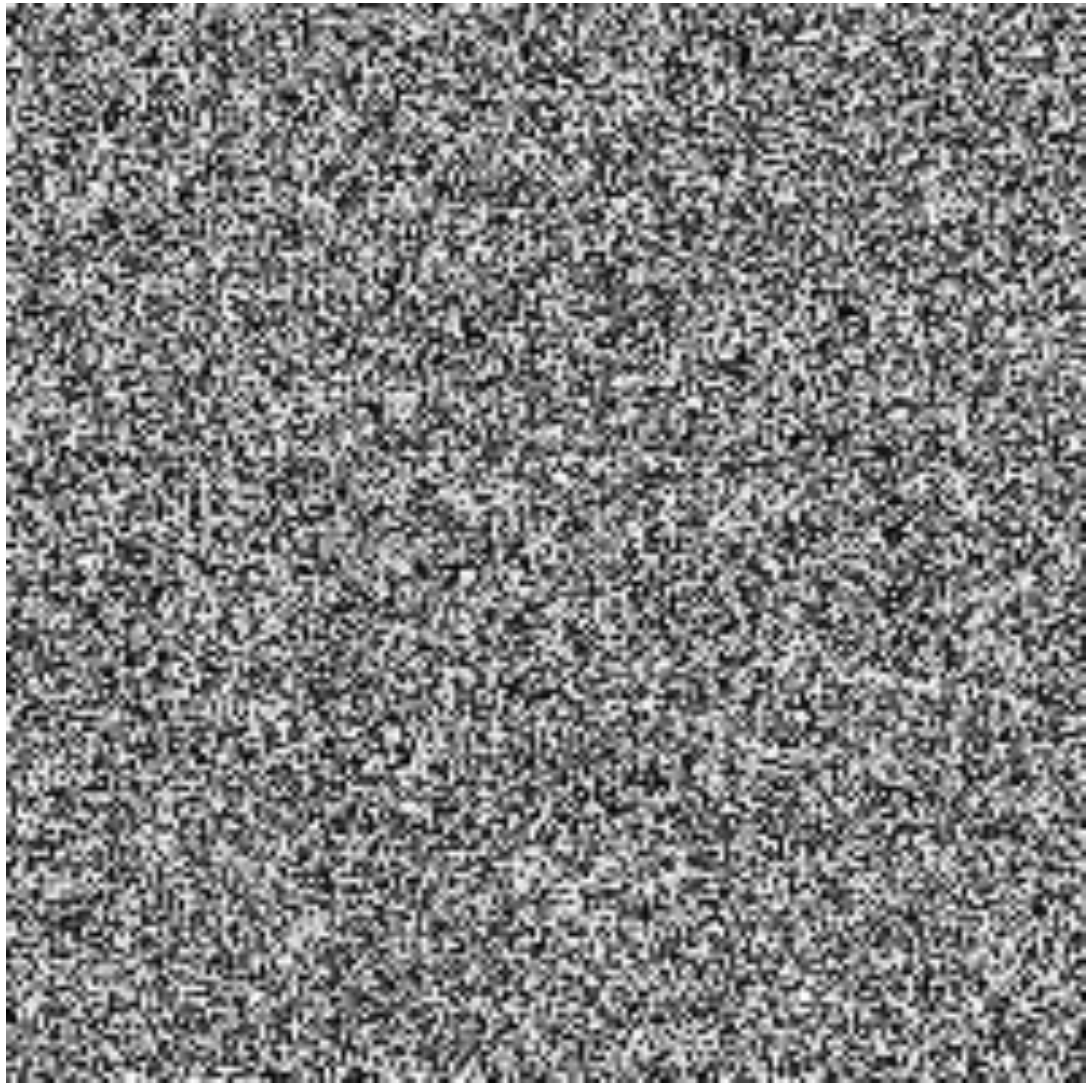
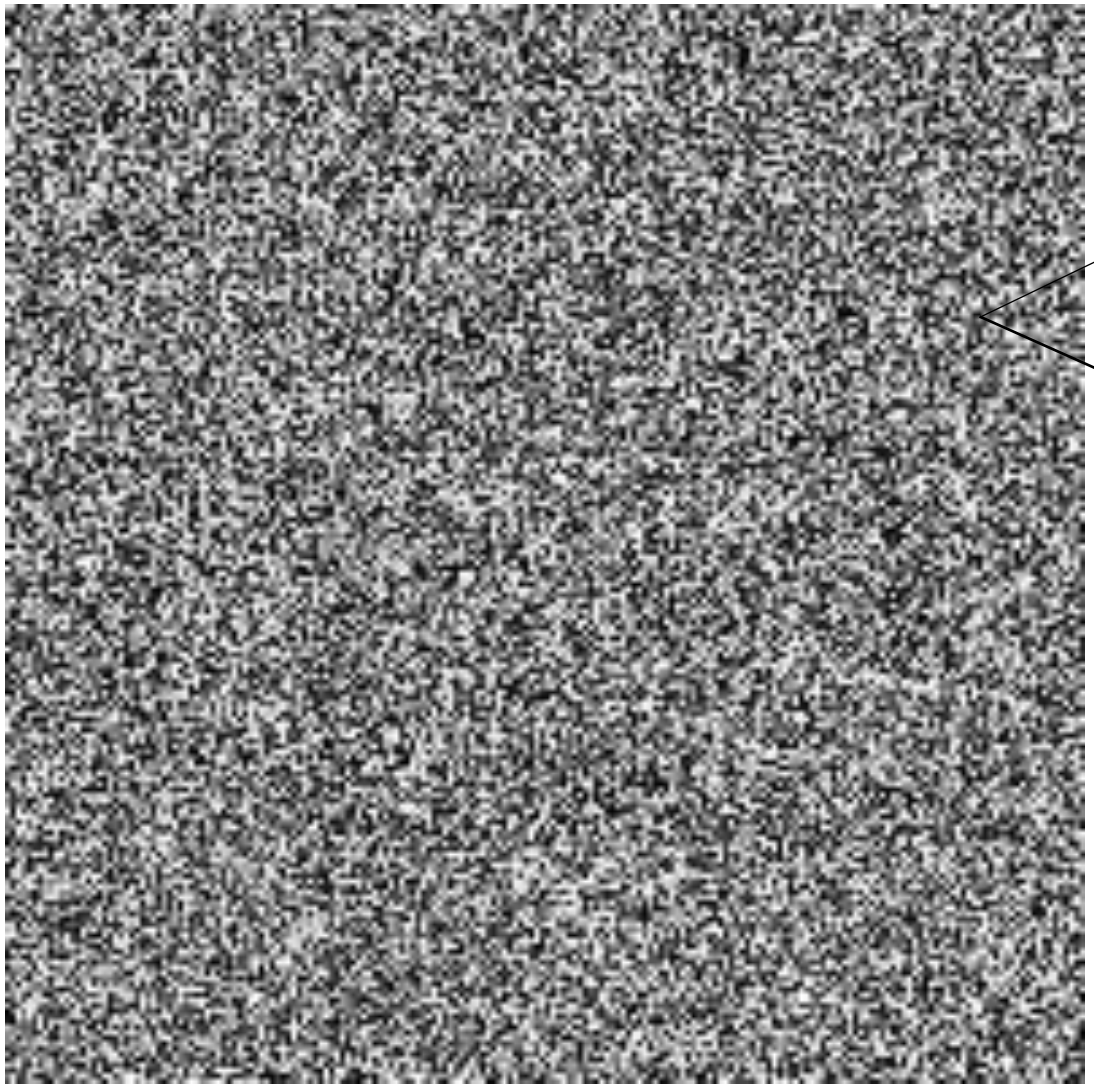
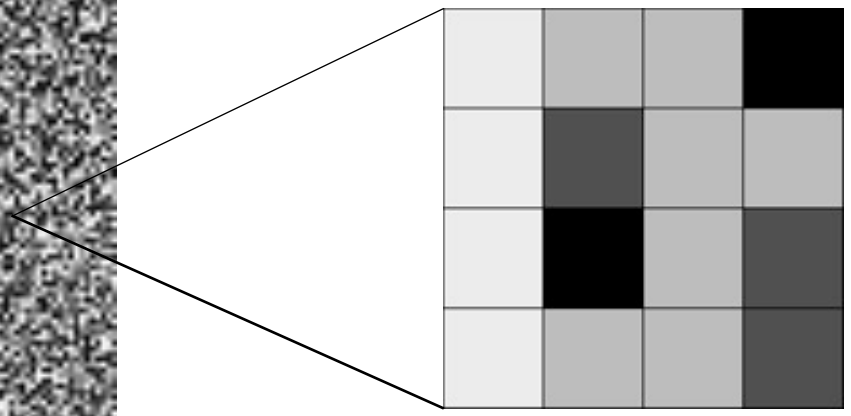
Lesson 8

2 Dimensional Array – Nested Lists

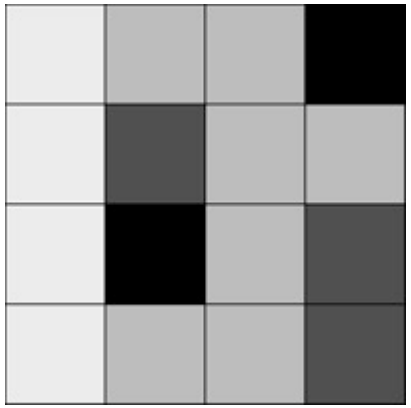






A table with 4 rows and 4 columns

Representing . . .



A table with 4 rows and 4 columns

255	185	185	0
255	96	185	185
255	0	185	96
255	185	185	96

We can represent a table with **n** rows and **m** columns into a **n by m** matrix.

$$\begin{pmatrix} 255 & 185 & 185 & 0 \\ 255 & 96 & 185 & 185 \\ 255 & 0 & 185 & 96 \\ 255 & 185 & 185 & 96 \end{pmatrix}$$

n rows (horizontal)
by
m columns (vertical)

A Matrix to a 2 dimensional array

$$\begin{pmatrix} 255 & 185 & 185 & 0 \\ 255 & 96 & 185 & 185 \\ 255 & 0 & 185 & 96 \\ 255 & 185 & 185 & 96 \end{pmatrix}$$

```
[[255,185,185,0],
 [255,96,185,185],
 [255,0,185,96],
 [255,185,185,96]]
```

Nested lists in Python: a list in a list

Lesson Objective

- manipulate two dimensional array : nested lists
  - using matrix as example

Notation of Matrix

Notation

A matrix is usually shown by a **capital letter** (such as A, or B)

Each entry (or "element") is shown by a **lower case letter** with a "subscript" of **row,column**:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{bmatrix}$$

Operations and Methods to Matrices

- print
- finding maximum
- addition , subtraction
- scalar multiplication
- transpose
- matrix multiplication (if possible)

creating a null matrix : matrix with all 0

creating a **m** row by **n** column nu

```
def createNull(m,n):
    return [[0]*n]*m
```

- if m = n, the matrix will be squa..
- a matrix need not be squar

```
def createNull(m,n):
    return result
```

this does not work. Why? so how?

printing a matrix

printing a matrix row by row:

```
def printM(mat):
    for row in mat:
        print(row)
```

finding maximum value in a matrix :

for a **1 by n** matrix:

```
def maxM(mat):
    result = mat[0][0]
    col = len(mat[0])
    for i in range(col):
        if mat[0][i] > result:
            result = mat[0][i]
    return result
```

How to generalize it to a **m by n** matrix ?

Matrix Addition

Example: Given $\mathbf{A} = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 7 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 4 & 1 \end{bmatrix}$, find A + B.

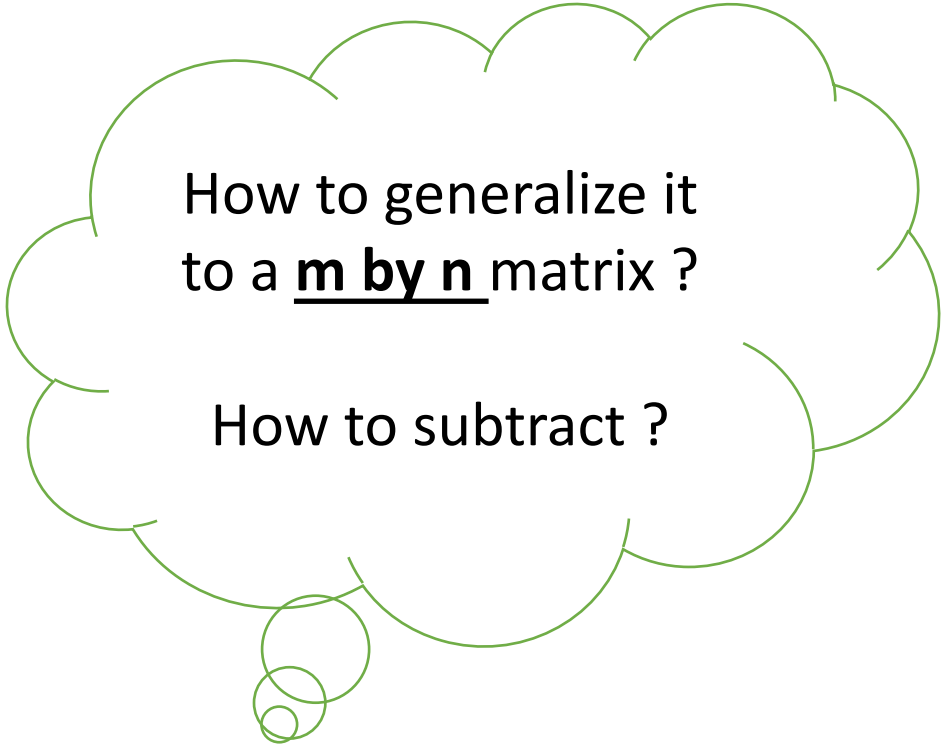
Solution:

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 7 \end{bmatrix} + \begin{bmatrix} 4 & 2 & 3 \\ -1 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4 & 3+2 & 4+3 \\ 1-1 & 5+4 & 7+1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 5 & 7 \\ 0 & 9 & 8 \end{bmatrix}$$

Matrix Addition



for a **m by 1** matrix:

```
def addM(matA,matB):
    row = len(matA)
    result = createNull(row,1)
    for i in range(row):
        result[i][0] = matA[i][0]+matB[i][0]
    return result
```

Scalar Multiplication

MULTIPLICATION OF A MATRIX BY A SCALAR:

If $A = [a_{ij}]_{mxn}$ is a matrix and k is a scalar, then k.A is another matrix which is obtained by multiplying each element of A by the scalar k, that is,

$k.A = k[a_{ij}]_{mxn} = [k(a_{ij})]_{mxn}$ which means that (i, j)th element of kA is ka_{ij}.

Example:

If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \end{bmatrix}$ then find 2A.

Solution:

$$2A = 2\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ -2 & 0 & 8 \end{bmatrix}$$

Write a function `scalarM` that takes in a matrix and a scalar and outputs the scalar multiplication ?

Transpose of a Matrix

If $A = [a_{ij}]$ be a matrix of order m x n, then the matrix obtained by interchanging the rows and columns of A is known as the transpose of A. It is represented by A^T.

Hence if $A = [a_{ij}]_{mxn}$, then $A^T = [a_{ji}]_{mxn}$

Example: If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then $A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$

Write a function `transpose` that takes in a matrix and returns the transpose of the matrix.

Matrix Multiplication

If $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then find AB.

Solution:

1st row and 1st column

$$\mathbf{A} \times \mathbf{B} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 1 + 3 \times 3 & 2 \times 2 + 3 \times 4 \\ 4 \times 1 + 5 \times 3 & 4 \times 2 + 5 \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 9 & 4 + 12 \\ 4 + 15 & 8 + 20 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 16 \\ 19 & 28 \end{bmatrix}$$

Hence $\mathbf{AB} = \begin{bmatrix} 11 & 16 \\ 19 & 28 \end{bmatrix}$

For more examples:
https://www.mathsisfun.com/algebra/matrix-multiplying.html

If A is a **p by q** matrix and B is a **q by r** matrix, AB is a **p by r** matrix

If A is a **3 by 4** matrix and B is a **4 by 2** matrix, AB is a **3 by 2** matrix

Write a function `multiplication` that takes in two matrices and returns the multiplication of the two matrices.

Work to do . . .

- 8A – List
- 8B – List 2
- Programming Assignment 7 : Matrix 1