XINMIN SECONDARY SCHOOL
新民中学
SEKOLAH MENENGAH XINMIN
Preliminary Examinations 2022

CANDIDATE NAME: Mark Scheme (Draft 3)

CLASS

INDEX NUMBER

# ADDITIONAL MATHEMATICS

4049/01

**Paper 1**

29 August 2022

2 hour 15 minutes

Secondary 4 Express/ 5 Normal Academic
Setter : Mr Johnson Chua
Vetter : Mrs Loh Si Lan
Moderator: Ms Pang Hui Chin

Candidates answer on the Question Paper.
No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the paper, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **90.**

| Errors | Qn No. | Errors | Qn No. |
|---|---|---|---|
| Accuracy | | Simplification | |
| Brackets | | Units | |
| Geometry | | Marks Awarded | |
| Presentation | | Marks Penalised | |

| For Examiner's Use |
|---|
| 90 |

Parent's/Guardian's Signature:

This document consists of **22** printed pages.

[Turn over

1 Solve the equation $\sqrt{2x^2-5x+3}-2x=0$. [3]

$2x^2-5x+3=4x^2$ — (M1)

$2x^2+5x-3=0$

$x=\dfrac{-5\pm\sqrt{5^2-4(2)(-3)}}{2(2)}$

$= \underbrace{-3 \text{ (rej).}}_{\text{(A1)}}$ or $\underbrace{\tfrac{1}{2}}_{\text{(A1)}}$

2 (a) Express $3x^2-24x+53$ in the form $a(x+b)^2+c$ where $a$, $b$ and $c$ are constants. [2]

$3(x^2-8x)+53$

$=3(x^2-8x+4^2)+53-3(4)^2$ — (M1)

$=3(x-4)^2+5$ — (A1)

Alternative: $3(x^2-8x+\frac{53}{3})$

$=3[(x-\frac{8}{2})^2-(\frac{8}{2})^2+\frac{53}{3}]$ — (M1)

$=3[(x-4)^2+1\frac{2}{3}]$

$=3(x-4)^2+5$ — (A1)

(b) Hence explain why there is no intersection between the graph $y=3x^2-24x+53$ and the line $y=4$. [1]

Min. pt of graph $y=3x^2-24x+53$ is $(4,5)$, which is above the line $y=4$. Hence, there will be no intersection.

Also accept: Since the min. val. of the graph is 5, which is more than $y=4$, there is no intersection between the curve and the line.

3 Given that $f'(x) = \dfrac{6}{3x+5}$ and that $f(x)$ passes through the origin, find the equation of $f(x)$.

[3]

$$f(x) = \int \frac{6}{3x+5}\,dx$$

$$= \frac{6 \ln(3x+5)}{3} + c \quad \text{— (M1)}$$

$$= 2\ln(3x+5) + c$$

Since f(x) passes through the origin,

$$\underbrace{0 = 2\ln 5 + c}_{\text{(M1)}} \Rightarrow c = -2\ln 5.$$

$$\therefore f(x) = 2\ln(3x+5) - 2\ln 5.$$

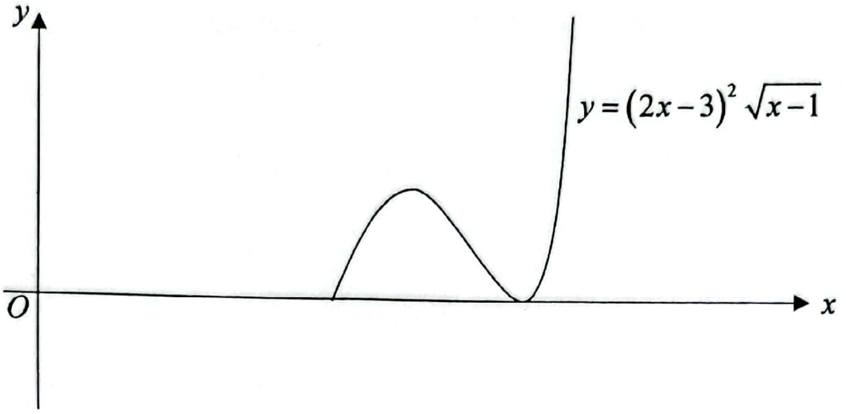
$$= 2[\ln(3x+5) - \ln 5]$$

$$= 2\ln\left(\frac{3x+5}{5}\right)$$

$$= \ln\left(\frac{3x+5}{5}\right)^2$$

(A1, accept either).

4




The diagram shows part of the graph $y=(2x-3)^2\sqrt{x-1}$.

Find the range of values of $x$ for which $y$ is increasing. [5]

$$\frac{dy}{dx} = 2(2x-3)(2)\sqrt{x-1} + (2x-3)^2\left(\tfrac{1}{2}\right)(x-1)^{-\frac{1}{2}} \quad \text{— (M1)}$$

$$= 4(2x-3)\sqrt{x-1} + \frac{(2x-3)^2}{2\sqrt{x-1}}$$

$$= \frac{8(2x-3)(x-1)+(2x-3)^2}{2\sqrt{x-1}}$$

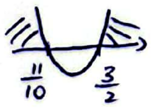
$$\frac{8(2x-3)(x-1)+(2x-3)^2}{2\sqrt{x-1}} > 0 \quad \text{— (M1)}$$

$8(2x-3)(x-1)+(2x-3)^2 > 0$ and $2\sqrt{x-1} > 0$.

$(2x-3)(8x-8+2x-3) > 0$ $\qquad \sqrt{x-1} > 0$

$\qquad\qquad\qquad\qquad\qquad\qquad x-1 > 0$

$(2x-3)(10x-11) > 0$ $\qquad \underbrace{x > 1}_{\text{(M1)}}$




$x < \frac{11}{10}$ and $x > \frac{3}{2}$

Since $x > 1$,

$\underbrace{1 < x < \frac{11}{10}}_{\text{(A1)}}$ and $\underbrace{x > \frac{3}{2}}_{\text{(A1)}}$

only

Alternative

$x$-int. of graph:

$(2x-3)^2\sqrt{x-1} = 0$ — (M1)

$(2x-3)^2 = 0$ or $\sqrt{x-1} = 0$

$2x-3 = 0$ $\qquad x-1 = 0$

$x = \frac{3}{2}$ $\qquad x = 1$

$$\frac{dy}{dx} = \underbrace{2(2x-3)(2)\sqrt{x-1} + (2x-3)^2\left(\tfrac{1}{2}\right)(x-1)^{-\frac{1}{2}}}_{\text{(M1)}}$$

$\vdots$

$$= \frac{8(2x-3)(x-1)+(2x-3)^2}{2\sqrt{x-1}}$$

$$\frac{8(2x-3)(x-1)+(2x-3)^2}{2\sqrt{x-1}} = 0 \quad \text{— (M1)}$$

$8(2x-3)(x-1)+(2x-3)^2 = 0$.

$(2x-3)(8x-8+2x-3) = 0$

$2x-3=0$ or $10x-11=0$.

$x = \frac{3}{2}$ or $\frac{11}{10}$

$\therefore$ Ans: $\underbrace{1 < x < \frac{11}{10}}_{\text{(A1)}}$ and $\underbrace{x > \frac{3}{2}}_{\text{(A1)}}$

**5** **(a)** Solve $3^{2x-1} = 4^{2-x}$ and show that $x = \dfrac{\lg 48}{2\lg 6}$. [3]

$3^{2x}(3^{-1}) = 4^2(4^{-x})$ — (M1)

$\dfrac{3^{2x}}{3} = \dfrac{16}{4^x}$

$9^x(4^x) = 16(3)$

$36^x = 48$ — (M1)

$\lg 36^x = \lg 48$

$x \lg 36 = \lg 48$

$x = \dfrac{\lg 48}{\lg 36}$

$= \dfrac{\lg 48}{2\lg 6}$ — (A1)

**(b)** In order to obtain a graphical solution of the equation $x = \ln\left(\dfrac{7-x}{5}\right)^2$, a suitable straight line can be drawn on the same axes as the graph $y = 5e^{\frac{x}{2}} - 4$. Find the equation of this line. [2]

$x = 2\ln\left(\frac{7-x}{5}\right)$

$\frac{x}{2} = \ln\left(\frac{7-x}{5}\right)$

$e^{\frac{x}{2}} = \frac{7-x}{5}$ — (M1)

$5e^{\frac{x}{2}} = 7-x$

$5e^{\frac{x}{2}} - 4 = 7-x-4$

$5e^{\frac{x}{2}} - 4 = 3-x$

$\therefore$ Equation: $y = 3-x$. — (A1)

6 Solve the equation $3\sin A = \operatorname{cosec} A - 5\cot A$ for $-180° < A < 180°$. [5]

$$3\sin A = \frac{1}{\sin A} - \frac{5\cos A}{\sin A} \quad \text{— (M1)}$$

$$3\sin^2 A = 1 - 5\cos A$$

$$3\sin^2 A + 5\cos A - 1 = 0$$

$$3(1 - \cos^2 A) + 5\cos A - 1 = 0. \quad \text{— (M1)}$$

$$3 - 3\cos^2 A + 5\cos A - 1 = 0.$$

$$-3\cos^2 A + 5\cos A + 2 = 0$$

$$3\cos^2 A - 5\cos A - 2 = 0$$

$$(\cos A - 2)(3\cos A + 1) = 0 \quad \text{— (M1)}$$

$\cos A = 2$ or $\cos A = -\frac{1}{3}$

$$\text{B.A}: \cos^{-1}\left(\frac{1}{3}\right)$$

$$= 70.529°$$

$$A = 180° - 70.529°, -180° + 70.529°$$

$$= \underbrace{109.5°}_{\text{(A1)}}, \underbrace{-109.5°}_{\text{(A1)}} \text{ (1dp)}$$

7 Express $\dfrac{3x^3+x^2+13x+8}{x\left(x^2+4\right)}$ in partial fractions. [6]

$$= 3 + \frac{x^2+x+8}{x(x^2+4)} \quad \text{— (M1)}$$

$$\frac{x^2+x+8}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4} \quad \text{— (M1)}$$

$$x^2+x+8 = A(x^2+4) + (Bx+C)x$$

when $x=0$,

$$8 = A(4) \Rightarrow A = 2$$

$$x^2+x+8 = 2(x^2+4) + Bx^2 + Cx$$

$$= (2+B)x^2 + Cx + 8$$

By comparing coefficient,

$$2+B = 1 \Rightarrow B = -1$$

$$C = 1.$$

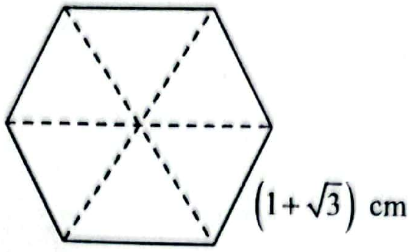
M1 for applying either subst. or comparing coeff. method.

B1: for at least 2 correct,

B2: for all correct.

$$\therefore \frac{3x^3+x^2+13x+8}{x(x^2+4)} = 3 + \frac{2}{x} + \frac{1-x}{x^2+4} \quad \text{— (A1)}$$

**8** The diagram below shows the base of a hexagonal pyramid.
The base is a regular hexagon, made up of six equilateral triangles of sides $\left(1+\sqrt{3}\right)$ cm.




**(a)** Show that the area of the hexagon is $\left(6\sqrt{3}+9\right)$ cm$^2$. [3]

$6 \times \frac{1}{2}(1+\sqrt{3})^2 \sin 60^\circ$ — (M1)

$= 3(4+2\sqrt{3})\left(\frac{\sqrt{3}}{2}\right)$ — (M1: $\sin 60^\circ = \frac{\sqrt{3}}{2}$)

$= 3\sqrt{3}(2+\sqrt{3})$

$= 6\sqrt{3}+9$ — (A1)

Alternative

⊥ height of Δ: $\sqrt{(1+\sqrt{3})^2 - \left(\frac{1+\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{3}{4}(1+\sqrt{3})^2}$

Area of 1Δ: $\frac{1}{2} \times (1+\sqrt{3}) \times \sqrt{\frac{3}{4}(1+\sqrt{3})^2}$ — (M1)

$= \frac{1+\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}(1+\sqrt{3})$

Area of Hexagon: $6 \times \frac{\sqrt{3}}{2} \cdot \frac{(1+\sqrt{3})^2}{2}$

$= \underbrace{\frac{3\sqrt{3}(1+\sqrt{3})^2}{2}}_{(M1)} = \frac{3\sqrt{3}(4+2\sqrt{3})}{2}$

$= 6\sqrt{3}+9$ — (A1)

**(b)** Given that the volume of the hexagonal pyramid is $\left(9+4\sqrt{3}\right)$ cm$^3$, find, the exact height of the hexagonal pyramid.
(Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height) [3]

$9+4\sqrt{3} = \frac{1}{3}(6\sqrt{3}+9) \times h$

$h = \frac{27+12\sqrt{3}}{6\sqrt{3}+9}$

$= \frac{(27+12\sqrt{3})(6\sqrt{3}-9)}{(6\sqrt{3})^2 - 81}$ — (M1 for multiplying conjugate base)

$= \frac{162\sqrt{3} - 243 + 216 - 108\sqrt{3}}{27}$ — (M1 for correct exp. of numerator)

$= \frac{54\sqrt{3} - 27}{27}$

$= \frac{27(2\sqrt{3}-1)}{27} = 2\sqrt{3} - 1$ — (A1)

**9** The function $f(x) = 2x^3 + ax^2 + bx + 2$, where $a$ and $b$ are constants, is exactly divisible by $(2x-1)$. Given that $f(x)$ leaves a remainder of 9 when divided by $(x+1)$,

**(a)** find the value of $a$ and $b$. [4]

$f(\frac{1}{2}) = 0$

$2(\frac{1}{2})^3 + a(\frac{1}{2})^2 + b(\frac{1}{2}) + 2 = 0$ — (M1)

$\frac{1}{4} + \frac{a}{4} + \frac{b}{2} + 2 = 0$

$1 + a + 2b + 8 = 0$

$a + 2b = -9$ — ①

$f(-1) = 9$

$2(-1)^3 + a(-1)^2 + b(-1) + 2 = 9$ — (M1)

$-2 + a - b + 2 = 9$

$a - b = 9$ — ②

① − ②: $3b = -18$

$\underbrace{b = -6}_{(A1)}$

Sub $b = -6$ into ② : $a - (-6) = 9$

$\underbrace{a = 3}_{(A1)}$

**(b)** Using the value of $a$ and $b$ found in **(a)**, solve the equation
$2x^3 + ax^2 + bx + 2 = 0$. [4]

$2x^3 + 3x^2 - 6x + 2 = 0$.

$(2x-1)(x^2+2x-2) = 0$ — (M1)

$\underbrace{x = \frac{1}{2}}_{B1}$ or $x^2 + 2x - 2 = 0$

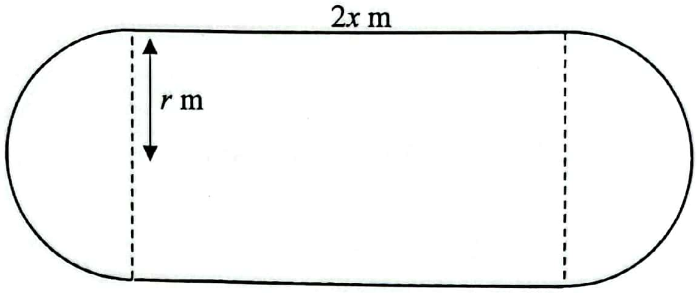
$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-2)}}{2}$ — (M1)

$= \frac{-2 \pm \sqrt{12}}{2}$

$= -1 \pm \sqrt{3}$

also accept : $-2.73$ or $0.732$ (3sf) } (A1)

**10** A gardener wants to use 40 m of fence to form a flower bed, which is made up of a rectangle of length $2x$ m and two identical semicircles of radius $r$ m.




**(a)** Express $r$ in terms of $x$. [2]

$2\pi r + 4x = 40$ — (M1)

$2\pi r = 40 - 4x$

$r = \dfrac{40 - 40x}{2\pi}$

$= \dfrac{20 - 2x}{\pi}$ — (A1)

**(b)** Show that the total area, $A$ m$^2$, of the flowerbed is given by

$$A = \frac{400 - 4x^2}{\pi}.$$ [2]

$A = \pi r^2 + 2x(2r)$

$= \pi\left(\dfrac{20-2x}{\pi}\right)^2 + 2x(2)\left(\dfrac{20-2x}{\pi}\right)$ — (M1)

$= \pi\left(\dfrac{400 - 80x + 4x^2}{\pi^2}\right) + \dfrac{4x(20-2x)}{\pi}$

$= \dfrac{400 - 80x + 4x^2 + 80x - 8x^2}{\pi}$

$= \dfrac{400 - 4x^2}{\pi}$ — (A1)

**(c)** Given that $x$ can vary, find the value of $x$ which gives a stationary value of $A$. [2]

$$\frac{dA}{dx} = -\frac{4}{\pi}(2x)$$
$$= -\frac{8x}{\pi} \quad \text{— (M1)}$$

$$-\frac{8x}{\pi} = 0 \Rightarrow x = 0. \text{ — (A1)}$$

**(d)** The gardener's wife claimed that he can optimise the length of the fencing to obtain a maximum area by forming only a circular flowerbed of radius $\frac{20}{\pi}$ m.

Do you agree with her? Explain your answer with relevant workings. [2]

$$\frac{d^2A}{dx^2} = -\frac{8}{\pi} \ (<0)$$

Since $\frac{d^2A}{dx^2} < 0$, area is a max. } (M1).

When $x = 0$, $r = \frac{20 - 2(0)}{\pi}$.

$$= \frac{20}{\pi}$$

Yes, I agree. Area is a max when $x = 0$ and $r = \frac{20}{\pi}$. } (A1).

---

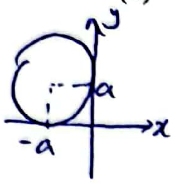
Alternative: Given $A = \frac{400 - 4x^2}{\pi}$, $A$ is max when $4x^2 = 0$.
$\therefore x = 0$.
When $x = 0$, shape of flowerbed is circular. } (M1)

$\therefore A = \pi r^2$

$$\frac{400}{\pi} = \pi r^2$$
$$r^2 = \frac{400}{\pi^2}$$
$$r = \pm\sqrt{\frac{400}{\pi^2}} = \frac{20}{\pi} \text{ or } -\frac{20}{\pi} \text{ (rej)}$$ } (A1)

**11** The negative $x$-axis and positive $y$-axis are tangents to circle $C$. Given that the radius of the circle is $a$ units and that centre of circle is $(-a, a)$,

**(a)** write down the equation(s) of the tangent(s) to the circle. [2]



$x = 0$ — (B1)

$y = 0$ — (B1)

*do not accept if students state $x$-axis & $y$-axis.

$P(-1,8)$ and $Q(-8,25)$ lie on the circumference of circle $C$.

**(b)** Find the equation of the perpendicular bisector of $PQ$. [3]

$m_{PQ} = \frac{25-8}{-8+1} = \frac{17}{-7}$.

$m_{\perp \text{ bisector}}: -1 \div \left(\frac{17}{-7}\right) = \frac{7}{17}$. — (M1)

midpt of $PQ$: $\left(\frac{-1+(-8)}{2}, \frac{8+25}{2}\right) = \left(-\frac{9}{2}, \frac{33}{2}\right)$

Eqn of $\perp$ bisector: $y - \frac{33}{2} = \frac{7}{17}\left(x + \frac{9}{2}\right)$ — (M1)

$y = \frac{7}{17}x + \frac{312}{17}$

$17y = 7x + 312$

} A1 for either.

**(c)** Show that the equation of the circle is $x^2+26x+y^2-26y+169=0$. [3]

Centre of circle lies on : $y=-x$ — ①

$y=\frac{7}{17}x+\frac{312}{17}$ — ②

Sub ① into ②: $-x=\frac{7}{17}x+\frac{312}{17}$ — (M1)

$-17x=7x+312$

$-24x=312$

$x=-13$

$\therefore y=13$.

Alternative

$\perp$ bisector of PQ passes through centre, $(-a,a)$

Sub $(-a,a)$ into $17y=7x+312$

$17a=-7a+312$ — (M1)

$24a=312$

$a=13$.

$\therefore$ Centre : $(-13, 13)$
radius : 13 units. } M1 for either.

$\therefore (x+13)^2+(y-13)^2=13^2$

$x^2+26x+169+y^2-26y+169=169$

$x^2+26x+y^2-26y+169=0$ — (A1, provided std. form is correct)

**(d)** Explain why $R(-14,27)$ lies outside the circle. [2]

Centre $(-13,13)$

$R(-14,27)$

Dist. b/w R and centre: $\sqrt{(-14+13)^2+(27-13)^2}$ — (M1)

$=14.035\ (>13)$

Since dist. b/w R and centre is greater than radius of circle, pt. R lies outside the circle. } (A1)

**12** **(a)** Find $\frac{\mathrm{d}}{\mathrm{d}x}(x\sin 2x)$ . [1]

$= \sin 2x + x(\cos 2x)(2)$

$= \sin 2x + 2x\cos 2x$ — (B1)

**(b)** Hence, find $\int 2x\cos 2x \, \mathrm{d}x$ . [1]

$\int \sin 2x + 2x\cos 2x \, dx = x\sin 2x + C$

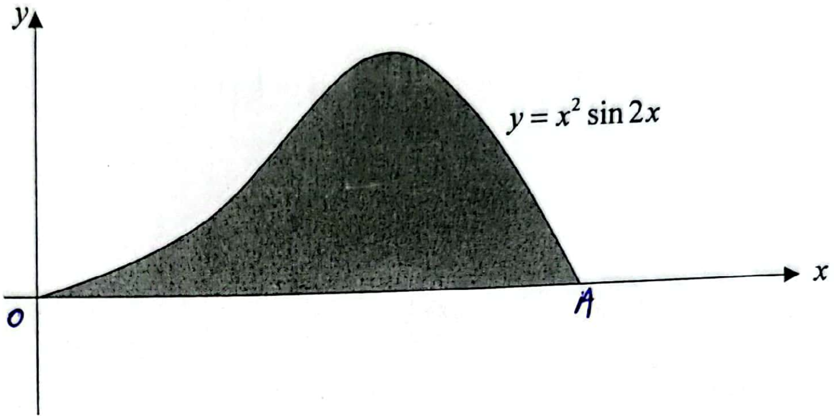
$\int 2x\cos 2x \, dx = x\sin 2x - \int \sin 2x \, dx + c$

$= x\sin 2x - \left(\frac{-\cos 2x}{2}\right) + d$

$= x\sin 2x + \frac{1}{2}\cos 2x + d$ — (A1)

Diagram below shows the graph of $y=x^2\sin 2x$, which passes through the origin and $A$.

(c) Differentiate $x^2\cos 2x$ with respect to $x$, and using the result found in part **(b)**, find the area of the shaded region, which is bounded by the curve $y=x^2\sin 2x$ and the $x$–axis.




[6]

$$\frac{d}{dx}\left(x^2\cos 2x\right) = 2x\cos 2x + x^2(-\sin 2x)(2) \quad \text{— (M1)}$$

$$= 2x\cos 2x - 2x^2\sin 2x$$

$$x^2\sin 2x = 0 \quad \text{— (M1)}$$

$$x=0 \quad \text{or} \quad \sin 2x = 0$$

$$x = 0, \tfrac{\pi}{2}, \pi, \ldots$$

$$= 0, \tfrac{\pi}{2}.$$

$$\int_0^{\frac{\pi}{2}} 2x\cos 2x - 2x^2\sin 2x \, dx = \left[x^2\cos 2x\right]_0^{\frac{\pi}{2}}$$ — [M1 for limit, M1 for $\int \frac{d}{dx} x^2\cos 2x \, dx$]

$$-2\int_0^{\frac{\pi}{2}} x^2\sin 2x \, dx = \left[x^2\cos 2x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2x\cos 2x \, dx$$

$$= \left(\tfrac{\pi}{2}\right)^2(-1) - \left[x\sin 2x + \tfrac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$$

$$= -\frac{\pi^2}{4} - \left[\tfrac{1}{2}(-1) - \tfrac{1}{2}(1)\right]$$

$$= \underline{\underline{-\frac{\pi^2}{4} + 1}} \quad \text{— (M1 for either)}$$

$$\int_0^{\frac{\pi}{2}} x^2\sin 2x \, dx = -\tfrac{1}{2}\left(-\frac{\pi^2}{4} + 1\right)$$

$$= \frac{\pi^2}{8} - \frac{1}{2} \quad \text{(also accept: 0.734)} \quad \text{— (A1)}$$

13 A motorcycle is rode along a straight horizontal road. As it passes a point $A$, the brakes are applied and the motorcycle slows down, with its velocity halved as it passes $B$.
For the journey from $A$ to $B$, the displacement, $s$ m, of the motorcycle from $A$, $t$ seconds after passing $A$, is given by $s = 400\left(1 - e^{-\frac{t}{10}}\right)$.

**(a)** Find the exact time taken for the journey from $A$ to $B$. [4]

$S = 400 - 400e^{-\frac{t}{10}}$

$V = \frac{ds}{dt}$

$= -400\left(-\frac{1}{10}\right)e^{-\frac{t}{10}}$ — (M1)

$= 40e^{-\frac{t}{10}}$

at $t=0$, $V = 40$.

at B, $V = 20$.

$\therefore 40e^{-\frac{t}{10}} = 20$ — (M1)

$e^{-\frac{t}{10}} = \frac{1}{2}$

$-\frac{t}{10} = -\ln 2$

$t = 10\ln 2$ — (A1)

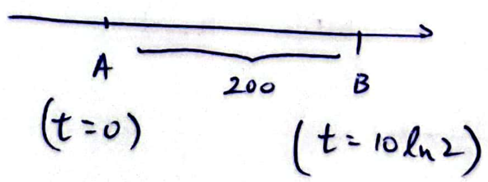
**(b)** Find the average speed of the motorcycle for the journey from $A$ to $B$. [2]

at $t = 10\ln 2$, $S = 400\left(1 - e^{-\frac{10\ln 2}{10}}\right)$

$= 400\left(1 - e^{-\ln 2}\right)$

$= 400\left(1 - \frac{1}{2}\right) = 200$ — (M1)

at $t=0$, $S = 400\left(1 - e^{0}\right) = 0$.




$\therefore$ total dist: 200m.

Average speed: $\frac{200\text{m}}{(10\ln 2)\text{s}} = 28.9 \text{ m/s}$ — (A1)
(3sf)

**(c)** Find the exact acceleration of the motorcycle at $B$. [2]

$a = \frac{dv}{dt}$

$= 40(-\frac{1}{10})e^{-\frac{t}{10}}$

$= -4e^{-\frac{t}{10}}$ — (M1)

at B, $a = -4e^{-\frac{1}{10}(10\ln 2)}$

$= -4(\frac{1}{2})$

$= -2$ — (A1)

Ans: $-2 m/s^2$

**14** **(a)** Show that $\dfrac{\cos(A+B)-\cos(A-B)}{\sin(A+B)-\sin(A-B)}=-\tan A$. [3]

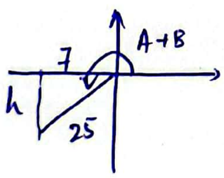
LHS: $\dfrac{\cos A\cos B-\sin A\sin B-(\cos A\cos B+\sin A\sin B)}{\sin A\cos B+\cos A\sin B-(\sin A\cos B-\cos A\sin B)}$ — M1 for applying addition formula correctly. — M1 for applying addition formula correctly

$= \dfrac{-2\sin A\sin B}{2\cos A\sin B}$

$= -\tan A$ } (A1)

**(b)** Given that $A$ and $B$ are both obtuse, and $\cos(A+B)=-\dfrac{7}{25}$, find the value of $\sin(A+B)$. [2]

$(A+B)$ lies in 3rd quadrant.




$h=\sqrt{25^2-7^2}$ — (M1)

$=24$

$\therefore \sin(A+B) = -\dfrac{24}{25}$ — (A1)

**(c)** **(i)** Given that $\sin(A-B)=\frac{8}{17}$ and $\cot(A-B)=\frac{15}{8}$, state the value of $\cos(A-B)$. [1]

$$\frac{\cos(A-B)}{\sin(A-B)} = \frac{15}{8}$$

$$\cos(A-B) = \frac{15}{8}(\sin(A-B))$$

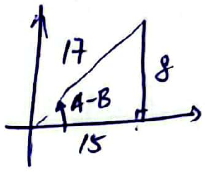
$$= \frac{15}{8}\left(\frac{8}{17}\right)$$

$$= \frac{15}{17} \quad \text{—(B1)}$$

<u>Alternative</u>

$$\tan(A-B) = \frac{8}{15}$$

Since $\sin(A-B)$ and $\tan(A-B)$ are +ve, $(A-B)$ lies in first quad.




$$\cos(A-B) = \frac{15}{17} \quad \text{—(B1)}$$

**(ii)** Explain why $A$ is larger than $B$. [1]

All trigo ratios of $(A-B)$ are positive.
This means that $(A-B)$ lies in first quadrant, suggesting that $A$ is larger than $B$. —(B1)

**(d)** Using the identity from part **(a)**, show that $\tan A = -\frac{13}{16}$. [2]

$$\tan A = -\frac{\cos(A+B) - \cos(A-B)}{\sin(A+B) - \sin(A-B)}$$

$$= -\frac{\left(\frac{-7}{25} - \frac{15}{17}\right)}{\left(\frac{24}{25} - \frac{8}{17}\right)} \quad \text{—(M1, allow ECF)}$$

$$= -\frac{13}{16} \quad \text{—(A1)}$$

[Turn over

**(e)** Hence, find the exact value of $\tan B$. [3]

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \quad \text{— (M1)}$$

$$\frac{8}{15} = \frac{-\frac{13}{16} - \tan B}{1 + (-\frac{13}{16})\tan B} \quad \text{— (M1)}$$

$$\frac{8}{15} - \frac{8}{15}\left(\frac{13}{16}\right)\tan B = -\frac{13}{16} - \tan B$$

$$\frac{17}{30}\tan B = -\frac{13}{16} - \frac{8}{15}$$

$$\tan B = -\frac{19}{8} \quad \text{— (A1)}$$

(also accept $-2\frac{3}{8}$)

--- End of Paper---