2023 Sec 4E/5N Prelim Math Paper 2 Marking Scheme

QN	Solution	Marks	AO Level
1a(i)	$p=\frac{4}{25}$ or 0.16	B1	N5 AO1
1a(ii)	$p=\frac{2+f}{4f-1}$ $p(4f-1)=2+f$ $4pf-p=2+f$ $4pf-f=2+p$ $f(4p-1)=2+p$ $f=\frac{2+p}{4p-1}$	M1 (x multiply) A1	N5 AO1
1b	$\frac{3}{x-2}-\frac{4}{2x+3}$ $=\frac{3(2x+3)-4(x-2)}{(x-2)(2x+3)}$ $=\frac{6x+9-4x+8}{(x-2)(2x+3)}$ $=\frac{2x+17}{(x-2)(2x+3)}$	M1 (common denominator A1	N5 AO1
1c	$3x-2y=56$----(1) $3y+5x=1.5$----(2) (1)x3, (1)x2 $9x-6y=168$----(3) $6y+10x=3$----(4) (3)+(4) $19x=171$ $x=9,\ y=-14.5$	 M1 (for subst/elim method) A1, A1	N7 AO1

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1d	$\frac{y+2}{2}-\frac{10}{3y+2}=0$ $\frac{y+2}{2}=\frac{10}{3y+2}$ $(y+2)(3y+2)=20$ $3y^2+2y+6y+4-20=0$ $3y^2+8y-16=0$ $(3y-4)(y+4)=0$ $3y-4=0$ or $y+4=0$ $y=\frac{4}{3}$ $y=-4$	M1 (Form Quadratic Eqn) A1, A1	N7 AO1
2a	CPF funds $0.20\times 6000=\$\,1200$ Ordinary Account $63\%\times 4800=\$\,756$	B1 B1	N3 AO1
2b	Amount Petrol used $4.4\times 12.5=55l$ Petro Cost $55\times 2.70=\$\,148.50$	B1/M1 B1/A1	N4 AO1
2c	Return after 1st yr 0.025×10000 $=\$\,250$ Return after 2nd yr $0.0865\times 10000-250$ $=\$\,615$ Let x % be rate of interest for 2nd year $\frac{10250x}{100}=615$ $x=6$	M1 A1	N3 AO2

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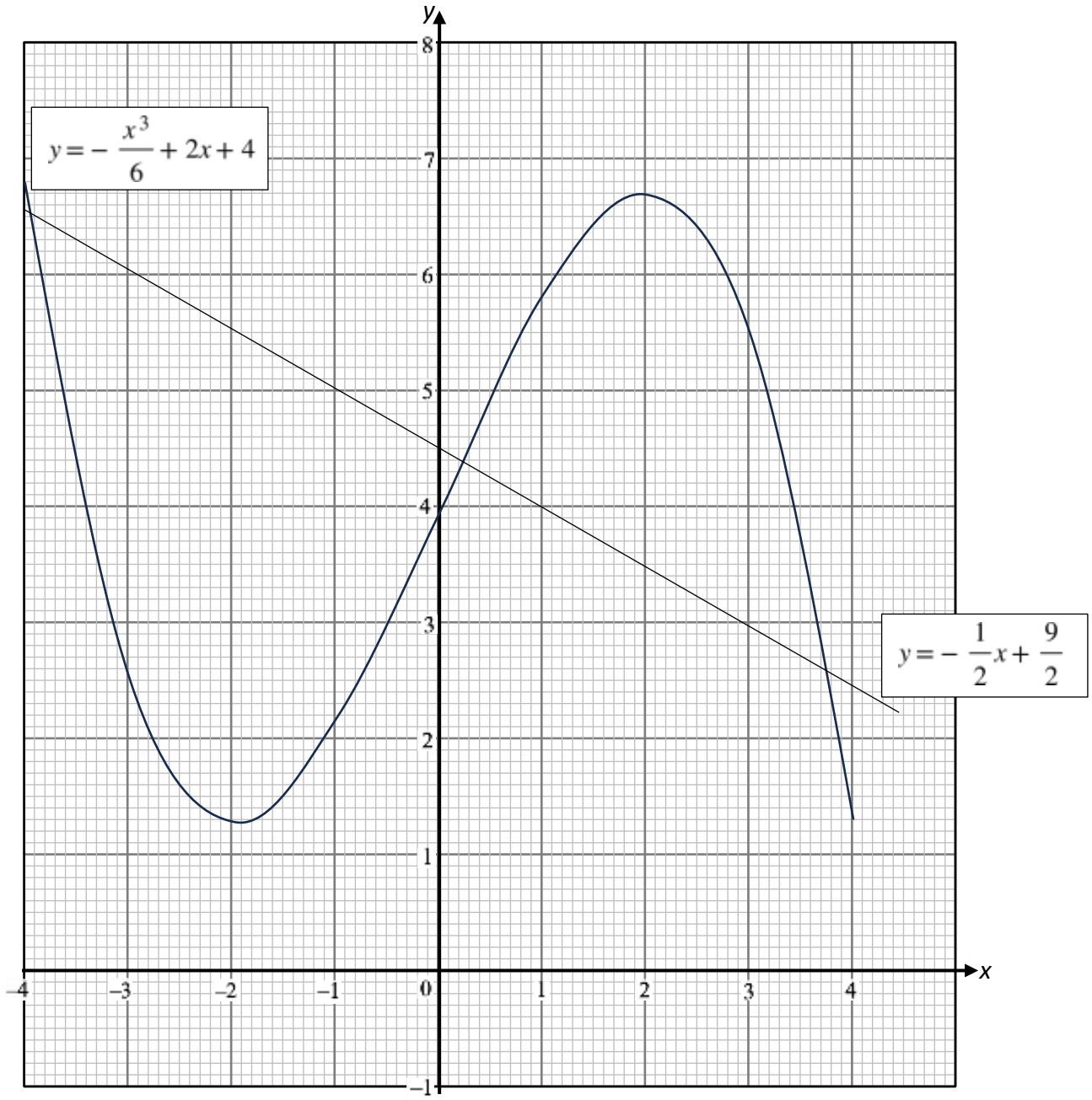
2d	Option B $1.015\left(\frac{50000}{108}\right)$ $= \$\ 469.907$ $= \$\ 469.91\ (2dp)$ Option A because the total charge for Option B is higher than Option A	M1 (converting to SGD) M1 (1.5% fee) A1 (only if Option B is calculated correctly	N10 AO3
3a(i)	150 min	B1	S1/AO1
3a(ii)	LQ = 138 UQ = 160 IQR = 160 – 138 = 22	M1 for LQ/UQ A1	S1 AO1
3b	$200\times 2 = 400$ mins $= 6$ hr 40 mins	B1	S1 AO1
3c	Between 140 mins 160 mins Airlines A has 90 – 34 = 56 movies Airlines B has 106 – 14 = 92movies Airlines B because there are more movies for Ali to choose.	M1 for finding no of movies A1	S1 AO3
3d	Prob of first movie is still playing $= \frac{120-13}{120} = \frac{107}{120}$	B1	S2 AO2
3e	Prob of choosing at least 2 comedy movies $\frac{60\times 59\times 60}{120\times 119\times 118}\times 3 + \frac{60\times 59\times 58}{120\times 119\times 118}$ $= \frac{1}{2}$	M1 A1	S2 AO1
4a	5.8 (1dp) Graph_	B1 P2 all points plotted correct P1 for 7 points plotted correct else P0 C1	N6/AO1

4a



QN	Solution	Marks	AO Level
4b	$1.3 < k < 6.7 \ \pm 0.1$	B1	N6 AO2
4c(i)	$x^3 - 15x + 3 = 0$ $-x^3 + 15x - 3 = 0$ $-x^3 + 12x - 3 = -3x$ $-x^3 + 12x + 24 = -3x + 27$ $\frac{-x^3 + 12x + 24}{6} = \frac{-3x + 27}{6}$ $\frac{-x^3}{6} + 2x + 4 = -\frac{1}{2}x + \frac{9}{2}$ $a = -\frac{1}{2},\ b = \frac{9}{2}$	B1, B1	N6 AO2
4c(ii)	Refer to graph $x = -3.9 \pm 0.1$ $x = 0.25 \pm 0.1$ $x = 3.75 \pm 0.1$	M1 for drawing $y = -\frac{1}{2}x + \frac{9}{2}$ A2 for all correct A1 for 1 or 2 correct	N6 AO2
5a(i)	$\begin{pmatrix} -2 \\ 8 \end{pmatrix}$	B1	G7 AO1
5a(ii)	$\overrightarrow{PQ} = \sqrt{(-2)^2 + 8^2}$ $= \sqrt{68}$ $= 8.246211$ $= 8.25\ (3sf)$	B1	G7 AO1

QN	Solution	Marks	AO Level
5a(iii)	$\overrightarrow{PR}=m\overrightarrow{RQ}$ $\overrightarrow{OR}-\overrightarrow{OP}=m(\overrightarrow{OQ}-\overrightarrow{OR})$ $\begin{pmatrix}2\\k\end{pmatrix}-\begin{pmatrix}8\\-4\end{pmatrix}=m\left(\begin{pmatrix}6\\4\end{pmatrix}-\begin{pmatrix}2\\k\end{pmatrix}\right)$ $\begin{pmatrix}-6\\k+4\end{pmatrix}=m\begin{pmatrix}4\\4-k\end{pmatrix}$ $-6=4m$ $m=-\frac{3}{2}$ $k+4=\left(-\frac{3}{2}\right)(4-k)$ $k=20$ $\overrightarrow{OR}=\begin{pmatrix}2\\20\end{pmatrix}$	M1 A1	G7 AO2
5b(i)	$\overrightarrow{BD}=-4\boldsymbol{a}+8\boldsymbol{b}$ $\overrightarrow{BE}=\frac{3}{4}\overrightarrow{BD}$ $\overrightarrow{BE}=\frac{3}{4}(-4\boldsymbol{a}+8\boldsymbol{b})$ $=-3\boldsymbol{a}+6\boldsymbol{b}$	M1 A1 –1M from whole qn if there is no vector notation	G7 AO1
5b(ii)	$\overrightarrow{BF}=\overrightarrow{BE}+\overrightarrow{EF}$ $=-3\boldsymbol{a}+6\boldsymbol{b}+2\boldsymbol{a}-2\boldsymbol{b}$ $=4\boldsymbol{b}-\boldsymbol{a}$ $\overrightarrow{BC}=2(4\boldsymbol{b}-\boldsymbol{a})$ $=8\boldsymbol{b}-2\boldsymbol{a}$	M1 A1	G7 AO1

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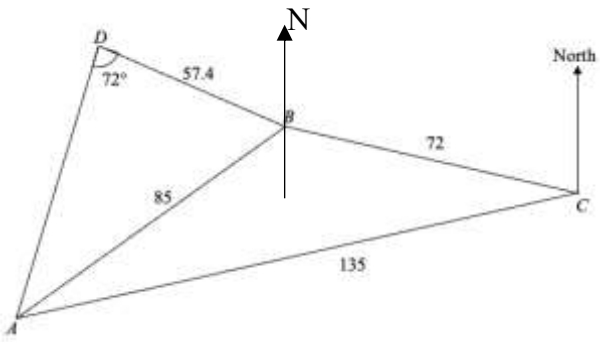
5b(iii)	$\overrightarrow{DC} = \overrightarrow{DB} + \overrightarrow{BC}$ $= 4a - 8b + 8b - 2a$ $= 2a$ $\overrightarrow{AB} = 2\overrightarrow{DC}$ Since vector *AB* is a scalar multiple of vector *DC*, *AB* is parallel to *DC*. *ABCD* is <u>trapezium</u> because <u>*AB* is parallel to *DC*</u>.	M1 M1 A1 Must explain scalar multiple therefore *AB* is // *DC*	G7 AO3
6(a) (i)(a)	angle *EBC* = 64º (angles in the same segment)	B1	G3 AO1
6(a) (i)(b)	angle *EOC* = 2 × 64º = 128º (angles at centre = 2 × angles at circumference) reflex angle *EOC* = 360º-128º = 232º (angles at a point)	 B1	G3 AO1
6(a) (i)(c)	angle *EBC* = 64º (angles in the same segment) angle *EDC* = 180º – 64º = 116º (angles in opposite segments) angle *EDG* = 180º – 116º – 46º =18º (adj angles on a str line)	 M1 A1 For Qn 6a(i) minus 1 mark if no/wrong reason given	G3 AO1
6(a) (ii)	*CH* = *DH* (tangent from ext point) angle *CHD* = 180º–2(46º) = 88º Since angle *CHD* does not form a right angle in a semi circle, therefor a semicircle with *CD* as diameter will not pass through *H*.	 M1 A1	G3 AO3

QN	Solution	Marks	AO Level
6b(i)	A: angle *ORS* = angle *PRQ* (common angle) Angle *OSR* = x^o (base angle of isosceles triangle) Angle *OSP* = $180^o - x^o$ (adj angles on a str line) Angle *PQR* = x^o (angles in opposite segments) A: Angle *PQR* = angle *OSR* = x^o By AA, triangles *ORS* and *PRQ* are similar.	M1 showing both corr angles No mark if no/wrong reason A1	G2 AO3
6b(ii)	Since $x^o = 60^o$, triangle *ORS* is equilateral *OR* : *QR* 1 : 2 triangle *ORS* : triangle *PRQ* 1 : 4 area of triangle *ORS* and quadrilateral *PQOS*. 1 : 3	 B1 B1 or B2	G2 AO1
7a	$(4x+10)(x+20) = 13550$ $4x^2 + 80x + 10x + 200 - 13550 = 0$ $4x^2 + 90x - 13350 = 0$ $2x^2 + 45x - 6675 = 0$	M1 M1 simplification AG1	N7 AO3
7b	$2x^2 + 45x - 6675 = 0$ $x = \frac{-45 \pm \sqrt{45^2 - 4(2)(-6675)}}{2(2)}$ $x = \frac{-45 \pm \sqrt{55425}}{4}$ $x = 47.606$ or $x = -70.106$ $x = 47.61\ (2dp)$ or $x = -70.11\ (2dp)$	 M1 A1, A1	N7 AO1

7c	Area of paper $=4(47.606) \times 47.606$ $=9065.3249\ cm^2$ $=0.90653249\ m^2$ Cost of paper $=63 \times 0.90653249$ $=\$\ 57.1115$ $=\$\ 57.11\ (2dp)$	M1 for Area M1 for conversion to m^2 A1	G5 AO2

8a	 $135^2 = 85^2 + 72^2 - 2(85)(72)\cos\angle ABC$ $18225 = 7225 + 5184 - 12240\cos\angle ABC$ $-12240\cos\angle ABC = 18225 - 7225 - 5184$ $\cos\angle ABC = \frac{5816}{-12240}$ $\angle ABC = \cos^{-1}\left(\frac{5816}{-12240}\right)$ $\angle ABC = 118.36998°$ $\angle ABC = 118.4°\ (1dp)$ Angle *NBC* = 180º – (360º – 290º) =110º Bearing of *A* from *B* = 118.4º + 110º =228.4º (1dp) OR $\frac{\sin\angle DAB}{57.4} = \frac{\sin 72°}{85}$ $\angle DAB = \sin^{-1}\left(\frac{\sin 72°}{85} \times 57.4\right)$ $= 39.959°$ $\angle DBA = 180° - 72° - 39.959°$ $= 68.041°$ Bearing of *A* from *B* = 360º – (70º + 68.041º) =222.0º (1dp)	M1 M1 A1 B1 OR M1 A1 M1 A1	G4 AO2

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8b	$\frac{\sin\angle DAB}{57.4}=\frac{\sin 72^\circ}{85}$ $\sin\angle DAB=\frac{\sin 72^\circ}{85}\times 57.4$ $\angle DAB=\sin^{-1}\left(\frac{\sin 72^\circ}{85}\times 57.4\right)$ $\angle DAB=39.95926^\circ \ or\ 180^\circ-39.95926^\circ\ (rej)$ $\angle ABD=180^\circ-39.95926^\circ-72^\circ=68.04074^\circ$ Area of ABD $=\frac{1}{2}\times 57.4\times 85\times \sin 68.04074^\circ$ $=2262.614527$ $=2260\ m^2\ (3sf)$	M1 A1 M1 A1	G4 AO2

8c	Let DX be the shortest distance from D to AB $\sin\angle ABD = \frac{DX}{57.4}$ $DX = \sin 68.04074° \times 57.4$ $DX = 53.235629\ m$ Or $\frac{1}{2}(85)(DX) = 2262.614527$ $DX = \frac{2\times 2262.614527}{85}$ $= 53.2379889$ Let largest angle of depression from the top of the mast to point along the path AB be θº $\tan\theta = \frac{40}{DX}$ $\tan\theta = \frac{40}{53.235629}$ $\theta = \tan^{-1}\left(\frac{40}{53.235629}\right)$ $\theta = 36.9203$ $\theta = = 36.9°\ (1dp)$	M1 OR M1 M1 A1	G4 AO2

QN	Solution	Marks	AO Level
9(a)	Measure distance = 11.9 to 13 cm 1 : 4 050 000 1 cm: 4 050 000 cm 1 cm : 4 050 0 m 1 cm : 40.5 km Driving distance = 12.4 × 40.5 km = 502 km (nearest km)	M1 for measuring in cm A1 accept 482 to 527 km	N2 AO1
9b	Osaka to rest stop 1 $dist = 90 \times 2 = 180\ km$ Rest stop 1 to 2 $time = \frac{200}{100} = 2hr$ Rest stop 2 to Tokyo $time = \frac{502.2 - 180 - 200}{110} = 1.1109hr$ Total duration $2 + 2 + 1.1109 + 0.5 + 0.5$ $= 6.11\ hr$ $= 6\ hrs\ (\ nearest\ hour)$	 M1 502.2 or their A1	N10 AO1

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9c	Fuel per way 5.022* × 8.2 = 41.1804*l* Total fuel used 41.1804*l* × 2 = 82.3608*l* Fuel top up cost = 82.3608 × 170 =¥14 001.336 <u>Parking</u> ¥6000 × 5 =¥30 000 <u>Toll Charges</u> ¥13 500 × 2 =¥27 000 <u>Total Cost</u> ¥90 000 + ¥14 001.336+¥30 000+¥27 000 =¥161 001.336 <u>Bullet Train</u> 1.45(¥18 000 × 2 × 3) =¥156 600 Conclusion Cheaper to travel by bullet train	*their distance M1 B1 B1 M1 Car Rental must be correct their Fuel their Parking their Toll Charges B1 C1 only if previous 5M are awarded.	AO3