TANJONG KATONG GIRLS' SCHOOL
PRELIMINARY EXAMINATION
SECONDARY FOUR EXPRESS

CANDIDATE NAME

CLASS | | E | |

INDEX NUMBER | | |

MATHEMATICS

Paper 1

4048/01

31 August 2022

2 hours

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's use

Setters: Mdm Ng
Markers: Mr Ang, Mdm Lim, Mdm Murni, Mdm Ng and Mrs Pang

This document consists of **24** printed pages, including this page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 Write the following numbers in order of size, starting with the **largest**.

$$1\frac{1}{9}, -\frac{22}{7}, \left(\frac{\sqrt{2}}{2}\right)^4, -\pi$$

Answer ..... [1]

---

2 Simplify $16(a^{-2}b^4)^{-\frac{3}{4}}$ and leave your answer in positive index form.

Answer ..... [2]

---

3 Show and explain that $(5n-1)^2+4$ is a multiple of 5 for all integers of n.

Answer .....

..... [2]

---

4 Factorise

(a) $36y^2 - 25(x-1)^2$

Answer ..... [1]

(b) $12x^2 - 2y^2 + 5xy$

Answer ..... [1]

---

5 Rearrange the formula to make x the subject.

$$y = \sqrt{\frac{x^2+1}{x^2-4}} \text{ where } y > 0.$$

Answer ..... [3]

6 One solution of the equation $5x^2 + (k - 23)x + k = 0$ is $x = 3$.

(a) Find the value of k.

Answer $k =$ ..... [1]

(b) Find the second solution of the equation.

Answer $x =$ ..... [2]

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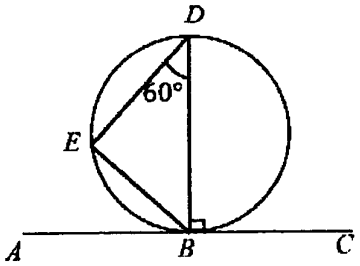
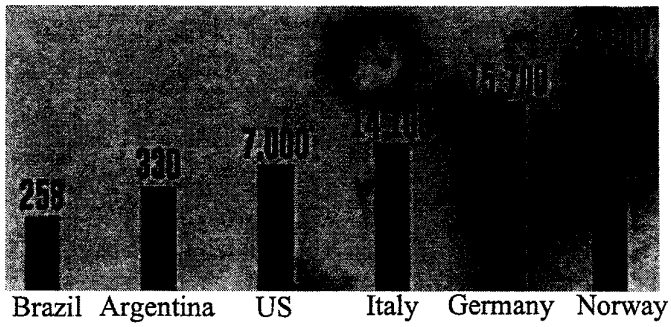
7 Given ABC is a straight line, B, D and E lie on the circumference of the circle, $\angle DBC = 90°$ and $\angle EDB = 60°$. Find $\angle DEB$, giving reason(s) for your answer.



Answer $\angle DEB =$ .....° [2]

8 **Number of Covid-19 Tests per million people**



After this bar graph was shown on Argentinian TV channel C5N, some statisticians claimed that the TV channel was misrepresenting the terrible number of COVID-19 tests in Argentina. Explain which feature of this bar graph is misleading and how it leads to the misrepresentation. Justify your answer with reference to the bar graphs.

Answer .....

.....

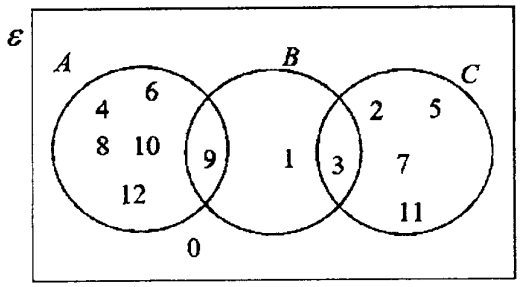
.....

.....

.....

..... [2]

9 ε = {non-negative integers, $x: 0 \leq x \leq 12$}.
The Venn diagram shows the elements of ε and the three sets A, B and C.
A is the set of composite numbers and C is the set of prime numbers.



(a) Describe the elements of set B.

Answer ..... [1]

.....

(b) List the elements contained in the set $(A \cup B')'$.

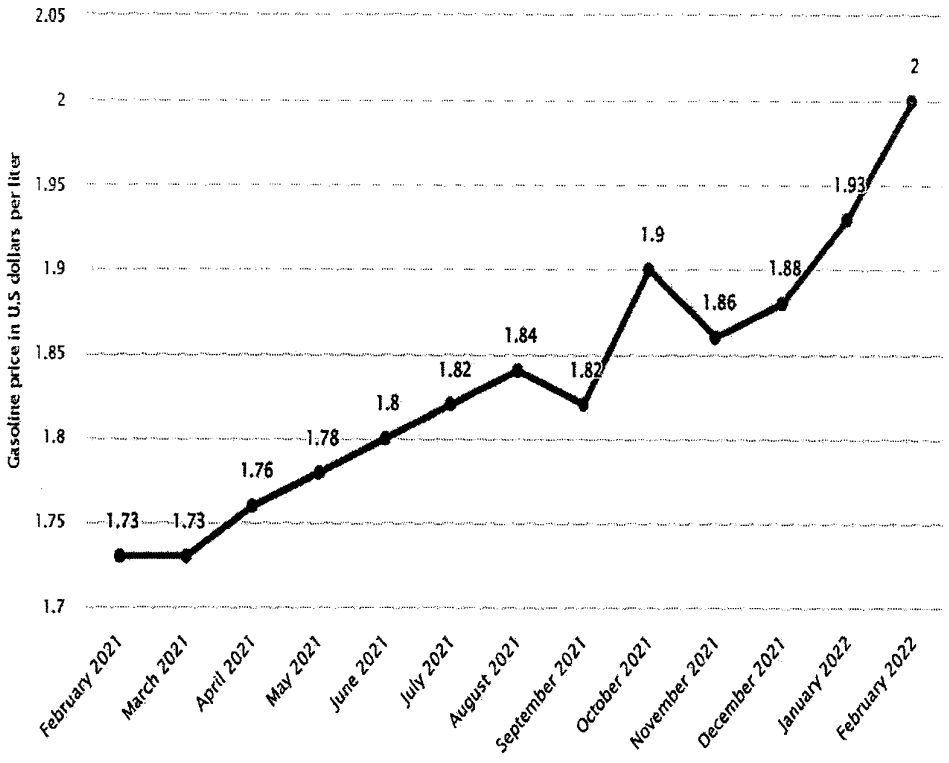
Answer $(A \cup B')'$ = ... [1]

(c) Underline the **correct** statements from the list below.

$A' \cap B' \neq \emptyset$ $\quad n[(A \cup B)'] = 5$ $\quad \{3\} \subset A \cup B$ $\quad \{9\} \notin A \cap C$ [2]

10 The line graph shows the average monthly prices of gasoline in Singapore from February 2021 to February 2022 (in U.S. dollars per litre).

Average Monthly Prices of Gasoline in Singapore from Feb 2021 to Feb 2022



(a) Find the percentage increase in gasoline price from February 2021 to February 2022.

Answer .....% [1]

(b) Jane claimed that the current gasoline price of US$2.34 per litre is 30% more expensive than before. Which month and year is she comparing the current gasoline price with?

Answer ..... [1]

(c) If the gasoline price increases by 1.5% every month starting February 2022, calculate the gasoline price 1 year later, in February 2023. Give your answer rounded to the nearest cent.

Answer US$...../litre. [2]

11 A The table shows the mass of a fruit, m g, with a diameter of x cm. Determine whether m is directly proportional to x^3.

Diameter, x cm	7	8	9	10
Mass, m g	61.74	92.16	131.22	180

Answer ..... [2]

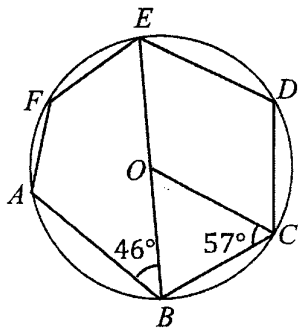
.....

---

12 y is inversely proportional to $\sqrt[3]{x}$. When y is increased by 10%, calculate the percentage decrease in x.

Answer .....% [2]

13 The diagram shows a circle with centre O. BOE is a straight line.
Given that $\angle ABO = 46°$ and $\angle BCO = 57°$, find, stating your reason(s) clearly,



(a) $\angle ADE$

Answer .....° [1]

(b) $\angle AFE$

Answer .....° [1]

(c) $\angle BEC$.

Answer .....° [1]

14 A shop sells Beauty Bath Soap that comes in two geometrically similar bottles of size 125 ml and 1000 ml as shown.

(a) Find the ratio of the height of the smaller bottle to that of the larger bottle in the form $1 : n$.

Answer ..... [1]

(b) At a sales, the price of the bottle with the content of 125 ml and 1000 ml are priced at \$1.90 and \$9.90 respectively. Explain which bottle provides better value for money. Support your answer with calculations.

Answer ..... [1]

.....

(c) Vouchers and posters were used to advertise this product. The ratio of the lengths of the vouchers to that of the posters were 1 : 4. If the size of each voucher is 24.75 cm^2, find the size of the posters in $\mathbf{m^2}$, written in standard form.

Answer ..... m^2 [2]

15 In an n-sided polygon, the sum of interior angles is 1080°.

(a) Find the value of n.

Answer $n =$ ..... [1]

(b) The interior angles are $x, x + 10°, x + 20°, ..., [x + (n - 1)10°]$. Find the smallest exterior angle.

Answer .....° [2]

16 Two numbers 528 and 756 written as product of their prime factors are

$$528 = 2^4 \times 3 \times 11$$
$$756 = 2^2 \times 3^3 \times 7$$

Find

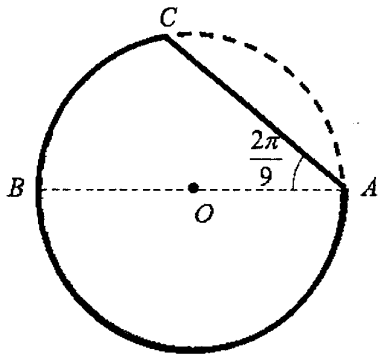
(a) the smallest positive integer x for which $528x$ is a multiple of 756.

Answer ..... [2]

(b) the smallest positive integer y for which $\frac{528}{y}$ is a factor of 756.

Answer ..... [2]

17 The figure below shows a major segment of a circle ABC where radius $OA = 6$ cm and $\angle OAC = \frac{2\pi}{9}$ radians.



(a) Show that the length of the arc BC is $\frac{8\pi}{3}$ cm. [1]

Calculate

(b) the perimeter of the major segment ABC,

Answer ..... cm [3]

(c) the area of the minor segment that was cut off.

Answer .....cm^2 [2]

18 Given A is the point $(2, -3)$ and B is the point $(5, m)$, $\overrightarrow{CD} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$.

(a) Express $\overrightarrow{AB}$ as a column vector in terms of m.

Answer ..... [1]

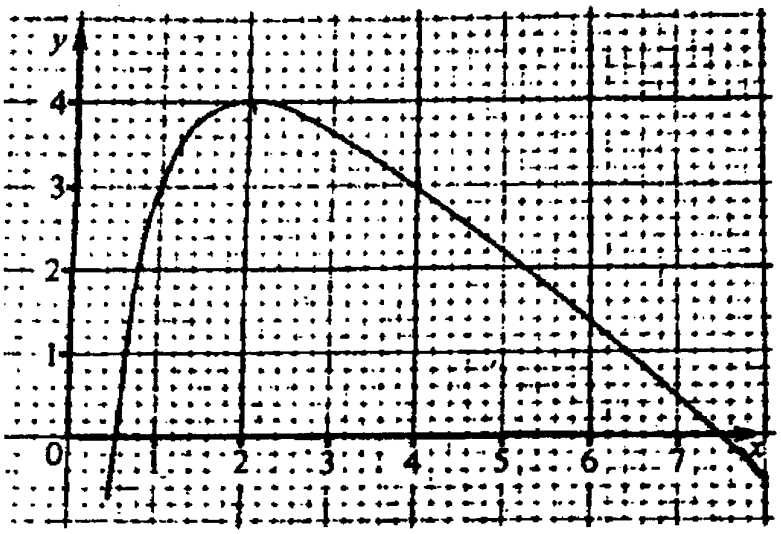
(b) If $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$, find the value of m.

Answer $m =$ ..... [2]

(c) If $|\overrightarrow{AB}| = \frac{1}{2}|\overrightarrow{CD}|$, find $|\overrightarrow{AB}|$.

Answer $|\overrightarrow{AB}|$= .....units [1]

19 Part of the graph of $y = 8 - x - \frac{4}{x}$ is drawn on the grid.



Solve each equation below for x in the range $0 < x < 8$, by drawing another straight line on the graph. Leave your answer(s) in **1 decimal place**.

(a) $x + \frac{4}{x} = 5$

Answer $x =$ ..... or ..... [2]

(b) $7 - \frac{4}{x} = \frac{x}{2}$

Answer $x =$ ..... [3]

20 The heights, in cm, of 15 boys and 18 girls were recorded in the Back-to-back Stem-and-Leaf diagram below.

Boys								Girls							
						9	14	8	8						
			9	8	6	5	15	2	x	x	8	8	9	9	
							16	y	y	y	z	z			
8	6	5	4	2	1	1	17	2	3	4	5				
				4	2	0	18								

Key (Boys)
9|15 means 159 cm

Key (Girls)
14|8 means 148 cm

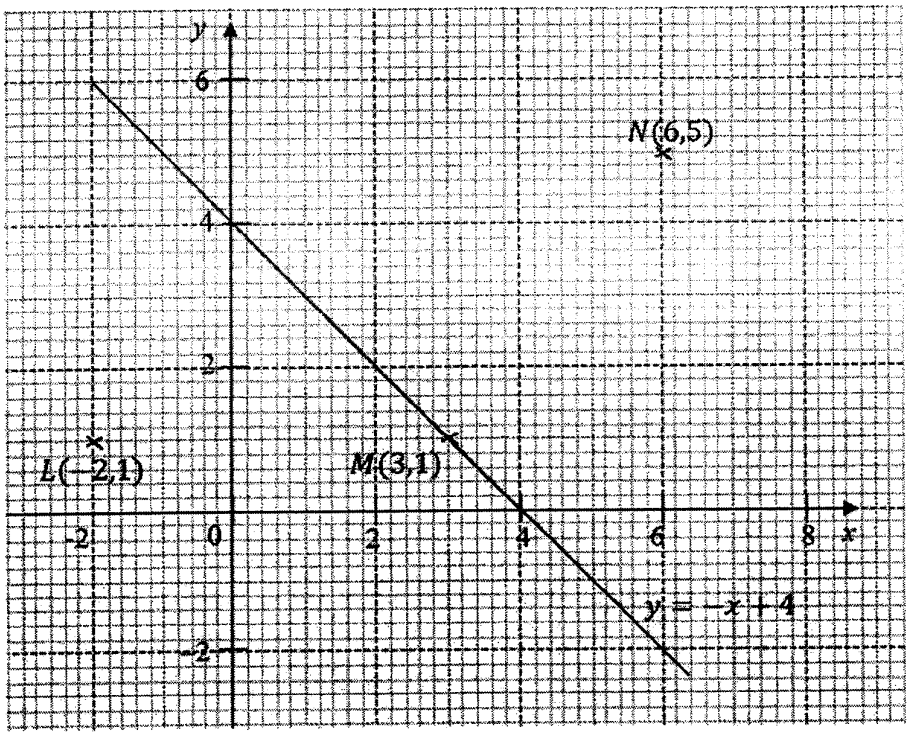
(a) If the median height of the girls is 161 cm, write down the value of y.

Answer $y =$ ..... [2]

(b) Find the interquartile range of the boys' heights.

Answer ..... cm [2]

21 The diagram shows three points $L(-2,1)$, M(3,1) and N(6,5) and the line $y = -x + 4$.



(a) Calculate the exact value of $cos\angle LMN$.

Answer ..... [2]

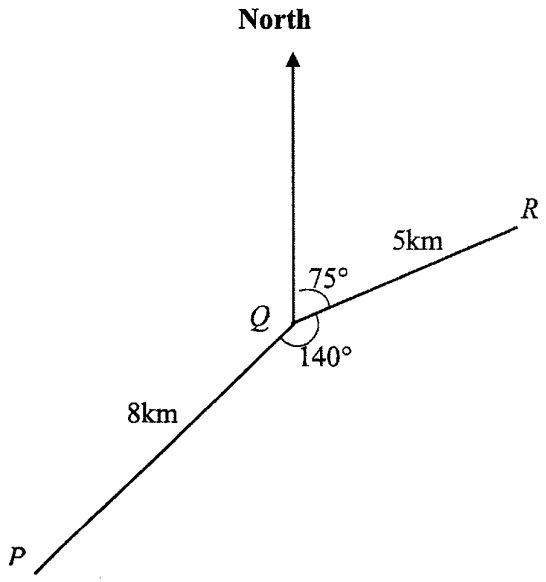
(b) P is the point $(3, k)$ and the area of triangle LMP is 15 square units. Find the possible value(s) of k.

Answer $k =$ ..... [3]

(c) Given that $y = -x + 4$ is the line of symmetry of triangle LMQ, write down the coordinates of Q.

Answer $Q($ ....., ...) [1]

22 A ship sails 8 km from port P to port Q. It then sails 5 km from port Q to port R on a bearing of 075°.



(a) Given that $\angle PQR = 140°$, calculate

(i) the bearing of Q from P,

Answer ..... ° [1]

(ii) how far Q is east of P.

Answer ..... km [2]

(b) An island X is located at a bearing of 085° from P and 160° from Q.

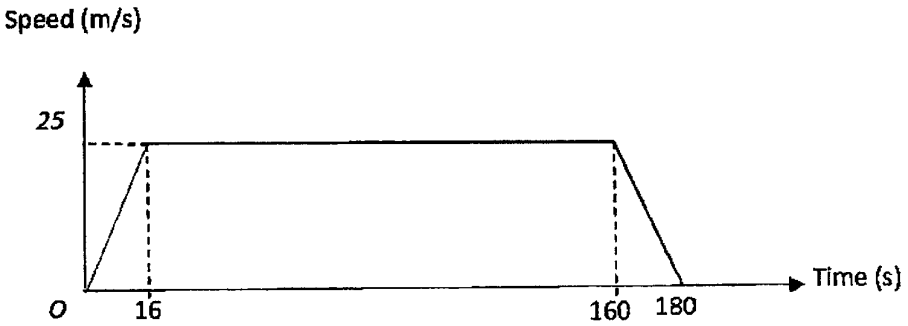
(i) Find $\angle PQX$.

Answer ..... ° [1]

(ii) If a boat travels from P to X at a speed of 40km/h, calculate the time, in minutes, that the boat takes to reach X.

Answer ..... minutes [3]

23 The diagram shows the speed-time graph of a car as it travelled from point ***A*** **to** ***B***.



(a) Find the speed of the car at 165 seconds.

Answer.....m/s [2]

(b) A stationary motorcycle started travelling at the same time as the car, taking the same route in opposite direction from point ***B*** **to** ***A***. Given that the motorcycle travelled at a constant acceleration of 3 m/s^2 and it passed by the car at T seconds where $16 < T < 160$, show that

$$1.5T^2 + 25T - 4250 = 0.$$

[3]

End of Paper

Sec 4 Prelim Math Paper 2

1 **(a)** Express as a single fraction in its simplest form

(i) $\dfrac{24q^2}{63p^3} \div \dfrac{9q^5}{21p}$, [1]

(ii) $\dfrac{1}{m-4} + \dfrac{2m}{m^2-16}$. [2]

(b) Simplify $\dfrac{3x-9}{2x-xy+3y-6}$. [3]

(c) Solve the equation $(x+2)(x-5) = (x-5)(4x-7)$. [3]

2	In 2019, Alan and Bala decided to start a business together. Alan invested \$210 000 and Bala invested \$140 000. They agreed that all profit should be divided in the same ratio as the sums of the money they invested.		
	(a)	In 2019, the profit was \$20 000. Calculate Alan's share of the profit.	[2]
	(b)	Due to the pandemic, the total profit in 2020 dropped to \$12 500. Calculate the percentage decrease in profit from 2019 to 2020.	[1]
	(c)	To expand their business, they decided to borrow \$100 000 from a bank. The bank charged an interest rate of 2.4% per annum compounded half yearly. Calculate how much interest they need to pay after 5 years. Give your answer correct to the nearest dollars.	[3]
	(d)	Alan and Bala can choose to import their raw materials which cost RM40 000 in Malaysia or NT\$265 000 in Taiwan. The exchange rate between Singapore and Malaysia is S\$1 = RM\$3.20 and the exchange rate between Taiwan and Singapore is NT\$100 = S\$4.60. There is a freight charge of 2% for the raw materials from Taiwan only. Determine which country they should import their raw materials from.	[4]

3 A wholesaler supplies snacks and delivers to two stalls. The matrix, **S**, shows the number of each type of snacks per delivery made to Stalls *A* and *B*. In a week, the wholesaler delivers 5 times to Stall *A* and 7 times to stall *B*.

$$\mathbf{S} = \begin{matrix} \\ \text{Stall } A \\ \text{Stall } B \end{matrix} \begin{matrix} \begin{matrix} \text{sandwich} & \text{cake} & \text{pie} \end{matrix} \\ \begin{pmatrix} 25 & 20 & 13 \\ 40 & 18 & 21 \end{pmatrix} \end{matrix}$$

(a) The wholesaler charges the stalls \$2.00, \$0.70 and \$1.50 each for sandwich, cake and pie respectively. Represent these prices in column matrix **P**. [1]

(b) Evaluate the matrix **C** = **SP**. [2]

(c) State what each of the elements of **C** represents. [1]

(d) The amount collected by the wholesaler in a week from Stall *A* and Stall *B* respectively is represented by a 2×1 matrix, **W**. Using **only** matrix multiplication, find **W**. [2]

(e) Hence, find the total amount collected by the wholesaler in a week. [1]

4 **(a)** These are the first four terms in a sequence.

$-2 \quad 1 \quad 4 \quad 7$

Find an expression, in terms of n, for the nth term of the sequence. [1]

(b) Study the following number pattern of the Pythagorean Triples.

Row	Pythagorean Triples
1	$5^2 = 3^2 + 4^2$
2	$13^2 = 5^2 + 12^2$
3	$25^2 = 7^2 + 24^2$
4	$41^2 = 9^2 + 40^2$
5	$p^2 = q^2 + 60^2$
⋮	⋮
N	$P_N^{\,2} = Q_N^{\,2} + R_N^{\,2}$

(i) Write down the value of p and of q in Row 5. [2]

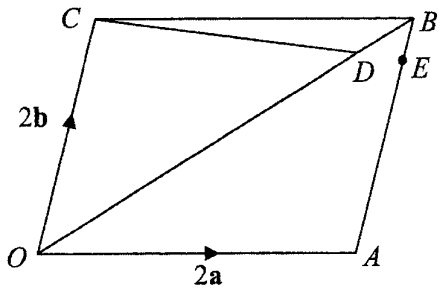
(ii) Write down the Pythagorean Triples in Row 10 when $P_{10} = 221$. [1]

(iii) When $Q_N = 111$, find N. [2]

(iv)	Given that $R_N = aN^2 + bN$, find the value of a and of b.	[4]

(v) Explain with reason why it is not possible for 2021 to be a number of R_N. [2]

5 **(a)**



The diagram shows a parallelogram $OABC$. The point D on OB is such that $OD = 5DB$. The point E on AB is such that $AB : EB = 5 : 1$. Given that $\overrightarrow{OA} = 2\mathbf{a}$ and $\overrightarrow{OC} = 2\mathbf{b}$.

(i) Express in terms of **a** and **b**, giving each of your answers in its simplest form.

(a) $\overrightarrow{OB}$, [1]

(b) $\overrightarrow{CD}$. [2]

(ii) Show that C, D and E are collinear. [3]

(iii)	Find the numerical value of $\dfrac{\text{Area of } \Delta ODC}{\text{Area of parallelogram } OABC}$	[1]

(b) It is given that $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ and the coordinates of R are $(4, 0)$. Find the coordinates of the point S such that $PQRS$ is a parallelogram. [2]

6 An aircraft flew from Town A to Town B and made a return trip to Town A from Town B. The total distance covered was 1200 km. The speed of the aircraft in still air is 200 km/h. The aircraft flew against the wind when flying from Town A to Town B, and flew wind assisted when flying back to Town A from Town B.

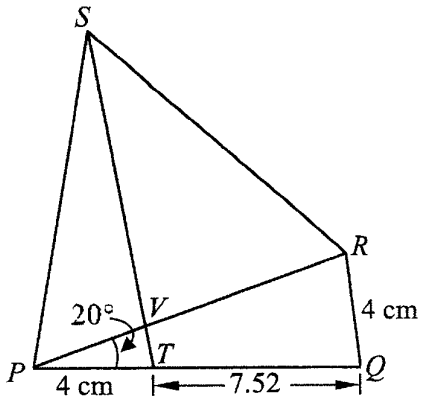
(a) The speed of the wind, which is constant throughout, is x km/h. The time taken by the aircraft, in hours, to fly from Town A to Town B is $\frac{600}{200-x}$. Write down an expression, in terms of x, the time taken by the aircraft, in hours, to fly from Town B to Town A. [1]

(b) The time taken to fly against the wind is 10 minutes longer than when it took to fly wind assisted. Write down an equation in terms of x and show that it reduces to $x^2 + 7200x - 40\,000 = 0$. [3]

(c) Showing your working clearly, solve the equation $x^2 + 7200x - 40\,000 = 0$, giving your solutions correct to 2 significant figures. [4]

(d) Find the time taken for the whole trip. [2]

7



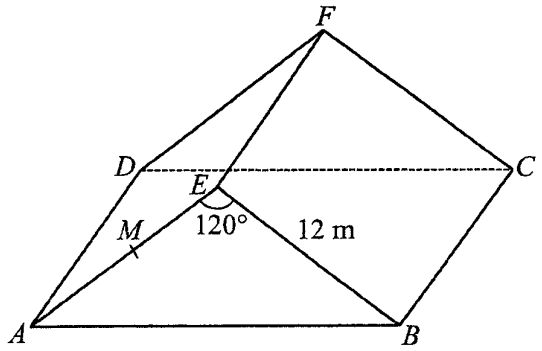
ΔPQR is an isosceles triangle with $\angle QPR = 20°$. S is a point outside ΔPQR such that ΔPRS is an equilateral triangle and SVT is a straight line that meets PR and PQ at V and T respectively. It is given that $QR = TP = 4$ cm and $TQ = 7.52$ cm.

(a)	Show that ΔPQR is congruent to ΔSTP. Give a reason for each statement you make.	[3]

(b)	Show that ΔSTR is an isosceles triangle.	[2]

(c)	Find $\angle STR$.	[2]

8 A roof in the shape of a triangular right prism is constructed as shown below such that $ABCD$ is a rectangle, $ADFE$ and $BCFE$ are squares. $AE = BE = 12$ m, $\angle AEB = 120°$ and M is the midpoint of AE.



Find

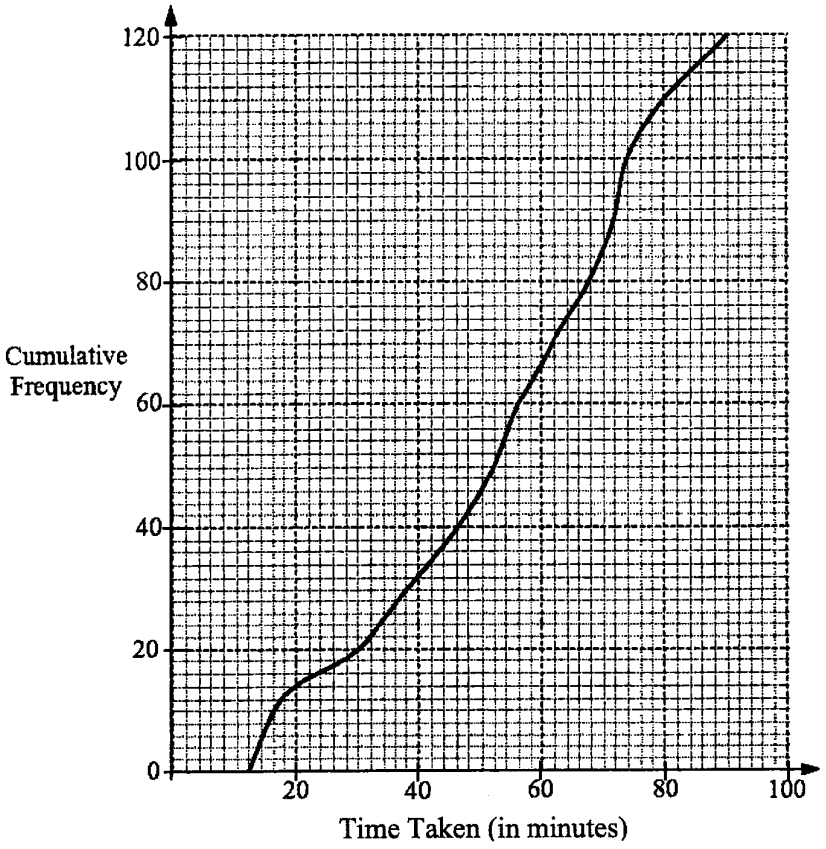
(a) the area of triangle ABE, [2]

(b) AB^2, [2]

(c) AC, [2]

(d) $\angle AEC$, [3]

9 **(a)** The cumulative frequency curve below shows the distribution of the time taken (in minutes) by 120 students in School *A* to complete an assignment.



Below is the grouped frequency table for the time taken by the students.

Time taken (in minutes)	$0 < x \leq 20$	$20 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 80$	$80 < x \leq 100$
Frequency	14	***a***	34	***b***	10

(i) Find the value of ***a*** and of ***b***. [1]

(ii) Estimate the mean time taken by the students. [1]

(iii) Estimate the standard deviation of the time taken by the students. [1]

(iv) 120 students from School *B* completed the same assignment, and the analysis of their time taken is represented in the table below.

Mean time taken	60
Standard deviation	13.6

Make two comments comparing the time taken by the students from the 2 schools. [2]

(b) Ali, Bryan and Chandra took part in a game of dart throwing. The probabilities that Ali, Bryan and Chandra will hit the target in a single throw are $\frac{1}{6}$, $\frac{1}{5}$ and $\frac{1}{4}$ respectively.

(i) For the first game, all three of them throw the dart at the target at the same time. Find the probability that all of them hit the target. [2]

(ii) In the second game, they each make a single throw of the dart at the target in the order of Ali, Bryan and Chandra. For this game, once the target is hit, the game will end. Find the probability the target is hit. [3]

10 Daryl owns a concert hall with a full capacity of 120 seats. He conducted a survey to find out how much to charge for tickets. The detail of the survey is below:

Price of one ticket	Number of people who will attend the concert
\$6.00	120
\$7.50	110
\$9.00	100
\$10.50	90
⋮	⋮

(a) Write down the revenue he will get if all 120 seats are sold. [1]

(b) Daryl noticed that for every \$1.50 increase in the price of one ticket, the number of people who attend the concert drops by 10.

(i) Find the revenue if he makes **three** \$1.50 increases to the price from \$6. [1]

(ii) Let n be the number of \$1.50 increase in the price of the tickets, explain why the revenue R, in dollars is given by $720+120n-15n^2$. [3]

(iii) Explain why the number of \$1.50 increase in price should be less than 12. [1]

(iv) By drawing a suitable graph for $n < 12$ on the grid opposite, work out how much should Daryl charge his ticket to maximum revenue. [4]

Answer Keys

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1	$1\frac{1}{9}, \left(\frac{\sqrt{2}}{2}\right)^4, -\pi, -\frac{22}{7}$		
2	$\frac{16a^{\frac{3}{2}}}{b^3}$		
3	$5(5n^2 - 2n + 1)$ is a multiple of 5 for all integers of n		
4(a)	$(6y + 5x - 5)(6y - 5x + 5)$	4(b)	$(4x - y)(3x + 2y)$
5	$x = \pm\sqrt{\frac{4y^2 + 1}{y^2 - 1}}$		
6(a)	$k = 6$	6(b)	The other solution is $x = \frac{2}{5}$
7	Angle $DEB = 90°$		
8	Misleading feature: The heights of the bars are not proportional to the number of covid-19 tests per million people. Effect of misleading feature: The heights of the bars suggest that Argentina tests about three-quarter the number of people per million as the *USA. However, Argentina tests 330 people per million while the USA tests 7000 people per million, which is about 21 times. *Accept correct comparison with other countries (Italy approx. 4/5 vs 43 times, Germany 2/3 vs 48 times or Norway ½ vs 68 times)		
9(a)	Elements of set B are factors of 9.	9(b)	$(A \cup B')' = A' \cap B = \{1, 3\}$
9(c)	$A' \cap B' \neq \emptyset$ $n[(A \cup B)'] = 5$ $\{3\} \subset A \cup B$ $\{9\} \notin A \cap C$		
10(a)	15.6%	10(b)	June 2021
10(c)	US$2.39		
11	Since $\frac{m}{x^3} = k$, where k=0.18 is a non-zero constant, m is directly proportional to x^3.		
12	24.9%		
13(a)	46°	13(b)	134°
13(c)	33°		
14(a)	1 : 2	14(c)	3.96×10^{-2} m^2
14(b)	1000ml bottle cost $0.0053 less per ml than 125 ml bottle. The 1000 ml bottle provides better value for money. OR 1000ml bottle provides 35.221 more ml per $1 than 125 ml bottle. The 1000 ml bottle provides better value for money.		

15(a)	$n = 8$	15(b)	$10°$
16(a)	smallest positive integer $x = 63$	16(b)	smallest positive integer $y = 44$
17(b)	36.4 cm	17(c)	13.7 cm^2
18(a)	$\begin{pmatrix} 3 \\ m+3 \end{pmatrix}$	18(b)	$m = -\frac{3}{4}$
18(c)	5 units		
19(a)	$x = 1.0$ or 4.0 (accept 3.9)	19(b)	$x = 0.6$
20(a)	$y = 3$	20(b)	IQR of boy's height = $178 - 168 =$ 10 cm
21(a)	$cos \angle LMN = -\frac{3}{5}$	21(b)	$k = 7$ or -5
21(c)	$Q(3, 6)$		
22(a)(i)	$035°$	22(a)(ii)	4.59km
22(b)(i)	$55°$	22(b)(ii)	$10.2\ min$
23(a)	Speed = 18.75 m/s		

Sec 4 Prelim Math Paper 2 Solutions

1 (a) Express as a single fraction in its simplest form

(i) $\dfrac{24q^2}{63p^3} \div \dfrac{9q^5}{21p}$, [1]

Solutions	Skills/Concept
$= \dfrac{24q^2}{63p^3} \times \dfrac{21p}{9q^5}$ $= \dfrac{8}{9p^2q^3}$	Take reciprocal: $\dfrac{24q^2}{63p^3} \times \dfrac{21p}{9q^5}$ Laws of indices: $a^{m+n} = a^m \times a^n$ $a^{m-n} = a^m \div a^n$

(ii) $\dfrac{1}{m-4} + \dfrac{2m}{m^2-16}$. [2]

Solutions	Skills/Concept
$= \dfrac{1}{m-4} + \dfrac{2m}{(m-4)(m+4)}$ $= \dfrac{(m+4)+2m}{(m-4)(m+4)}$ $= \dfrac{3m+4}{(m-4)(m+4)}$	Quadratic Identity: $a^2-b^2 = (a+b)(a-b)$ Express as single fraction

(b) Simplify $\dfrac{3x-9}{2x-xy+3y-6}$. [3]

Solutions/Alternative Methods	Skills/Concept
$= \dfrac{3x-9}{(2x-xy)+(3y-6)} = \dfrac{3x-9}{x(2-y)+3(y-2)}$ $= \dfrac{3(x-3)}{x(2-y)-3(2-y)}$ $= \dfrac{3(x-3)}{(x-3)(2-y)}$ $= \dfrac{3}{2-y}$	Factorisation by grouping Change of sign $-\dfrac{3}{y-2}$ also acceptable

(c) Solve the equation $(x+2)(x-5) = (x-5)(4x-7)$. [3]

Solutions/Alternative Methods	Skills/Concept
$(x+2)(x-5)-(4x-7)(x-5) = 0$ $(x-5)[(x+2)-(4x-7)] = 0$ $(x-5)(9-3x) = 0$ $x = 3$ or $x = 5$	Factorisation of quadratic function Solving quadratic equation

2 In 2019, Alan and Bala decided to start a business together. Alan invested \$210 000 and Bala invested \$140 000. They agreed that all profit should be divided in the same ratio as the sums of the money they invested.

(a) In 2019, the profit was \$20 000. Calculate Alan's share of the profit. [2]

Solutions/Alternative Methods	Skills/Concept
Alan's share of profit $= \frac{3}{5} \times 20\,000$ $= \$12\,000$	Ratio: $\frac{210000}{210000+140000} = \frac{3}{5}$

(b) Due to the pandemic, the total profit in 2020 dropped to \$12 500. Calculate the percentage decrease in profit from 2019 to 2020. [1]

Solutions/Alternative Methods	Skills/Concept
percentage decrease in profit $= \frac{20\,000 - 12\,500}{20\,000} \times 100\%$ $= 37.5\%$	Percentage decrease $= \frac{\text{difference}}{\text{original}} \times 100$

(c) To expand their business, they decided to borrow \$100 000 from a bank. The bank charged an interest rate of 2.4% per annum compounded half yearly. Calculate how much interest they need to pay after 5 years. Give your answer correct to the nearest dollars. [3]

Solutions/Alternative Methods	Skills/Concept
$\text{Interest} = 100\,000\left[1+\frac{\left(\frac{2.4}{2}\right)}{100}\right]^{5\times 2} - 100\,000$ $= \$12\,669.1778$ $= \$12\,669$	$n = 5 \times 2$ $r = 2.4 \div 2$ Round up to nearest dollars

(d) Alan and Bala can choose to import their raw materials which cost RM40 000 in Malaysia or NT\$265 000 in Taiwan. The exchange rate between Singapore and Malaysia is S\$1 = RM\$3.20 and the exchange rate between Taiwan and Singapore is NT\$100 = S\$4.60. There is a freight charge of 2% for the raw materials from Taiwan only. Determine which country they should import their raw materials from. [4]

Solutions/Alternative Methods	Skills/Concept
Amount paid in S\$ for Malaysia import $= \frac{40\,000}{3.2}$ $= \$12\,500$	Exchange rate for Malaysia Ringgit
Amount paid in S\$ for Taiwan import $= \frac{102}{100}\left(\frac{265\,000}{100} \times 4.6\right)$ $= \$12\,433.80$	Exchange rate for Taiwan dollars Include 2% freight charge
They should import their raw materials from Taiwan because the total amount paid is S\$66.20 lower compared to Malaysia.	Compare with difference in values

3 A wholesaler supplies snacks and delivers to two stalls. The matrix, **S**, shows the number of each type of snacks per delivery made to Stalls *A* and *B*. In a week, the wholesaler delivers 5 times to Stall *A* and 7 times to stall *B*.

$$\begin{matrix} & \text{sandwich} & \text{cake} & \text{pie} & \\ \mathbf{S} = & \begin{pmatrix} 25 & 20 & 13 \\ 40 & 18 & 21 \end{pmatrix} & & & \begin{matrix} \text{Stall } A \\ \text{Stall } B \end{matrix} \end{matrix}$$

(a) The wholesaler charges the stalls \$2.00, \$0.70 and \$1.50 each for sandwich, cake and pie respectively. Represent these prices in column matrix **P**. [1]

Solutions/Alternative Methods	Skills/Concept
$\mathbf{P} = \begin{pmatrix} 2.00 \\ 0.70 \\ 1.50 \end{pmatrix}$	Column matrices 3×1

(b) Evaluate the matrix **C = SP.** [2]

Solutions/Alternative Methods	Skills/Concept
$\mathbf{C} = \begin{pmatrix} 25 & 20 & 13 \\ 40 & 18 & 21 \end{pmatrix} \begin{pmatrix} 2.00 \\ 0.70 \\ 1.50 \end{pmatrix}$ $= \begin{pmatrix} 83.50 \\ 124.10 \end{pmatrix}$	Multiplying matrices, Order: $(2 \times 3) \times (3 \times 1) = (2 \times 1)$

(c) State what each of the elements of **C** represents. [1]

Solutions/Alternative Methods	Skills/Concept
The elements represent the amount collected by the wholesaler from the sales of snacks per delivery from Stall *A* and Stall *B* respectively.	

(d) The amount collected by the wholesaler in a week from Stall *A* and Stall *B* respectively is represented by a 2×1 matrix, **W.** Using **only** matrix multiplication, find **W.** [2]

Solutions/Alternative Methods	Skills/Concept
$\mathbf{W} = \begin{pmatrix} 5 & 0 \\ 0 & 7 \end{pmatrix} \begin{pmatrix} 83.50 \\ 124.10 \end{pmatrix}$ or $= \begin{pmatrix} 83.50 & 0 \\ 0 & 124.10 \end{pmatrix} \begin{pmatrix} 5 \\ 7 \end{pmatrix}$ $= \begin{pmatrix} 417.50 \\ 868.7 \end{pmatrix}$ $= \begin{pmatrix} 417.50 \\ 868.7 \end{pmatrix}$	Matrix multiplication of 2×2 with 2×1 to get 2×1

(e) Hence, find the total amount collected by the wholesaler in a week. [1]

Solutions/Alternative Methods	Skills/Concept
Total amount $= \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 417.50 \\ 868.7 \end{pmatrix}$ The total amount collected in a week is \$1286.20	Unit matrix (1 1)

4 **(a)** These are the first four terms in a sequence.

$-2 \quad 1 \quad 4 \quad 7$

Find an expression, in terms of n, for the nth term of the sequence. [1]

Solutions/Alternative Methods	Skills/Concept
$3(n-1)-2=3n-3-2$ $=3n-5$	General term: $a+(n-1)d$ a: 1st term, d: constant difference between terms

(b) Study the following number pattern of the Pythagorean Triples.

Row	Pythagorean Triples
1	$5^2=3^2+4^2$
2	$13^2=5^2+12^2$
3	$25^2=7^2+24^2$
4	$41^2=9^2+40^2$
5	$p^2=q^2+60^2$
N	$P_N^{\,2}=Q_N^{\,2}+R_N^{\,2}$

(i) Write down the value of p and of q in Row 5. [2]

Solutions/Alternative Methods	Skills/Concept
$p=61$ $q=11$	Number patterns

(ii) Write down the Pythagorean Triples in Row 10 when $P_{10}=221$. [1]

Solutions/Alternative Methods	Skills/Concept
$221^2=21^2+220^2$	Include P, Q & R

(iii) When $Q_N=111$, find N. [2]

Solutions/Alternative Methods	Skills/Concept
$Q_N=2(N-1)+3$ $=2N+1$ $2N+1=111$ $2N=110$ $N=55$	Find number pattern for Q_N Equate $2N+1$ to 111

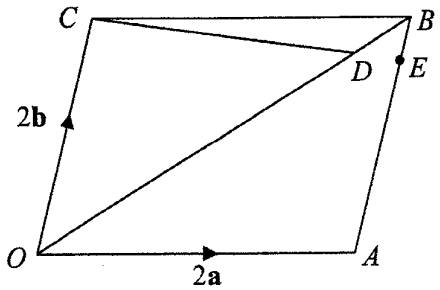
		(iv)	Given that $R_N = aN^2 + bN$, find the value of a and of b.	[4]

Solutions/Alternative Methods	**Skills/Concept**
$R_N = aN^2 + bN$	
When $N = 1$, $a(1)^2 + b(1) = 4$	Form 1st equation
$a + b = 4 \quad \cdots \quad (1)$	
When $N = 2$, $a(2)^2 + b(2) = 12$	Form 2nd equation
$4a + 2b = 12 \qquad (2)$	
$(1) \times 2$, $2a + 2b = 8 \qquad (3)$	
$(2) - (3)$, $2a = 4$	Solve simultaneous equations
$a = 2$	
Substitute $a = 2$ into (1), $(2) + b = 4$	Both a & b must be correct
$b = 2$	

		(v)	Explain with reason why it is not possible for 2021 to be a number of R_N.	[2]

Solutions/Alternative Methods	**Skills/Concept**
$2N^2 + 2N = 2(N^2 + 1)$	Make R_N a multiple of 2
Since $2N^2 + 2N = 2(N^2 + 1)$ is always even for all values of N, it is not possible for 2021 which is odd to be a number of R_N.	Multiples of 2 are even numbers

5 **(a)**



The diagram shows a parallelogram $OABC$. The point D on OB is such that $OD = 5DB$. The point E on AB is such that $AB : EB = 5 : 1$. Given that $OA = 2\mathbf{a}$ and $OC = 2\mathbf{b}$.

(i) Express in terms of **a** and **b**, giving each of your answers in its simplest form.

(a) OB, [1]

Solutions/Alternative Methods	Skills/Concept
$OB = OA + AB$ $= 2\mathbf{a} + 2\mathbf{b}$	Triangle Law of Vector Addition

(b) CD. [2]

Solutions/Alternative Methods	Skills/Concept
$CD = CO + OD$ $= CO + \frac{5}{6}OB$ $= -2\mathbf{b} + \frac{5}{6}(2\mathbf{a} + 2\mathbf{b})$ $= -2\mathbf{b} + \frac{5}{3}\mathbf{a} + \frac{5}{3}\mathbf{b}$ $= \frac{5}{3}\mathbf{a} - \frac{1}{3}\mathbf{b}$	Vector addition with $OD = \frac{5}{6}OB$

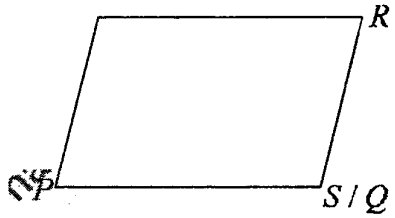
(ii) Show that C, D and E are collinear. [3]

Solutions/Alternative Methods	Skills/Concept
$CE = CB + BE$ $= 2\mathbf{a} + \frac{1}{5}BA$ $= 2\mathbf{a} + \frac{1}{5}(-2\mathbf{b})$ $= 2\mathbf{a} - \frac{2}{5}\mathbf{b}$ $CD = \frac{1}{3}(5\mathbf{a} - \mathbf{b})$ $CE = \frac{2}{5}(5\mathbf{a} - \mathbf{b})$ $\therefore CD = \frac{5}{6}CE$ Since CD is a <u>scalar multiple</u> of CE and C is a <u>common point</u>, $\therefore C$, D and E are collinear.	Find CE using vector addition Make CD a scalar multiple of CE Or any other scalar multiple $DE = \frac{1}{3}\mathbf{a} + \frac{1}{15}\mathbf{b}$ $= \frac{1}{15}(5\mathbf{a} - \mathbf{b})$ $= \frac{1}{5\times 3}(5\mathbf{a} - \mathbf{b})$ $DE = \frac{1}{5}CD$ Conditions for collinearity

		(iii)	Find the numerical value of $\dfrac{\text{Area of } \Delta ODC}{\text{Area of parallelogram } OABC}$	[1]

Solutions/Alternative Methods	**Skills/Concept**
$\dfrac{\text{Area of } \Delta ODC}{\text{Area of } \Delta OBC} = \dfrac{\frac{1}{2}\times 5\times h}{\frac{1}{2}\times 6\times h}$ $= \dfrac{5}{6}$ $\dfrac{\text{Area of } \Delta ODC}{\text{Area of parallelogram } OABC} = \dfrac{5}{6}\times\dfrac{1}{2}$ $= \dfrac{5}{12}$	Use of Area of triangle $= \dfrac{1}{2}\times \text{base}\times \text{height}$ to find ratio of 2 triangles with common height Or use counting method

	(b)	It is given that $PQ = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ and the coordinates of R are $(4, 0)$. Find the coordinates of the point S such that $PQRS$ is a parallelogram.	[2]

Solutions/Alternative Methods	**Skills/Concept**
Let the coordinates of S be (a, b) $PQ = SR$ $PQ = OR - OS$ $\begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} a \\ b \end{pmatrix}$ $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ $\therefore S(-1, -1)$	$PQ = SR$ for equal vectors in parallelogram  Clockwise or anti-clockwise for $PQRS$ Must be coordinates

6 An aircraft flew from Town A to Town B and made a return trip to Town A from Town B. The total distance covered was 1200 km. The speed of the aircraft in still air is 200 km/h. The aircraft flew against the wind when flying from Town A to Town B, and flew wind assisted when flying back to Town A from Town B.

(a) The speed of the wind, which is constant throughout, is x km/h. The time taken by the aircraft, in hours, to fly from Town A to Town B is $\frac{600}{200-x}$. Write down an expression, in terms of x, the time taken by the aircraft, in hours, to fly from Town B to Town A. [1]

Solutions/Alternative Methods	Skills/Concept
$\frac{600}{200+x}$	

(b) The time taken to fly against the wind is 10 minutes longer than when it took to fly wind assisted. Write down an equation in terms of x and show that it reduces to $x^2+7200x-40\,000=0$. [3]

Solutions/Alternative Methods	Skills/Concept
$\frac{600}{200-x}-\frac{600}{200+x}=\frac{10}{60}$	Forming quadratic equations
$\frac{600(200+x)-600(200-x)}{(200^2-x^2)}=\frac{1}{6}$	$(200+x)(200-x)=(200^2-x^2)$ as denominator
$600(200+x)-600(200-x)=\frac{1}{6}(200^2-x^2)$	
$1200x=\frac{1}{6}(200^2-x^2)$	
$7200x=200^2-x^2$	Simplify equation to required one
$x^2+7200x-40\,000=0$ (shown)	

(c) Showing your working clearly, solve the equation $x^2+7200x-40\,000=0$, giving your solutions correct to 2 significant figures. [4]

Solutions/Alternative Methods	Skills/Concept
$x^2+7200x-40\,000=0$	Solving of quadratic equation using formula or completing square **only**
$x=\frac{-7200\pm\sqrt{7200^2-4(1)(-40000)}}{2(1)}$	
$=5.55113$ or $=-7205.55113$	Leave answers in 2 sig fig
$=5.6$ or $=-7200$ (2 s.f.)	

Solutions/Alternative Methods	
$x^2+7200x-40\,000=0$	Solve by completing the square
$(x+3600)^2-(3600)^2-40\,000=0$	
$(x+3600)^2=13000000$	
$x=-3600\pm\sqrt{13000000}$	
$=5.55113$ or $=-7205.55113$	
$=5.6$ or $=-7200$ (2 s.f.)	Leave answers in 2 sig fig

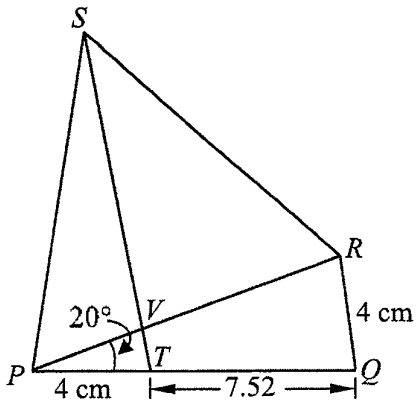
(d) Find the time taken for the whole trip. [2]

Solutions/Alternative Methods	Skills/Concept
Time taken $=2\times\frac{600}{200+5.55113}+\frac{10}{60}$	Use $x=5.55113$ to find time taken to the nearest hour
$=6.0046$	
$=6$ hours	

Solutions/Alternative Methods	

	Time taken $= 2 \times \frac{600}{200 - 5.55113} - \frac{10}{60}$ $= 6.0046$ $= 6$ hours	
Solutions/Alternative Methods		
	Time taken $= \frac{600}{200 - 5.55113} + \frac{600}{200 + 5.55113}$ $= 6.0046$ $= 6$ hours	

7



ΔPQR is an isosceles triangle with $\angle QPR = 20°$. S is a point outside ΔPQR such that ΔPRS is an equilateral triangle and SVT is a straight line that meets PR and PQ at V and T respectively. It is given that $QR = TP = 4$ cm and $TQ = 7.52$ cm.

(a) Show that ΔPQR is congruent to ΔSTP.
Give a reason for each statement you make. [3]

Solutions/Alternative Methods	Skills/Concept
$SP = PR$ (sides of equilateral triangle) $QR = TP$ (given)	Find 2 pairs of corresponding sides
$\angle PRQ = \frac{180 - 20}{2}$ (base angle of isosceles triangle) $= 80$ $\angle SPT = \angle SPR + \angle RPQ$ $= 60 + 20$ $= 80$	Find a pair of corresponding angles
$\therefore \angle PRQ = \angle SPT$ $\therefore \Delta PQR$ is congruent to ΔSTP (SAS congruent test)	Conditions for Congruency (SAS)

(b) Show that ΔSTR is an isosceles triangle. [2]

Solutions/Alternative Methods	Skills/Concept
$SP = SR$ (sides of equilateral triangle) Since ΔPQR is congruent to ΔSTP, $SP = ST$ (sides of isosceles triangle) $\therefore ST = SR$ ΔSTR is an iososceles triangle.	Use of congruence rules Know that sides of isosceles triangle are equal

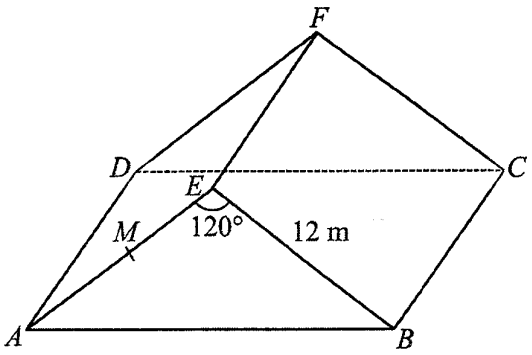
(c) Find $\angle STR$. [2]

Solutions/Alternative Methods	Skills/Concept
$\angle RST = \angle PSR - \angle PST$ $= 60 - 20$ $= 40$ Since ΔSTR is an isocesles triangle $\angle STR = \frac{180 - 40}{2}$ (base angles of isosceles triangle) $= 70$	Find $\angle RST$.

	(d)	Show that QR is parallel to ST, hence find the area of ΔPVT if area of triangle $\Delta PQR = 22.7\ \text{cm}^2$.	[4]

Solutions/Alternative Methods	**Skills/Concept**
Since ΔPQR is congruent to ΔSTP, $\angle PQR = \angle STP$ (corresponding $\angle$s of congruent triangles) $= 80°$ By converse of corresponding angles, QR is parallel to ST.	Use of congruent rule Corr. angles, parallel lines
OR $\angle TSR = 40°$ $\angle SRQ = 60° + 80° = 140°$ $\angle TSR + \angle SRQ = 40° + 140° = 180°$. By converse of interior angles, QR is parallel to ST.	
OR $\angle VRQ = 180° - 40° - 60° = 80°$ (angle sum in a triangle) $\angle SVR = \angle VRQ = 80°$. By converse of alternate angles, QR is parallel to ST.	
ΔPTV is similar to ΔPQR Area of $\Delta PTV = \left(\frac{4}{11.52}\right)^2 \times 22.7$ $= 2.7368\ \text{cm}^2$ $= 2.74\ \text{cm}^2$	Areas of similar triangles

8 A roof in the shape of a triangular right prism is constructed as shown below such that $ABCD$ is a rectangle, $ADFE$ and $BCFE$ are squares. $AE = BE = 12$ m, $\angle AEB = 120°$ and M is the midpoint of AE.



Find

(a) the area of triangle ABE, [2]

Solutions/Alternative Methods	Skills/Concept
area of triangle $ABE = \frac{1}{2} \times 12 \times 12 \times \sin 120$ $= 62.354$ $= 62.4 \text{ m}^2$	Area of triangle involving sine

(b) AB^2, [2]

Solutions/Alternative Methods	Skills/Concept
$AB^2 = 12^2 + 12^2 - 2(12)(12)\cos 120$ $= 432$	Cosine rule

(c) AC, [2]

Solutions/Alternative Methods	Skills/Concept
$AC = \sqrt{432 + 12^2}$ (Pythagoras' Theorem) $= 24$ m	Pythagoras' Theorem

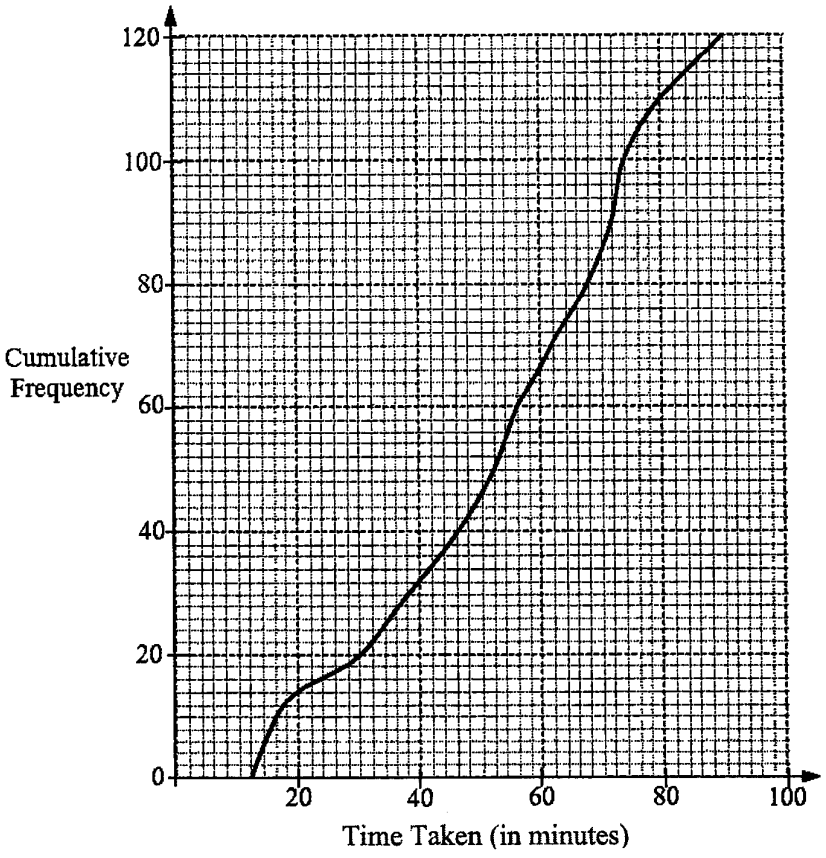
(d) $\angle AEC$, [3]

Solutions/Alternative Methods	Skills/Concept
$EC = \sqrt{12^2 + 12^2}$ (Pythagoras' Theorem) $= \sqrt{288}$ m	Pythagoras' Theorem
$\cos \angle AEC = \frac{12^2 + \left(\sqrt{288}\right)^2 - 24^2}{2(12)(\sqrt{288})}$ $= \frac{-144}{407.293506}$	Cosine rule
$\angle AEC = 110.7048$ $= 110.7$ (1 d.p.)	

	(e)	the largest angle of elevation of M viewed from a point along CD.	[3]

Solutions/Alternative Methods	Skills/Concept
Let the point directly below M at AB be P and let the point be Q on CD. $AM = 6$ m $\angle PAM = 30^\circ$ (base angle of isocesles triangle) $PM = 6\sin 30^\circ$ $= 3$ m $PQ = 12$ m $\tan \angle PQM = \frac{3}{12}$ $\angle PQM = \tan^{-1}\left(\frac{1}{4}\right)$ $= 14.036$ $= 14.0$ (1 d.p.)	Sine Trigo Ratio Tangent Trigo Ratio

9 **(a)** The cumulative frequency curve below shows the distribution of the time taken (in minutes) by 120 students in School *A* to complete an assignment.



Below is the grouped frequency table for the time taken by the students.

Time taken (in minutes)	$0 < x \leq 20$	$20 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 80$	$80 < x \leq 100$
Frequency	14	***a***	34	***b***	10

(i) Find the value of ***a*** and of ***b***. [1]

Solutions/Alternative Methods	**Skills/Concept**
$a = 18, \quad b = 44$	Convert Cumulative frequency to frequency

(ii) Estimate the mean time taken by the students. [1]

Solutions/Alternative Methods	**Skills/Concept**
$\text{mean time} = \dfrac{14\times10+18\times30+34\times50+44\times70+10\times90}{120}$ $= 53 \text{ min}$	Use calculator to find mean

(iii) Estimate the standard deviation of the time taken by the students. [1]

Solutions/Alternative Methods	Skills/Concept
$\text{SD} = \sqrt{\frac{14\times10^2+18\times30^2+34\times50^2+44\times70^2+10\times90^2}{120}-53^2}$ $= \sqrt{\frac{399200}{120}-2809}$ $= 22.752289$ $= 22.8$ min (3 s.f.)	Use calculator to find S.D

(iv) 120 students from School *B* completed the same assignment, and the analysis of their time taken is represented in the table below.

Mean time taken	60
Standard deviation	13.6

Make two comments comparing the time taken by the students from the 2 schools. [2]

Solutions/Alternative Methods	Skills/Concept
On average, students from school *B* took longer to complete the same assignment as their mean time taken of 60 minutes is 7 minutes longer than the mean time taken of 53 minutes by students from school *A*.	Comparing of data <u>in context</u> using mean by stating the difference
The spread of the time taken to complete the assignment for students from school *A* is wider compared to students from School *B* as their standard deviation of 22.8 minutes is 9.2 minutes higher than School *B*'s 13.6 minutes. The time taken by the students from School *B* is more homogeneous.	Comparing of data <u>in context</u> using S.D. by stating the difference

(b) Ali, Bryan and Chandra took part in a game of dart throwing. The probabilities that Ali, Bryan and Chandra will hit the target in a single throw are $\frac{1}{6}$, $\frac{1}{5}$ and $\frac{1}{4}$ respectively.

(i) For the first game, all three of them throw the dart at the target at the same time. Find the probability that all of them hit the target. [2]

Solutions/Alternative Methods	Skills/Concept
$\text{P(all of them missed)} = \frac{1}{6}\times\frac{1}{5}\times\frac{1}{4}$ $= \frac{1}{120}$	Probability of independent events

(ii) In the second game, they each make a single throw of the dart at the target in the order of Ali, Bryan and Chandra. For this game, once the target is hit, the game will end. Find the probability the target is hit. [3]

Solutions/Alternative Methods	Skills/Concept
$\text{P(hit the target)} = \frac{1}{6}+\left(\frac{5}{6}\times\frac{1}{5}\right)+\left(\frac{5}{6}\times\frac{4}{5}\times\frac{1}{4}\right)$ $= \frac{1}{2}$	Probability of independent events & mutually exclusive events

10 Daryl owns a concert hall with a full capacity of 120 seats. He conducted a survey to find out how much to charge for tickets. The detail of the survey is below:

Price of one ticket	Number of people who will attend the concert
\$6.00	120
\$7.50	110
\$9.00	100
\$10.50	90

(a) Write down the revenue he will get if all 120 seats are sold. [1]

Solutions/Alternative Methods	Skills/Concept
Revenue $=120\times 6$ $=\$720$	

(b) Daryl noticed that for every \$1.50 increase in the price of one ticket, the number of people who attend the concert drops by 10.

(i) Find the revenue if he makes **three** \$1.50 increases to the price from \$6. [1]

Solutions/Alternative Methods	Skills/Concept
Price after increase $=6.00+3(1.50)$ $=\$10.50$ From the table when ticket at \$10.50, 90 people will attend Revenue $=90\times 10.50$ $=\$945$	

(ii) Let n be the number of \$1.50 increase in the price of the tickets, explain why the revenue R, in dollars is given by $720+120n-15n^2$. [3]

Solutions/Alternative Methods	Skills/Concept
Amount increase $=1.50n+6$ Number of people who will attend $=120-10n$ Revenue $=(1.50n+6)(120-10n)$ $=180n-15n^2+720-60n$ $=720+120n-15n^2$ (shown)	Find amount increase The drop in number who attend Form expression for revenue

(iii) Explain why the number of \$1.50 increase in price should be less than 12. [1]

Solutions/Alternative Methods	Skills/Concept
When $n>12$, Number of people who will attend is $120-10n<0$ $\therefore$ the number of increase of \$1.50 should not >12.	

(iv) By drawing a suitable graph for $n<12$ on the grid opposite, work out how much should Daryl charge his ticket to maximum revenue. [4]

Solutions/Alternative Methods	Skills/Concept
Graph below: must include table of values & scales as they are not given in the question. From the graph, since revenue is maximum at $n=4$, He should charge $= 4(1.50)+6=\$12$	Sufficient points (at least 6) to draw a smooth curve Smooth curve passing through all points Know max revenue is at $n=4$

n	0	1	2	3	4	5	6	7	8	9	10	11
R	720	825	900	945	960	945	900	825	720	585	420	225