *A* and *B* have *n* sides and 2*n* sides respectively.

The size of each interior angle in polygon *A* is 300% greater than the size of each exterior angle in polygon *B*.

(i) Find the number of sides polygon *A* has.

$$\frac{180(n-2)}{n} = 4\left(\frac{360}{2n}\right)$$
$$180(n-2) = 720$$
$$n-2 = 4$$
$$n = 6$$

Polygon *A* has 6 sides.

Answer..……6..…… [2]

(ii) Hence, find the size of an interior angle in *B*.

Size of interior angle in *B*

$$= \frac{180°(12-2)}{12}$$
$$= 150°$$

Answer..……150…… ° [1]

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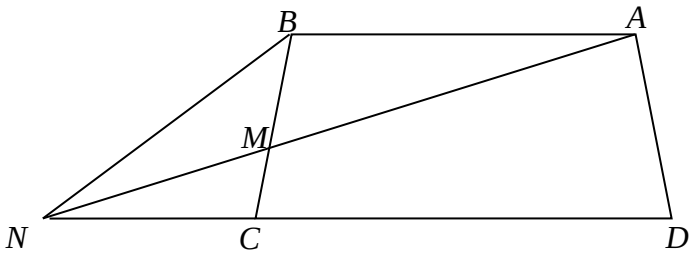
21 $ABCD$ is a trapezium, with side AB parallel to side DC. M is a point on BC such that $3CM = BC$. The lines DC and AM produced meets at N.



(a) Show that ΔABM and ΔNCM are similar. [2]

$\angle BAM = \angle CNM$ (alt.$\angle$s)

$\angle BMA = \angle CMN$ (vert. opp. $\angle$s)

$\therefore \Delta ABM$ is similar to ΔNCM

(b) Show that $2NC = AB$. [1]

Answer

Since ΔABM is similar to ΔNCM

$$\frac{NC}{AB} = \frac{CM}{MB} = \frac{1}{2}$$

Hence $2NC = AB$ (shown)

(c) Calculate the value of

(i) $\dfrac{\text{area of } \Delta ABM}{\text{area of } \Delta NCM}$,

$$\frac{\text{area of } \Delta ABM}{\text{area of } \Delta NCM} = \left(\frac{2}{1}\right)^2 = 4$$

Answer ……4…… [1]

(ii) $\dfrac{\text{area of } \Delta NBC}{\text{area of } \Delta NBA}$.

$$\frac{\text{area of } \Delta NBC}{\text{area of } \Delta NBA} = \frac{\text{area of } \Delta NBC}{\text{area of } \Delta BNM} \times \frac{\text{area of } \Delta BNM}{\text{area of } \Delta NBA}$$

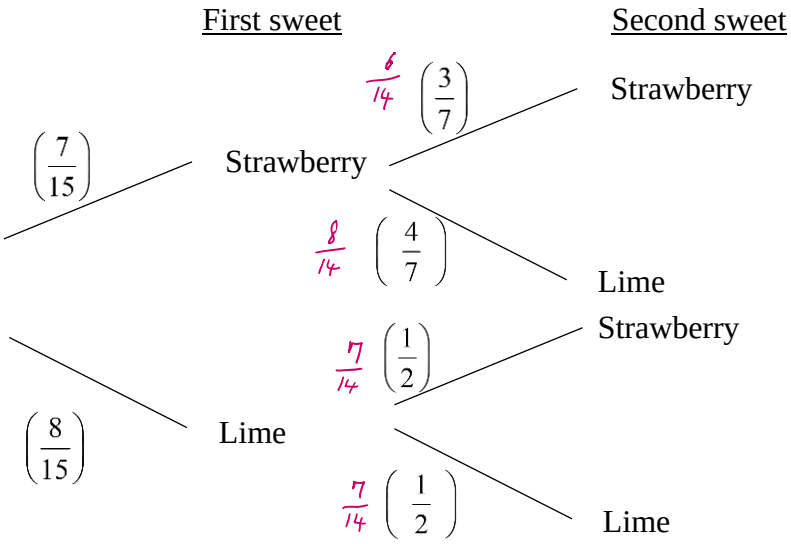
$$= \frac{3}{2} \times \frac{1}{3}$$

$$= \frac{1}{2}$$

Answer …… $\frac{1}{2}$ ……. [1]

22 A packet of 15 sweets contains 2 different flavours. There are 7 strawberry-flavoured sweets and 8 lime-flavoured sweets in the packet. Two sweets are taken from the packet at random, without replacement.

(i) Complete the tree diagram to show the probability of the possible outcomes. [2]



(ii) Hence, find the probability as a fraction in its simplest form, that

(a) both sweets drawn are the same flavours,

$$\begin{aligned}&\mathrm{P}(\text{same flavours})\\&=\frac{7}{15}\left(\frac{3}{7}\right)+\frac{8}{15}\left(\frac{1}{2}\right)\\&=\frac{7}{15}\end{aligned}$$

Answer ..... $\frac{7}{15}$ ..... [2]

(b) at least one sweet is strawberry-flavoured.

$$\begin{aligned}&\mathrm{P}(\text{at least one sweet is strawberry})\\&=1-\frac{8}{15}\left(\frac{1}{2}\right)\\&=\frac{11}{15}\end{aligned}$$

Answer ..... $\frac{11}{15}$ ..... [2]

23 Point A has the position vector $\begin{pmatrix}3\\7\end{pmatrix}$ and point B has position vector $\begin{pmatrix}0\\11\end{pmatrix}$.

(i) Calculate $\left|\overrightarrow{AB}\right|$.

$$\overrightarrow{AB} = \overrightarrow{DO} + \overrightarrow{OB}$$
$$= \begin{pmatrix}-3\\-7\end{pmatrix} + \begin{pmatrix}0\\11\end{pmatrix}$$
$$= \begin{pmatrix}-3\\4\end{pmatrix}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{(-3)^2 + 4^2}$$
$$= 5 \text{ units}$$

Answer ……5……units [2]

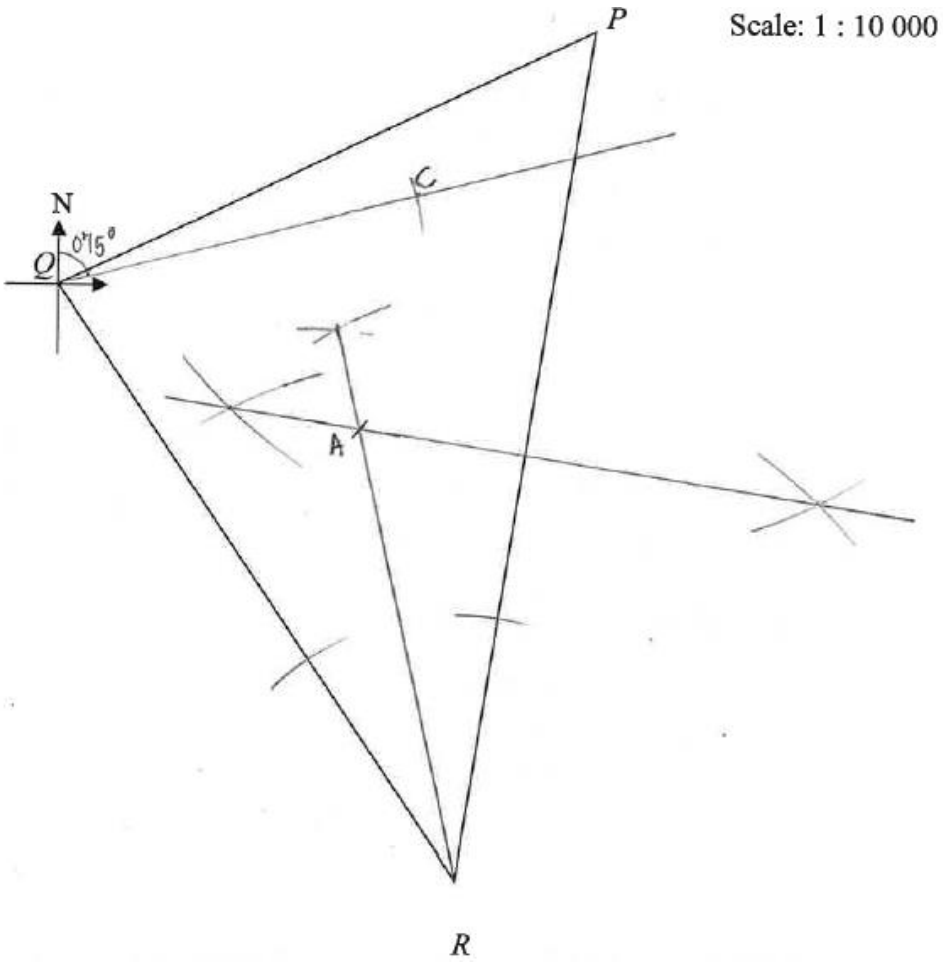
(ii) Given that the point C is $(-3,\ 15)$, using vectors, explain why B is the midpoint of AC. [3]

$$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$$
$$= \begin{pmatrix}-3\\-7\end{pmatrix} + \begin{pmatrix}-3\\15\end{pmatrix}$$
$$= \begin{pmatrix}-6\\8\end{pmatrix}$$

$$\overrightarrow{AB} = \begin{pmatrix}-3\\4\end{pmatrix}$$
$$= \frac{1}{2}\begin{pmatrix}-6\\8\end{pmatrix}$$
$$= \frac{1}{2}\overrightarrow{AC}$$

Since $\frac{AB}{AC} = \frac{1}{2}$, and A is a common point, A, B, C are collinear .
Hence B is the midpoint of AC.

24 In the scale drawing, triangle PQR represents the positions of three shops.



(a) A new shop is to be built at A where it is equidistant from the points P and R and equidistant from the line segments PR and QR.

(i) On the scale drawing, mark the point A. [2]

(ii) Find, in square metres, the actual area of triangle APR.

1 cm : 100 m

Size of ΔAPR

$$= \frac{1}{2}(2.3\times100)(11.7\times100)$$

$$= 134550 \text{ m}^2$$

$$127600 \leq x \text{ m}^2 \leq 141600$$

Answer ……134550……m^2 [2]

(b) A new community centre, C is to be built inside the triangle PQR such that the bearing of C is $075°$ and 500 m from Q.

Mark the point C on the diagram above. [1]

25 A car dealer priced a car at \$189 500 and offered the following 2 payment schemes:

Scheme A: Cash Payment with 10% discount.
Scheme B: 30% deposit and \$2519 monthly instalments for 5 years.

Peter bought the car under scheme B.

(a) Calculate the total amount of money Peter paid for the car.

$$\begin{aligned}\text{amt. paid} &= \frac{30}{100}(189500) + 2519(5)(12) \\ &= \$207990\end{aligned}$$

Answer \$……207990……..… [2]

(b) Peter claims that he had paid more buying the car using scheme B as compared to scheme A. Do you agree with Peter? Justify your answer.

$$\begin{aligned}\text{amt. (Scheme A)} &= \frac{90}{100}(189500) \\ &= \$170550 \\ \text{Difference} &= 207990 - 170550 \\ &= \$37440\end{aligned}$$

Answer

I **agree** with Peter because he would have spent \$37440 more as shown above.

[2]

(c) Calculate the simple interest rate per annum Peter was charged for scheme B.

$$\begin{aligned}\text{Total instament} &= \$(2519 \times 5 \times 12) \\ &= \$151140 \\ 151140 - \frac{70}{100}(189500) &= \frac{\frac{70(189500)}{100} \times R \times 5}{100} \\ \frac{18490 \times 100}{5 \times 132650} &= R \\ R &= 2.7877... = 2.79 \text{ (3sf)}\end{aligned}$$

Answer ……2.79……..…% [3]

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