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JURONGVILLE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2021
Secondary 4 Express

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| STUDENT NAME | |
| CLASS | |
| INDEX NUMBER | |

# ADDITIONAL MATHEMATICS 4049/02

Paper 2

**31 AUGUST 2021**

**2 hours 15 minutes**

Candidates answer on the Question Paper.
No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **ALL** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is 83.

**DO NOT OPEN THE BOOKLET UNTIL YOU ARE TOLD TO DO SO**

| For Examiner's Use |
|---|
| 83 |

Setter: Mr Billy Chew

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for* $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

**1** Represent the solution set of $(k + 2)^2 > 13k - 16$ on a number line. [4]

**2** The equation of a curve is $y=3\left(\frac{x}{4}+a\right)^{\frac{5}{6}}$. The normal to the curve at $x=\frac{1}{2}$ is parallel to the line $5y+4x=2$.

**(a)** Show that $a=-\frac{7}{64}$. [4]

**(b)** Find the equation of the tangent to the curve at $x=\frac{1}{2}$. [3]

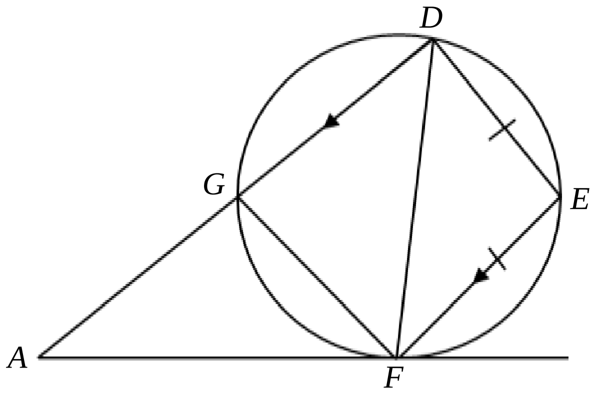
**3** The binomial expansion of $(1-3x)^n$, where $n>0$, in ascending powers of $x$ is [5]

$$1-6px+54x^2+qx^3+\dots.$$

**(a)** Find the value of $n$, $p$ and $q$.

**(b)** Find the coefficient of $x^3$ in $(1-3x)^n(1+2x)$ [2]

**4**



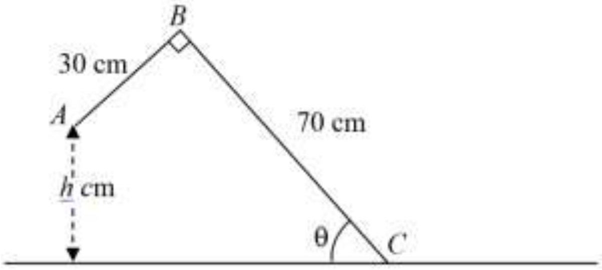

In the diagram, *DEFG* is a cyclic quadrilateral in which *DE* = *FE* and *DG* is parallel to *EF*.

The tangent to the circle at *F* meets *DG* produced at *A*.

**(a)** Show that angle *GFA* = angle *EDF*. [3]

**(b)** Show that triangle *FGA* is isosceles. [4]

**5** The diagram shows two rods, *AB* and *BC*, of lengths 30 cm and 70 cm respectively. The rods are fixed at *B* such that angle *ABC* = 90° and hinged at *C* so as to rotate in a vertical plane. The rod *BC* makes an acute angle $\theta$ with horizontal ground.
The height of *A* above the ground is *h* cm.




**(a)** Show that the height, *h* cm, can be expressed in the form $p\sin\theta - q\cos\theta$ , where *p* and *q* are constants to be found. [3]

**(b)** Express *h* in the form $R\sin(\theta-\alpha)$, where $R > 0$ and $\alpha$ is an acute angle. [3]

**(c)** Find the value of $\theta$ for which $h = 20$ cm. [2]

**6** The table below shows some values of $x$ and $y$.

| $x$ | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| $y$ | 5.3 | 12.3 | 27.3 | 61.5 |

**(a)** On the grid provided, plot ln $y$ against $x$ and draw a straight line graph [3]

**(b)** Find the gradient of your straight line and hence express $y$ in the form of $ab^x$, where $a$ and $b$ are constants. [4]

**(c)** By drawing a suitable line on your graph, solve the equation $ab^x = e^{0.7x}$ [2]

**7** **(a)** Find the range of values of $c$ for which the curve $y = x(5x - 12)$ does not intersect the line $y = 2x - c$. [3]

**(b)** By expressing $x^2 - 4x + 7$ in the form of $(x - h)^2 + k$, where $h$ and $k$ are constants, explain why $x^2 - 4x + 7$ is always greater than or equal to 3. [3]

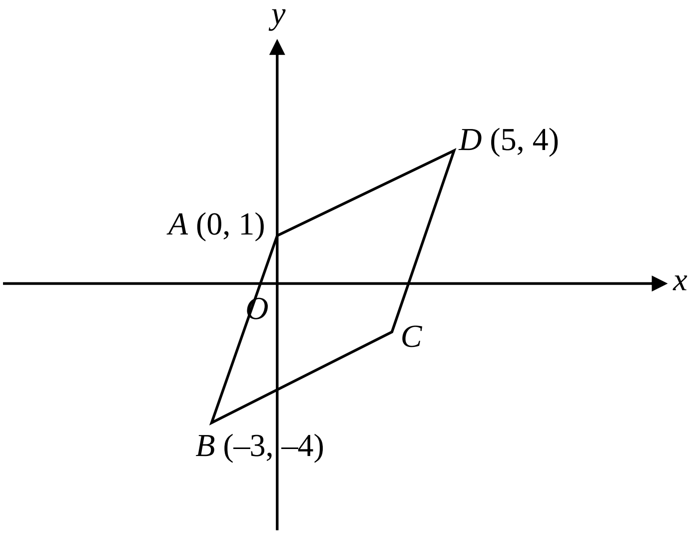
**(c)** Solve the simultaneous equations [3]

$$y = 2x^2 - 6x - 4$$
$$y + 2x = 12$$

**8** **(a)** Solve $\log_2 x^2 - 3\log_x 4 = \log_4 2 \times \log_4 16$. [5]

**(b)** Solve the equation $64^{x-1} \times 8^x = \sqrt[3]{16^{6x}}$. [4]

**9**




The vertices of the rhombus are at the points $A$ (0, 1), $B$ (−3, −4), $C$ and $D$ (5, 4).

**(a)** Find the equation of $AC$. [3]

**(b)** Find the area of *ABCD*. [4]

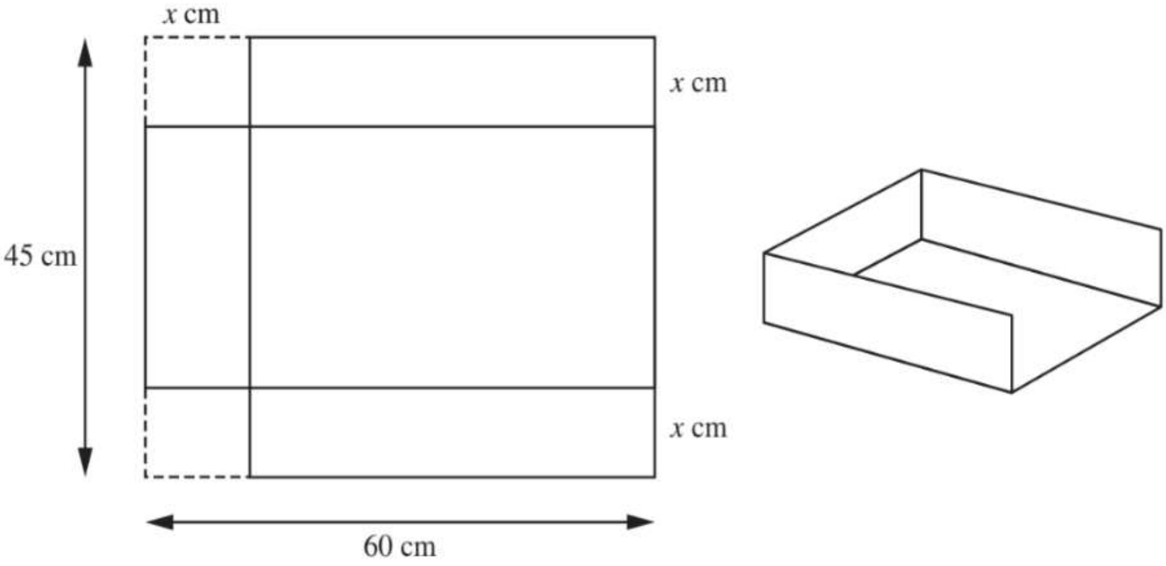
**(c)** The equation of line $p$ is $-5x + 3y = 7$. [2]
Show that the line $p$ does **not** intersect the line *AB*.

**10** **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x\cos^2 x) = \cos^2 x - 2x\cos x\sin x$. [3]

**(b)** Hence, find $\int x\sin 2x \, \mathrm{d}x$. [5]

**(c)** Find the value of $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} x \sin 2x \ dx$ [3]

**11**




A manufacturer uses a 60 cm by 45 cm of rectangular sheet of metal to make a scoop. The scoop is made by cutting out squares, of side $x$ cm, from the corners of the sheet and folding the remainder as shown.

**(a)** Show that the volume, $V$ cm$^3$, of the scoop is given by $V = 2x^3 - 165x^2 + 2700x$. [3]

**(b)** Given that $x$ can vary, find the value of $x$ which gives a stationary value of $V$. [4]

**(c)** Determine the nature of this stationary value and hence explain whether the manufacturer will be satisfied with the product. [3]

**End of Paper**