MEASUREMENTS

SI base units

In the SI system of units, there are **seven** base quantities and their units. These 7 quantities are assumed to be mutually independent.

Base Quantities	SI Base Units	
	Name	Symbol
Length	metre	m
Mass	kilogram	kg
Time	seconds	s
Current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous Intensity	candela	cd

Prefixes (Order of magnitudes)

Can be added to SI base units and derived units to make larger or smaller units. (e.g. 1 ms = 10^{-3} s, 1 km = 10^{3} m)

Name	Symbol	Factor
pico	p	$\times 10^{-12}$
nano	n	$\times 10^{-9}$
micro	μ	$\times 10^{-6}$
milli	m	$\times 10^{-3}$
centi	c	$\times 10^{-2}$
deci	d	$\times 10^{-1}$
kilo	k	$\times 10^{3}$
mega	M	$\times 10^{6}$
giga	G	$\times 10^{9}$
tera	T	$\times 10^{12}$

Homegeneity of Physical Equations

- Each term in an homogeneous equation has the same units.
- For any two quantities to be **equated, added or subtracted**, they must have the same dimension (or units).
- Homogeneous equation may not be correct (e.g. missing terms / coefficients) but a non-homogeneous equation must be wrong.

Derived Quantities

Derived quantities are physical quantities expressed in terms of one or more base quantities.

General rules for determining the units of derived quantities:

1) For addition / subtraction of two or more quantities, each quantity must have the SAME unit.
2) For multiplication / division, rules of algebraic multiplication and division apply.
3) Exponents (powers/indexes) are unitless.

Systematic Error & Random Error

Systematic errors have the same magnitude and sign when measurements are repeated.

Causes: Instrument errors (e.g. zero errors), Environmental conditions, Poor experimental techniques (e.g. parallax error).

Systematic error can be eliminated if the source of the error is known.

Random errors have different magnitudes and signs (i.e. varying both magnitude and direction) when measurements are repeated.

Causes: Variations in environmental conditions, Irregularity of the quantity being measured, Limitation of equipment.

Random error cannot be completely eliminated but can be minimised by finding the average of repeated measurements.

Precision & Accuracy

Precision refers to how close the repeated measured values are to each other, without regard to the true value of the quantity. Repeated measurements which are very close to one another are precise measurements. Thus an experiment which has small random errors (i.e. small spread of readings) is said to have high precision.

Accuracy refers to how close a measured value is to the true value of a quantity. An experiment which has small systematic errors is said to have high accuracy. The average value is close to the true value.

Errors / Uncertainties

When making any measurement, there is always some ***uncertainty / error***.

Experimental uncertainty or error = Measured value – True value

Absolute error / uncertainty – e.g. error / uncertainty in measuring length, $\Delta\ell$

Fractional error / uncertainty – e.g. $\frac{\Delta\ell}{\ell}$ **Percentage error / uncertainty** – e.g. $\frac{\Delta\ell}{\ell}\times 100\%$

General rules for recording measurement with its uncertainty (i.e. $\ell \pm \Delta\ell$):

- Round off errors / uncertainties / absolute uncertainties to 1 s.f. (i.e. $\Delta\ell$ rounded to 1 s.f.)
- Write the measured value to the same decimal place as its error / uncertainty / absolute uncertainty. (i.e. ℓ rounded to same d.p. as $\Delta\ell$)

Consequential Uncertainties

Method 1 - Numerical Substitution

1. Find the "best" value
2. Find the "maximum" value
3. Uncertainty = max value – best value

Method 2 - Formula Method

Addition/Subtraction

If $Y = nA \pm mB$

then $\Delta Y = n\Delta A + m\Delta B$

Multiplication/Division

If $X = nA^{\alpha}.mB^{\beta}$ or $X = \frac{nA^{\alpha}}{mB^{\beta}}$, where α, β, *m* and *n* are numbers,

then $\frac{\Delta X}{X} = \alpha\frac{\Delta A}{A} + \beta\frac{\Delta B}{B}$

These rules can also "stack up", e.g.:

if $Z = \frac{nA^{\alpha}B}{mC^{\beta}D}$, where α, β, *m* and *n* are numbers,

then $\frac{\Delta Z}{Z} = \alpha\frac{\Delta A}{A} + \frac{\Delta B}{B} + \beta\frac{\Delta C}{C} + \frac{\Delta D}{D}$

Important: To determine the error of a quantity, always <u>express the quantity as the subject</u> of the equation <u>first</u>.

Scalars & Vectors

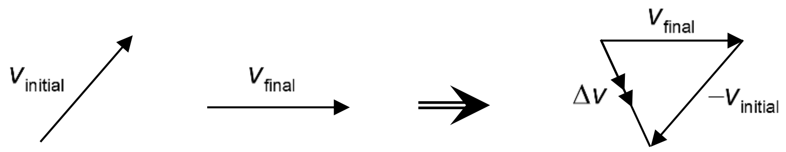
A **<u>scalar quantity</u>** is one with <u>magnitude only</u>.

A **<u>vector quantity</u>** is one that has <u>a magnitude and direction</u> (e.g. displacement, velocity, acceleration, momentum, etc).

Vector addition

- Use to determine the resultant of two vectors.
- Use parallelogram or vector triangle.

Change of a vector (e.g. Δv) refers to final vector – initial vector. (e.g. $\Delta v = v_{final} - v_{initial}$)



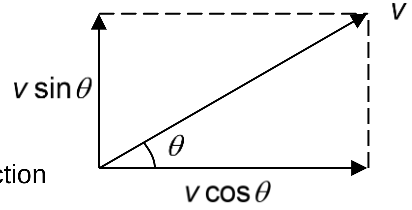
Relative velocity refers to the velocity of one object relative to another moving object.

Suppose body 1 has velocity $\vec{v}_1$ and body 2 has velocity $\vec{v}_2$. Relative velocity of 1 with respect to 2 is represented as $\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$.

Resolution of Vector into two perpendicular components:

Remember:

- The component on the side ***adjacent*** to the angle – **cosine** function
- The component on the side ***opposite*** to the angle – **sine** function



Note: 2 perpendicular vectors are independent of each other.

To determine the **vector sum** of 3 or more vectors:

1. Identify two perpendicular axes to determine components of each vector.
2. Determine the components of each vector.
3. Determine the vector sum of each component for all vectors.
4. Find the resultant of the two net perpendicular components using Pythagoras theorem.

KINEMATICS

Physical Quantities Representing Motion

Scalars:

- **Distance** (x) is the total length moved by an object irrespective of the direction of motion.
- **Speed** of an object is defined as the rate of change of distance travelled.

  → average speed $= \frac{\Delta x}{\Delta t}$

  → instantaneous speed $= \frac{dx}{dt}$

Vectors:

- **Displacement** (s) is the shortest linear distance of the position of a moving object from a given reference point.
- **Velocity** of an object is defined as the rate of change of its displacement.

  → average velocity $= \frac{\Delta s}{\Delta t}$

  → instantaneous velocity $= \frac{ds}{dt}$

- **Acceleration** of an object is defined as the rate of change of its velocity.

  → average acceleration $= \frac{\Delta v}{\Delta t}$

  → instantaneous acceleration $= \frac{dv}{dt}$

  → Acceleration can refer to increase or decrease in velocity. Negative acceleration DO NOT necessary indicate that object is slowing down. To determine if object is speeding up or slowing down (decelerating), compare the directions of the velocity and acceleration vectors.
  If both velocity vector and acceleration vector are in the SAME direction, object speeds up.
  If velocity vector and acceleration vector are in OPPOSITE direction, object slows down.

Graphical Representations of Motion

Features	***s* – *t* graph**	***v* – *t* graph**	***a* – *t* graph**
Axes	Displacement	Instantaneous velocity	Instantaneous acceleration
Gradient	Instantaneous velocity $v = \frac{ds}{dt}$	Instantaneous acceleration $a = \frac{dv}{dt}$	----
Area under graph	----	Net change in displacement	Net change in velocity

Rectilinear Motion (1-D Motion)

Equations of Motion (*suvat*)

- ONLY applicable for motion
  → in a straight line
  → with constant acceleration

- Need to be able TO DERIVE from definitions of velocity and acceleration.

- (1) $v = u + at$

  (2) $v^2 = u^2 + 2as$

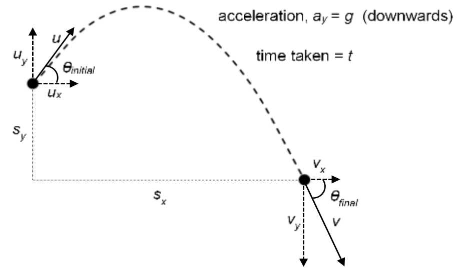
  (3) $s = ut + \frac{1}{2}at^2$

  (4) $s = \frac{1}{2}(u+v)t$

  Remember to take into account sign convention when applying these equations

Projectile Motion (2-D Motion)

- Object projected in a uniform gravitational field with negligible air resistance, results in a parabolic path



- Step 1:
  If you are given the initial velocity, **resolve it into its x and y components.**

- Step 2:
  Analyze the horizontal (x) and vertical (y) motion **separately**.

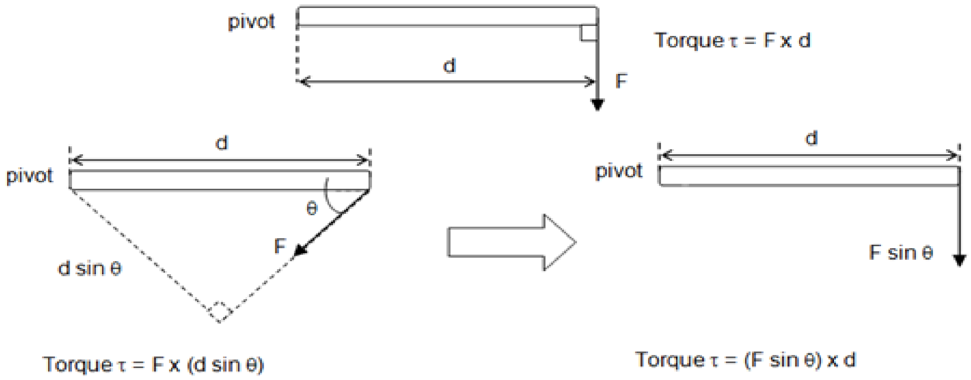
- Step 3:
  Recall that
  (i) time t links the x and y component motions
  (ii) $a_y = 9.81$ m s^{-2} and is directed downwards
  (iii) at max height, $v_y = 0$
  (iv) $v_x = u_x$ (since $a_x = 0$)

- Step 4 :
  Apply the relevant equations of motion (suvat).
  Remember to take sign conventions into consideration!
  *****Only vertical and horizontal components can be used for the equations

FORCES

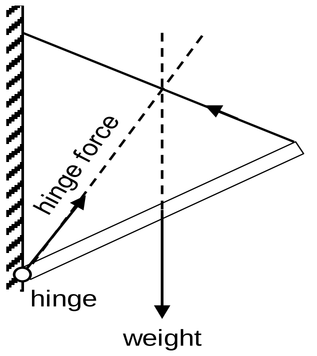
Moment or torque τ

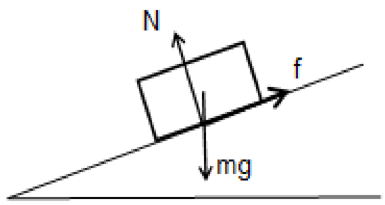
- τ = F d (find *perpendicular* distance *or* force)
- *Torque of a couple* is product one of the forces and the perpendicular distance between the lines of action of the forces



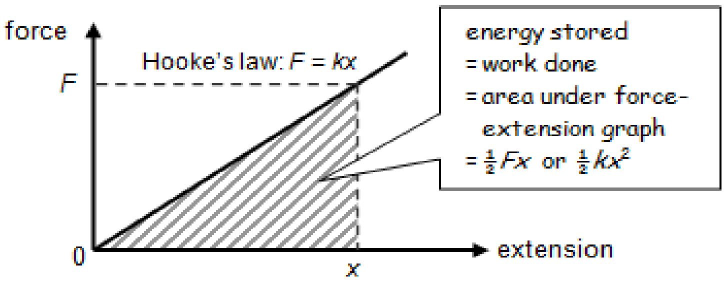
Free Body Diagram (FBD)

- Draw **all** forces acting **on** the body.
- Length & direction of arrow ⇒ Magnitude & direction of force
- Arrow starts from correct point/surface of contact
- **Concurrent point**: three non-parallel forces acting on body in equilibrium





Spring/Elastic force (for extension and compression)



Conditions for (static) equilibrium

1) Resultant force in any direction is zero (translational equilibrium, $\sum F = 0$)
2) Resultant torque about any point is zero (rotational equilibrium, $\sum \tau = 0$)

<u>Problem solving</u>

- Draw FBD
- Use **vector diagram** (closed vector triangle) to represent forces in equilibrium **or resolve forces** (horiz & vert *or* perpendicular & along slope)
- Take moments about *appropriate* point (e.g. where unknown force acts)
- Apply $F_{net} = 0$ & $\tau_{net} = 0$
- Calculate from vector diagram or from resolved components

Pressure due to fluid

$p = \rho g h$, $p_{total} = p_{atm} + \rho g h$

***H2 only*:**

Upthrust (buoyant force)

U = m_f g (wt of fluid displaced) = $\rho_f V_f$ g

Principle of flotation

Object floating in equilibrium: wt of object = upthrust

Drag / Viscous force

- Force resisting a body moving relative to a fluid (e.g. air resistance)
- Always oppose motion
- Magnitude depends on speed of the body, density of the fluid, cross-sectional area, the shape of the body

Dynamics
Newton's Laws of Motion

Newton's 1st Law

A body will continue in its state of rest or move with uniform velocity unless a resultant force acts on it.

The tendency for bodies to resist changes in motion is called *inertia*

Mass is the property of a body which resists change in motion, i.e. a measure of inertia of a body

Linear Momentum and Impulse

The linear momentum of a moving object is defined as the product of its mass and its velocity.

$$p = mv$$

Linear momentum is a *vector* quantity and its direction follows the direction of the velocity.

Impulse of a force is defined as the product of the force and the time during which the force acts (time of impact).

Impulse of a resultant force = $p_{final} - p_{initial}$
= Area under a (resultant) force-time graph

Impulse is a vector quantity.

Problem solving for $F_{net} = ma$ questions

1. Draw free body diagram(s) with the forces acting on the body
2. Identify and indicate the acceleration direction
3. Form an equation of $F_{net} = ma$
4. If need to resolve the forces, form equations for each perpendicular component separately

Newton's 2nd Law

The rate of change of momentum of a body is directly proportional to the resultant force acting on it and the change takes place in the direction of the resultant force.

$$F_{net} \propto \frac{dp}{dt} \quad \Rightarrow \quad F_{net} = k\frac{dp}{dt}$$

If quantities are in SI units, $F_{net} = \frac{dp}{dt}$

Thus, force acting on a body is defined as the rate of change of momentum of the body.

If mass is constant, $F_{net} = ma$

If velocity is constant, $F_{net} = v\frac{dm}{dt}$

Direction of the acceleration or the change in momentum is in the <u>same direction</u> of the resultant force.

Newton's 3rd Law

If body A exerts a force (action) on body B, then body B exerts a force of the same kind, equal in magnitude, but in the opposite direction on body A.

Action-Reaction Pairs:

1. The two forces must be of the same kind
2. The two forces act on different bodies

Weight

Weight of a body is the gravitational force acting on the body.

Apparent Weightlessness

A body is said to be experiencing apparent weightlessness if the resultant force acting on it is its weight (mg) and its acceleration, a, is equal to g.

Conservation of Linear Momentum and Collisions

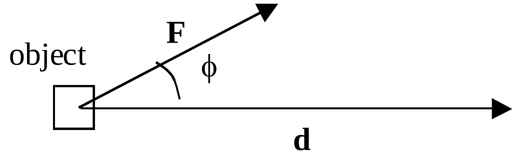
Principle of Conservation of Momentum:
The total momentum of a system of bodies remains constant provided no external resultant force acts on the system.

Type of Collision	Total Linear Momentum	Total Kinetic Energy	Remarks	Laws to Apply
Elastic	Conserved	Conserved		1. $\sum p_{initial} = \sum p_{final}$ 2. $u_1 - u_2 = v_2 - v_1$ <u>or</u> $\sum KE_{initial} = \sum KE_{final}$
Inelastic	Conserved	Not Conserved		$\sum p_{initial} = \sum p_{final}$
Completely Inelastic	Conserved	Not Conserved	bodies stick together after collision	$\sum p_{initial} = \sum p_{final}$

Work, Energy & Power

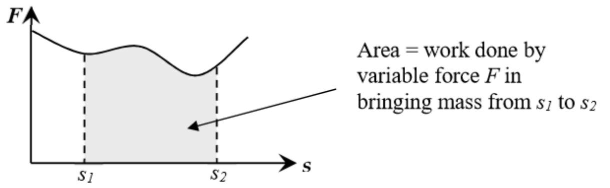
Work and energy are scalar quantities and therefore have no direction.

Work done by a constant force is defined as the product of the *force* and the *displacement* in the direction of the force.



$W = F\,d\cos\phi$

Work done by a variable force is determined by the area under the force displacement graph.



Positive and Negative W.D

Negative W.D. by a force means that energy is taken out of the system while positive W.D. means that energy is put into the system.

Joule

The **joule** (**J**) is defined as the amount of work done when a force of one Newton moves its point of application through a distance of one metre in the direction of the force.

Net work done

The net work done on an object is always equal to the change in K.E of the object.

Work done by gas

$W = P\,(V_f - V_i)$

Where P is a constant external pressure and $(V_f - V_i)$ is the change in volume of the gas.

Conservation of energy

Energy cannot be created or destroyed but converted from one form to another.

Therefore, By Conservation of Energy,

Total initial energy = Total final energy

Provided no external force acts on system (or no change in total energy for isolated system).

For ease of calculations, mechanical energy (See **) is commonly being used instead of total energy.

For Isolated system under conservative forces, free from the influence of a net external force.

Total initial Mechanical Energy
= Total final Mechanical Energy
OR
Total Change in Mechanical Energy = 0

For Non-isolated system, because of the presence of non-conservative forces, we have to account for $W_{n.c}$

Total initial Mechanical Energy + $W_{n.c}$
= Total final Mechanical Energy
OR
Total Change in Mechanical Energy = $W_{n.c}$

**Types of energy included in Mechanical energy

(Mechanical Energy is the Sum of P.E and K.E)

G.P.E = mgh, where h is the vertical height h, and only applicable when near the Earth's surface

K.E = $1/2\ mv^2$

E.P.E = strain/compression energy
= $1/2\ k\,x^2$ (k spring constant and x is the extension of the spring.)

Electrical potential energy is not discussed here but in the topic of Electric field.

Power – rate at which work is being done.

$P = W / t$

which can be derived to get

$P = F\,v$

where F and v are in the same direction.

Efficiency

= useful output power/input power x 100%

Additional notes for students.

Conservative forces are forces where the work done is dependent on position and independent of the path taken, examples are spring force and gravitational force.

For ease of calculations, the lowest position attained by the object is normally taken as zero G.P.E.

For calculations involving connected bodies, the system taken should be that of the bodies that are connected.

The benefits of work energy theorem is that it deals with only magnitude but not direction. Consider ball projected down and projected horizontally at 5 ms^{-1}. K.E is the same although their directions are different.

The derivation of K.E and G.P.E is a requirement in the A level syllabus and can be derived from equations of motion and work done by a force respectively.

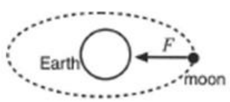
In the use of P = Fv, it is important to know to take note of the relevance of quantities. power delivered by **A** is the force exerted by **A** and the velocity of **A** in the direction of the force exerted by A.

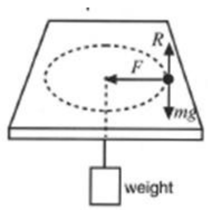
Circular Motion

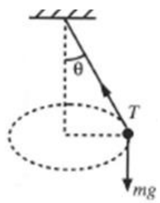
HORIZONTAL Uniform Circular Motion

- For uniform circular motion, there is NO work done by the centripetal force since the direction of the force is always perpendicular to the direction of displacement.

Examples:







Conditions:

- Constant ω and constant r
- $\sum F_{\text{vertical}} = 0$
- $\sum F_{\text{horizontal}} = F_c = ma_c$

Problem-solving:

- Draw the FBD (all individual forces) for the object undergoing circular motion.
  (do NOT draw the centripetal force)
- Locate the centre of the circular motion.
- Resolve the forces.
- Determine the force(s) that is(are) directed to the centre of the circular motion. These force(s) provides for centripetal force.
- Apply $\sum F_{\text{horizontal}} = F_c = ma_c$
- Manipulate the equation and solve for the unknown.

- For some questions, use $\sum F_{\text{horizontal}} = F_c = ma_c$ and $\sum F_{\text{vertical}} = 0$, then solve simultaneously.

Basic Terminology

Angular displacement, $\theta = \dfrac{s}{r}$

Angular velocity, $\omega = \dfrac{\Delta\theta}{\Delta t} = \dfrac{2\pi}{T} = 2\pi f$

Linear velocity, $v = r\omega$

Centripetal acceleration, $a = \dfrac{v^2}{r} = r\omega^2$

Centripetal force, $F = ma_c = \dfrac{mv^2}{r} = mr\omega^2$

Using Newton's Laws to explain why an **<u>object</u>** moving in a constant speed **<u>in a circle experiences a resultant force towards the centre of the circle</u>**.

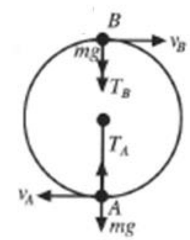
- Since object experiences a constant change in direction of motion, by N1L, there must be a resultant force on it.

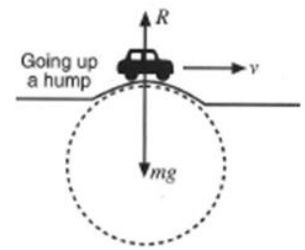
- Given the tangential speed remains constant, by N2L, the resultant force must act perpendicular to the velocity, hence in the radial direction, towards the centre of the circle.

VERTICAL Circular Motion

- Most examples of vertical circular motion are non-uniform, since the speed and angular velocity are not constant.
- Uniform vertical circular motion are usually forced to rotate at constant ω (e.g. ferris wheel)

Examples:





Conditions:

- $\Sigma F_{VERTICAL} = F_c = ma_c$

Problem-solving:

- Draw the FBD (all individual forces) for the object undergoing circular motion.
  (do NOT draw the centripetal force)
- Locate the centre of the circular motion.
- Resolve the forces.
- Determine the force(s) that is(are) directed to the centre of the circular motion. These force(s) provides for centripetal force.
- Apply $\sum F_{\text{vertical}} = F_c = ma_c$
- Manipulate the equation and solve for the unknown.

- For some questions in order to solve, will need to use conservation of energy
  $KE_{\text{top}} + GPE_{\text{top}} = KE_{\text{bottom}} + GPE_{\text{bottom}}$
  (if there are no frictional forces)

Gravitational Field

Gravitational Force F

Newton's law of gravitation states that the force of attraction between two point masses is proportional to the product of their masses and inversely proportional to the square of their separation

Gravitational Field strength g

Gravitational field strength at a point is defined as the gravitational force per unit mass acting on a small test mass placed at that point

$$F = \frac{GMm}{r^2} \quad F = -\frac{dU}{dr} \quad U = -\frac{GMm}{r}$$

$$F = m\,g \qquad U = m\,\phi$$

$$g = \frac{GM}{r^2} \quad g = -\frac{d\phi}{dr} \quad \phi = -\frac{GM}{r}$$

Gravitational Potential Energy U

Work done by an external force in bringing mass from infinity to point in gravitational field without a change in kinetic energy.

Gravitational Potential ϕ

WD per unit mass by an external force in bringing a small test mass from infinity to point in gravitational field without a change in kinetic energy.

Note: Gravitational potential (and energy) is negative because gravitational force is an attractive force and potential is taken to be zero (maximum) at infinity.

Minimum Escape Speed

(straight upwards from surface of Earth)

By Conservation of Energy, the total energy (E_K + E_P) at infinity is greater or equal to zero,

$$\frac{1}{2}mv^2 + \left(-\frac{GM_E m}{R_E}\right) \geq 0$$

$$v \geq \sqrt{\frac{2GM_E}{R_E}}$$

Orbits

When a satellite (m) orbits around a planet (M), the centripetal force is provided by the gravitational force.

$$\frac{GMm}{r^2} = m\frac{v^2}{r}$$

Kepler's 3rd law:

$$\frac{GMm}{r^2} = mr\omega^2$$

so $T^2 \propto r^3$

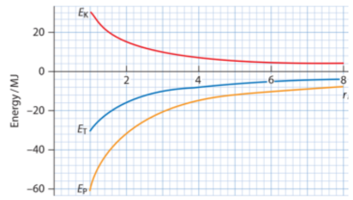
Energy in Orbits

Starting from $\frac{GMm}{r^2} = m\frac{v^2}{r}$

$$E_K = \frac{GMm}{2r}$$

$$E_P = -\frac{GMm}{r}$$

$$E_{Total} = E_K + E_P = -\frac{GMm}{2r}$$



Geostationary Satellite

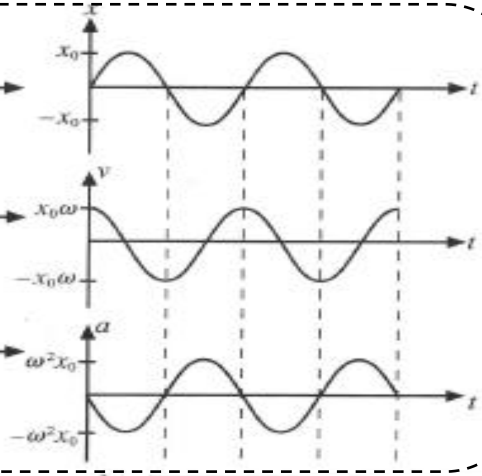
A geostationary satellite is a satellite in a geostationary orbit which allows it to remain stationary relative to an observer on Earth.

Conditions:

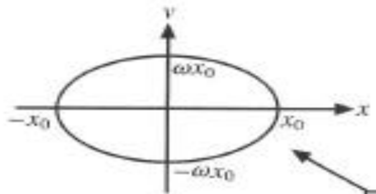
1. The period of the satellite orbit is 24 hours
2. The satellite is directly above the equator.
3. The satellite is orbiting in the same direction as the Earth's rotation, west to east.

OSCILLATIONS / SIMPLE HARMONIC MOTION

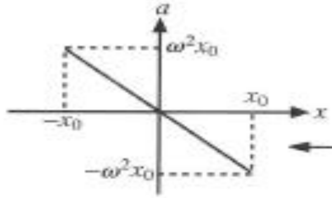
Variation of x
with time (t = 0, x = 0): $x = x_0 \sin \omega t$



Variation of v
with time: $v = x_0\omega \cos \omega t$
with x: $v = \pm\, \omega\sqrt{x_0^2 - x^2}$



Variation of a
with time: $x = -x_0\omega^2 \sin \omega t$
with x: $a = -\omega^2 x$



Terminology

$f = \frac{1}{T}$

$T = \frac{1}{f}$

$\omega = 2\pi f = \frac{2\pi}{T}$

means

Restoring force (acceleration) is **proportional to its displacement** from the equilibrium position and **opposite in direction to the displacement** (i.e. always **directed towards the equilibrium position**).

absence of external dissipative force

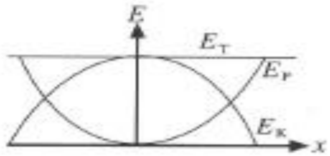
Total energy of the oscillating system is a constant

Kinetic Energy
$E_K = \frac{1}{2} m\omega(x_0^2 - x^2)$

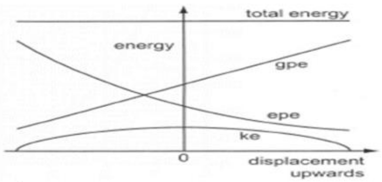
Potential Energy
$E_P = \frac{1}{2} m\omega^2 x^2$

Total Energy
$E_T = \frac{1}{2} m\omega^2 x_0^2$

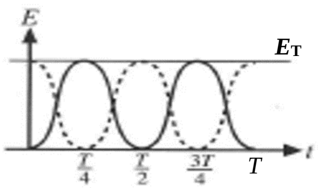
Energy variation with displacement



Energy of spring-mass system (vertical oscillation)



Energy variation with time



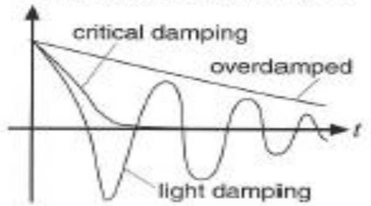
Velocity	Kinetic Energy	Total Energy	Potential Energy
$\omega x_0 \cos\omega t$	$\frac{1}{2} m\omega^2 x_0^2 \cos^2\omega t$	$\frac{1}{2} m\omega^2 x_0^2$	$\frac{1}{2} m\omega^2 x_0^2 \sin^2\omega t$
$\omega x_0 \sin\omega t$	$\frac{1}{2} m\omega^2 x_0^2 \sin^2\omega t$	$\frac{1}{2} m\omega^2 x_0^2$	$\frac{1}{2} m\omega^2 x_0^2 \cos^2\omega t$

presence of external dissipative force

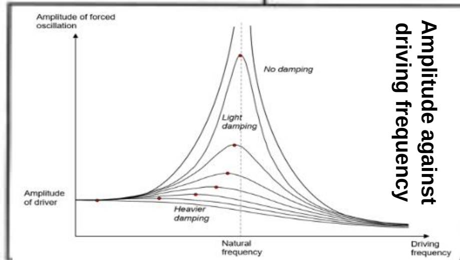
Damped Oscillations

- **Light Damping**
- **Heavy Damping**
- **Critical Damping**

Displacement against time



Amplitude against driving frequency



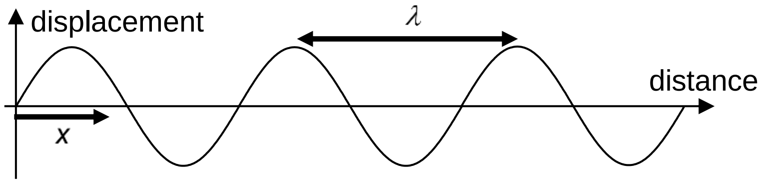
Waves

Waves can be described by **Terms**:

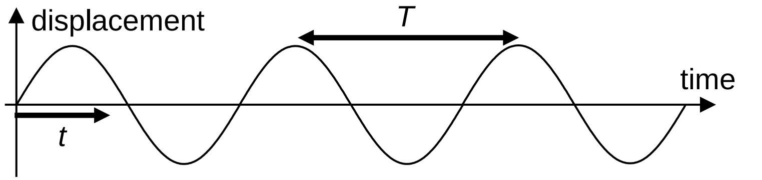
- amplitude (A)
- wavelength (λ) $\lambda = \frac{v}{f}$
- frequency (f) $f = \frac{v}{\lambda}$
- speed (v) $v = f\lambda$
- period (T) $T = \frac{1}{f}$
- phase (ϕ) $\phi = 2\pi \frac{x}{\lambda}$, $\phi = 2\pi \frac{t}{T}$

Waves can be represented by **Graphs**:

Shows how displacement varies with distance for all particles in a wave at one instant.



Shows how displacement varies with time for one particle in a wave.



Transverse Waves
Waves with direction of oscillation is perpendicular to direction of propagation of wave.

Longitudinal Waves
Waves with direction of oscillation is parallel to direction of propagation of wave.

Electromagnetic Waves
Speed in a vacuum $c = 3.00 \times 10^8 \text{ m s}^{-1}$

γ-ray	X-ray	Ultra-violet	Infra-red	Micro-wave	Radio-wave

Short λ → Long λ

High frequency → Low frequency

Transverse Waves can be **Polarised**

definition: **Polarisation**
Confine the oscillations of the electric vector of light waves to one direction.

Waves have **Intensity**

$\text{Intensity} = \frac{\text{Power}}{\text{Area}}$

for spherical waves: $I = \frac{P}{4\pi r^2}$

$\text{Intensity} = kA^2$

$\text{Intensity} \propto (\text{amplitude})^2$

(Source: Longman A-Level Course in Physics Volume 1, K.W. Loo & B. H. Ong)

Superposition

The Principle of Superposition states that when two or more waves of the same kind overlap, the resultant displacement at any point at any instant is the vector sum of the displacements that the individual waves would have separately produced at that point and at that instant.

Interference is the phenomenon which occurs when two or more waves of the same type overlap (superpose) according to the principle of superposition.

Constructive interference occurs when the phase difference is 0 rad and the waves are in phase. The component waves superpose to produce a resultant with a maximum amplitude and intensity.

Destructive interference occurs when the phase difference is π rad and the waves are in antiphase. The component waves superpose with each other to produce a resultant with a minimum amplitude and intensity

		Path Difference	
		$n\lambda$	$\left(n+\frac{1}{2}\right)\lambda$
Sources	in phase	Constructive	Destructive
	in antiphase	Destructive	Constructive

Conditions for Observable Interference Pattern

1. The waves must be coherent.
2. The waves should have equal or similar amplitudes.
3. For transverse waves, they must either be unpolarised, or polarised in the same plane.

Diffraction refers to the spreading of waves when they travel through a small opening or when they pass round a small obstacle.

Diffraction appears most significant when the size of the aperture (or obstacle) is of the same order of magnitude as the wavelength of the wave.

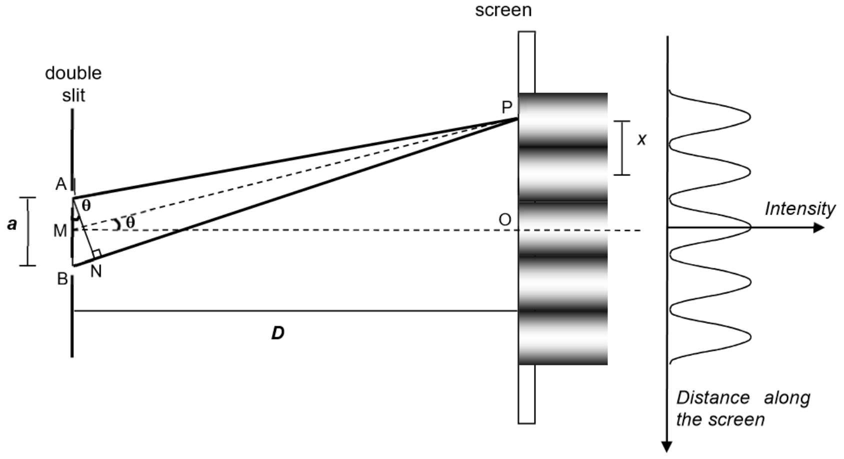
Young's Double Slit Experiment

The fringe separation (distance between successive bright fringes) x is given by the equation

$$x = \frac{\lambda D}{a}$$

where λ is the wavelength of the light,
D is the distance from the double slits to the screen,
a is the separation of the two slits (measure from centre to centre of slits)

Note: This formula is only valid when D is much larger than a ($D >> a$).

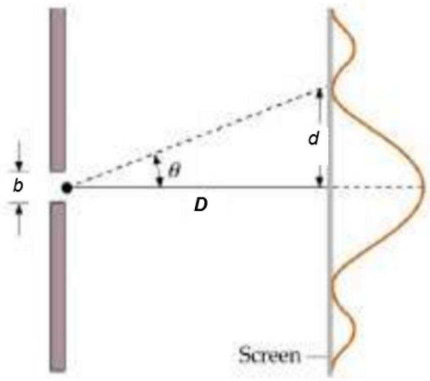


Single-slit Diffraction

The angle θ at which the first minima occurs using the relationship

$$\sin\theta = \frac{\lambda}{b}$$

where λ is the wavelength of the light,
b is the width of the single slit.



Rayleigh's criterion

When the central maximum of one image falls on the first minimum of another image, the images are distinguishable and said to be just resolved. This limiting condition of resolution is known as Rayleigh's criterion.

Mathematically, Rayleigh's criterion is expressed as

$$\theta_{\min} \approx \frac{\lambda}{b}$$

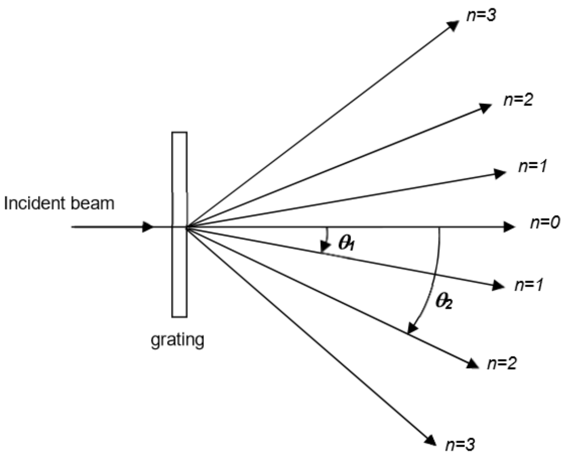
where θ is the limiting angle of resolution,
λ is the wavelength of the light,
b is the width of the single slit.

Diffraction Grating

The position (or angle) of the n^{th} order intensity maximum may be determined using:

$$d \sin\theta = n\lambda$$

where d = line spacing (i.e. slit separation) of the diffraction grating
n = order of diffraction, an integer
θ = angle between the n^{th} order beam and the normal to the grating
λ = wavelength of the incident beam

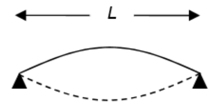
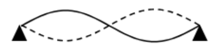


Stationary Waves

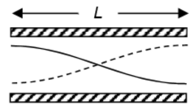
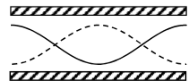
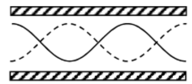
A stationary wave is the result of interference

- between two identical waves (same amplitude, frequency);
- travelling along the same line with the same speeds;
- but in opposite directions.

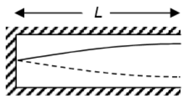
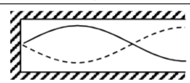
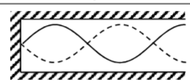
Stationary (Transverse) Wave on a Stretched String (2 fixed ends)

Harmonic series	Overtone	Mode of vibration	Wavelength	Frequency
1			$\lambda_1 = 2L$	$f_1 = \frac{v}{2L}$ fundamental
2	1		$\lambda_2 = L$	$f_2 = \frac{v}{L}$
3	2		$\lambda_3 = \frac{2L}{3}$	$f_3 = \frac{3v}{2L}$
n	n-1	All harmonics possible	$\lambda_n = \frac{2L}{n}$	$f_n = \frac{nv}{2L}$

Stationary (Sound) Wave within an Open Pipe (2 ends open)

Harmonic series	Overtone	Mode of vibration	Wavelength	Frequency
1			$\lambda_1 = 2L$	$f_1 = \frac{v}{2L}$ fundamental
2	1		$\lambda_2 = L$	$f_2 = \frac{v}{L}$
3	2		$\lambda_3 = \frac{2L}{3}$	$f_3 = \frac{3v}{2L}$
n	n-1	All harmonics possible	$\lambda_n = \frac{2L}{n}$	$f_n = \frac{n\,v}{2\,L}$

Stationary (Sound) Wave within a Closed Pipe (one end open, one end closed)

Harmonic series	Overtone	Mode of vibration	Wavelength	Frequency
1			$\lambda_1 = 4L$	$f_1 = \frac{v}{4L}$ fundamental
2	Not possible			
3	1		$\lambda_2 = \frac{4L}{3}$	$f_2 = \frac{3v}{4L}$
4	Not possible			
5	2		$\lambda_3 = \frac{4L}{5}$	$f_3 = \frac{5v}{4L}$
n		Only odd-numbered harmonics are possible	$\lambda_n = \frac{4L}{n}$	$f_n = \frac{nv}{4L}$

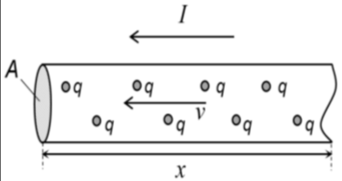
Current of Electricity

Electric Current

Electric current is the rate of flow of charge.

- For a steady current, $I = \dfrac{Q}{t}$
- Conventional current is defined as the direction of flow of positive charge (i.e. opposite flow of negative charge)

Derivation of $I = nAvq$



A: cross-sectional area
n: number density
v: drift velocity
q: charge on each charge carrier

total charge in volume = $nAxq$

time interval, $t = x / v$

$$\text{current, } I = \frac{\text{charge}}{\text{time}} = \frac{Q}{t} = \frac{nAxq}{x/v} = nAvq$$

Charge

$Q = It$ = Area under I - t graph
$= Nq$

e.m.f. and p.d.

The electromotive force **e.m.f.** of <u>a source</u> is the energy converted from other forms to electrical energy per unit charge.

The **potential difference** <u>between two points</u> in a circuit is the electrical energy converted to other forms of energy per unit charge.

$$V = \frac{W}{Q}$$

W: energy converted in joules
Q: *charge in coulombs*

Resistance

The **resistance** of a circuit component is the <u>ratio</u> of the potential difference across the component to the current flowing through it.

$$V = IR$$

Resistance and resistivity

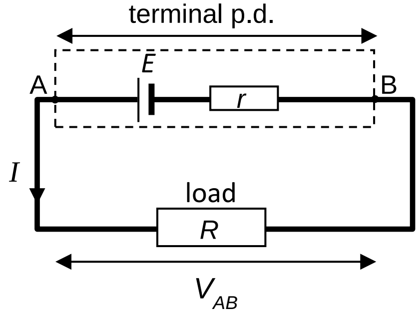
$$R = \frac{\rho l}{A}$$

A: cross-sectional area
l: length
ρ: *resistivity*

Power

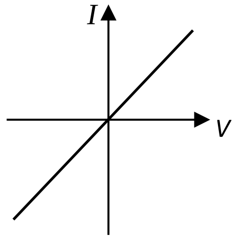
$$P = VI = I^2 R = \frac{V^2}{R}$$

Effects of internal resistance



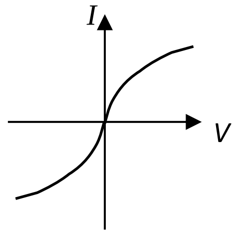
- **Terminal p.d.** $V_{AB} = IR = E - Ir$
- A battery delivers **maximum power** to a circuit when the <u>load resistance</u> of the circuit is <u>equal to the internal resistance</u> of the battery

I-V characteristics of electrical components



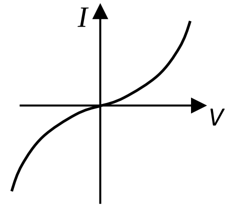
Metallic ohmic resistor at <u>constant temperature</u>

- magnitude of vibration of lattice ions remains the same
- rate of collision of electrons with lattice ions is constant
- Resistance is constant, so ratio of V to I is constant



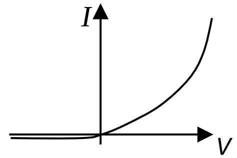
Filament lamp

- As V and I increase, energy dissipated as heat increases. Temperature increase.
- Lattice ions vibrate with greater amplitude. Rate of collision between free electrons and lattice ions increase.
- Resistance increases



Thermistor

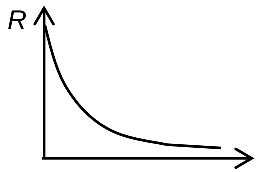
- As V and I increase, number of charge carriers increase at higher temperature.
- Effect of increase in number density of charge carriers outweighs the effect of increase in collision between electrons and lattice ions.
- Resistance decreases.



Semiconductor diode

- Low resistance when the diode is in forward bias
- Very high resistance when the diode is in reverse bias

Temperature characteristics of NTC thermistor



Resistance decreases with increasing temperature due to an increase in number of mobile charge carries

"**Potential**" refers to a value at single point. Across a circuit component, current will flow from a point with higher potential to a point with lower potential.

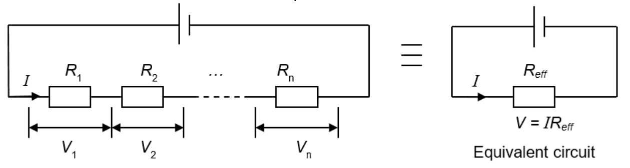
"**Potential difference**" (p.d.) refers to the difference when we compare the potential values across the two ends of a circuit component.

e.g. p.d. across a resistor, $V_{AB} = V_A - V_B$

If potential V_A > potential V_B, a current flows.

If $V_A = V_B$, $V_{AB} = 0$, no current flows.

For n **resistors in series**,



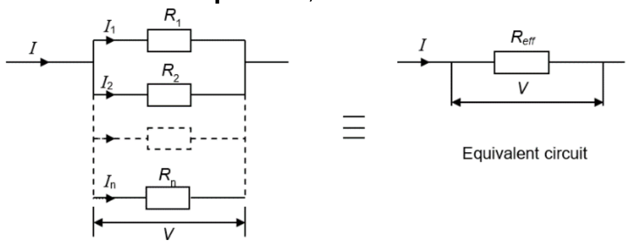
I is constant (same I flowing through each resistor)

$V = V_1 + V_2 + \ldots V_n$

$IR_{eff} = IR_1 + IR_2 + \ldots IR_n$

$R_{eff} = R_1 + R_2 + \ldots R_n$

For n **resistors in parallel**,



V is constant (same p.d. across each resistor)

$I = I_1 + I_2 + \ldots I_n$

$V/R_{eff} = V/R_1 + V/R_2 + \ldots V/R_n$

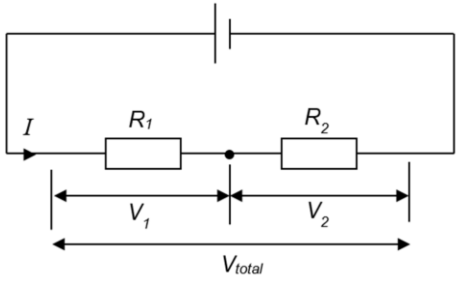
$1/R_{eff} = 1/R_1 + 1/R_2 + \ldots 1/R_n$

R_{eff} is always less than the value of smallest individual resistance R.

D.C. CIRCUITS

Potential Divider

(for resistors connected in series)



$$I = \frac{V_{total}}{R_{total}} = \frac{V_{total}}{R_1 + R_2}$$

$$V_1 = IR_1 = \frac{R_1}{R_{total}} V_{total} \;;\; V_2 = IR_2 = \frac{R_2}{R_{total}} V_{total}$$

Ratio of p.d. across the two resistors $\frac{V_1}{V_2} = \frac{R_1}{R_2}$

In general,

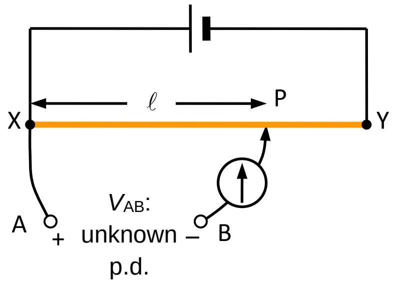
$$V_R = \frac{R}{R_{total}} V_{total} \quad \text{or} \quad \frac{V_R}{V_{total}} = \frac{R}{R_{total}}$$

Practical uses of potential divider circuit

Thermistor is a semiconductor whose resistance R_t changes with temperature. Most thermistors have a *negative temperature coefficient* (NTC), meaning its resistance *decreases* as temperature *increases*.

Light-Dependent Resistor (LDR) is a semiconductor whose resistance *decreases* as light intensity falling on them *increases*.

Potentiometer



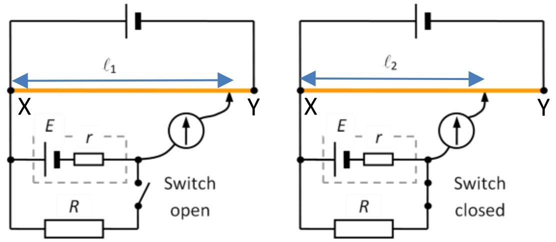
Galvanometer shows null deflection: $V_{XP} = V_{AB}$

Applying potential divider & $R \propto \ell$,

$$\frac{V_{XP}}{V_{XY}} = \frac{L_{XP}}{L_{XY}} \quad \text{or} \quad V_{XP} = \frac{V_{XY}}{L_{XY}} L_{XP}$$

Potentiometer is like an ideal voltmeter (which draws no current).

Determining unknown internal resistance r



Switch opened: Whether or not E has r and independent of value of r (since there is no current passing r), ℓ_1 will not change. $E = \frac{V_{XY}}{L_{XY}} \ell_1$

Switch closed: $V_R = \frac{ER}{R+r} = \frac{V_{XY}}{L_{XY}} \ell_2 \Rightarrow \frac{R}{R+r} = \frac{\ell_2}{\ell_1}$

THERMAL PHYSICS

HEAT

- **Specific heat capacity** (c) of a substance is the heat required per unit mass per unit temperature change to raise the temperature of the substance without a change in phase of the substance.
- When the temperature of a body of mass *m* and specific heat capacity *c* changes by ΔT, the heat absorbed or given out is $Q = mc\Delta T$
- The **specific latent heat of fusion** of a substance is the heat required per unit mass to change the phase of the substance between solid and liquid phase without a change in temperature.
- The **specific latent heat of vaporization** of a substance is the heat required per unit mass to change the phase of the substance between liquid and gas phase without a change in temperature.
- For a body with mass *m* and specific latent heat *L*, the latent heat required for it to change phase is $Q = mL$
- Using conservation of energy,
  → Power source present:
  $Q_{\text{supplied}} = Q_{\text{gain to melt}} + Q_{\text{gain to increase } T} + Q_{\text{gain to vaporise}}$
  → No power source present:
  $Q_{\text{gain}} = Q_{\text{lost}}$
- For questions involving constant power or constant rate of heat loss,
  Approach question using power instead of heat supplied.
  $P_{\text{supplied}} = P_{\text{gain to melt / gain to increase } T \text{ /gain to vaporise}} + P_{loss}$

TEMPERATURE

- **Temperature** is a physical quantity that measures the degree of hotness or coldness of a body as indicated on a calibrated scale.
- Temperature is a measure of the mean kinetic energy of the molecules within a body.
- **Heat** is the thermal energy that flows from a region of higher temperature to a region of lower temperature.
- Two bodies are in **thermal equilibrium** if there is no new flow of heat between them when they are in thermal contact. Objects in thermal equilibrium have the same temperature.
- The **absolute scale of temperature** does not depend on the property of any particular substance and has an absolute zero.
- The **absolute zero** (0 K) is the temperature at which all substances have a ***minimum*** internal energy.
- Convert Kelvin to degree Celsius:
  $T\,/\,\text{K} = T\,/\,°\text{C} + 273.15$

  [In calculations, the d.p in 273.15 is omitted because $T\,/\,°\text{C}$ given in the question has no d.p]

IDEAL GAS & KINETIC THEORY

- An **ideal gas** is a gas that obeys the equation $pV = nRT$ at all pressure *p*, volume *V* and temperature *T*.
- Ideal gas equation: $pV = nRT$ or $pV = NkT$
  where *n* = no. of moles of gas,
  N = no. of molecules of gas
- Basic assumptions of the kinetic theory of gases:
  1. A gas consists of a large number of molecules in continuous random motion.
  2. The molecules collide elastically with the container and with each other.
  3. The duration of collision is negligible compared to the time interval between collisions.
  4. There are no intermolecular forces except during collisions.
  5. The total volume of molecules is negligible compared to the volume of the container.
- Refer to lecture notes for derivation of pressure exerted by a gas.
- Pressure of an ideal gas
  $$p = \frac{1}{3}\frac{Nm}{V} < c^2 > \quad = \frac{1}{3}\rho < c^2 >$$
  where *m* = mass of 1 molecule of gas,
  $<c^2>$ = mean square speed, ρ = density
- Mean kinetic energy E_k of <u>a molecule</u>
  $$E_K = \frac{1}{2}m < c^2 > = \frac{3}{2}kT$$
  → $c_{rms} = \sqrt{< c^2 >} \propto \sqrt{T}$

INTERNAL ENERGY

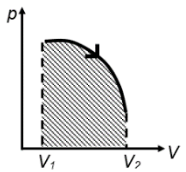
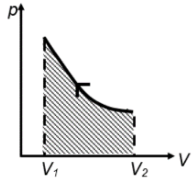
- Matter is made up of many molecules.
  The molecules are in constant motion, hence have **kinetic energy**.
  There may also be attractive and repulsive forces between molecules, hence have **potential energy** due to the interaction between them.
- The **internal energy** of a system is the sum of a random distribution of kinetic and potential energies associated with the molecules of the system.
- For an IDEAL (monatomic) GAS,
  no intermolecular P.E,
  so total internal energy,

  $U_{\text{ideal gas}}$ = total KE = $\frac{3}{2}NkT = \frac{3}{2}nRT = \frac{3}{2}pV$

  → $U \propto T$ and so $\Delta U \propto \Delta T$

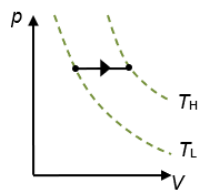
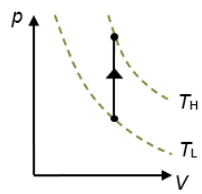
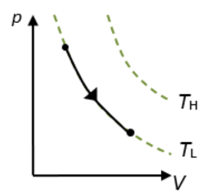
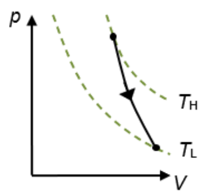
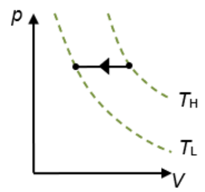
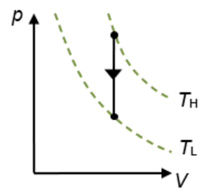
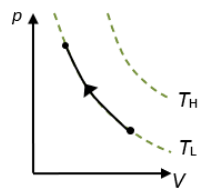
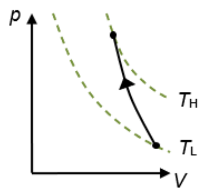
WORK DONE

- Work done on gas at **constant pressure**
  $W_{\text{on gas}} = -W_{\text{by gas}} = -p\Delta V$
- In general, the **area under a *p-V* graph** gives the work done on a gas (decreasing volume) or work done by a gas (increasing volume).
-

	Expansion $W_{\text{ON gas}}$ is negative $W_{\text{BY gas}}$ is positive
	Compression $W_{\text{ON gas}}$ is positive $W_{\text{BY gas}}$ is negative

FIRST LAW OF THERMODYNAMICS

- The **first law of thermodynamics** states that the increase in internal energy of a system is equal to the sum of the heat supplied to the system and the work done on the system.
- $\Delta U = Q + W$ *[applies not only to ideal gas but also to system in general such as ice, water etc.]*
- All *p*, *V*, *T* states on the *p-V* graphs follow ideal gas equation.
-

isobaric constant pressure	isochoric constant volume	isothermal constant temperature	adiabatic no heat flow
			
• $W = -p\Delta V$ = – ve (expansion) • ΔU = + ve (ΔT = + ve) • $Q = \Delta U - W$ = + ve	• $W = 0$ • ΔU = + ve (ΔT = + ve) • $Q = \Delta U$ = + ve	• $\Delta U = 0$ ($\Delta T = 0$) • W = – ve (expansion) • $Q = -W$ = + ve	• $Q = 0$ • W = – ve (expansion) • $\Delta U = W$ = – ve
			
• $W = -p\Delta V$ = + ve (compression) • ΔU = – ve (ΔT = – ve) • $Q = \Delta U - W$ = – ve	• $W = 0$ • ΔU = – ve (ΔT = – ve) • $Q = \Delta U$ = – ve	• $\Delta U = 0$ ($\Delta T = 0$) • W = + ve (compression) • $Q = -W$ = – ve	• $Q = 0$ • W = + ve (compression) • $\Delta U = W$ = + ve

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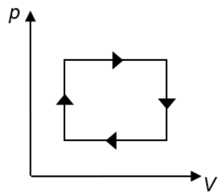
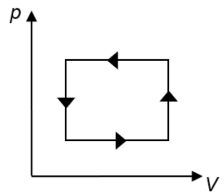
cyclic start & end at the same state		• $\Delta U = 0$ ($\Delta T = 0$) • W = – ve (area enclosed) • $Q = -W$ = + ve		• $\Delta U = 0$ ($\Delta T = 0$) • W = + ve (area enclosed) • $Q = -W$ = – ve

Electric Fields

Electric Field and Field Strength

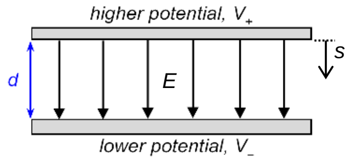
An **electric field** is a <u>region in space where a</u> stationary <u>charge experiences a</u>n electric <u>force</u>.

<u>Electric Field Strength, *E*</u>

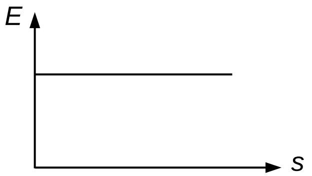
The electric field strength E at a point is the <u>electric force per unit positive charge</u> <u>acting on a</u> small stationary <u>charge placed at that point</u>.

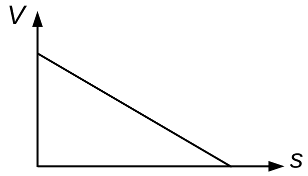
$$E = \frac{F}{q}$$

Uniform Electric Field



- field strength, $E = \frac{\Delta V}{d}$ is the same at all points
- potential, V decreases linearly along the field
- variation of E and V with distance s from the plate with higher potential:

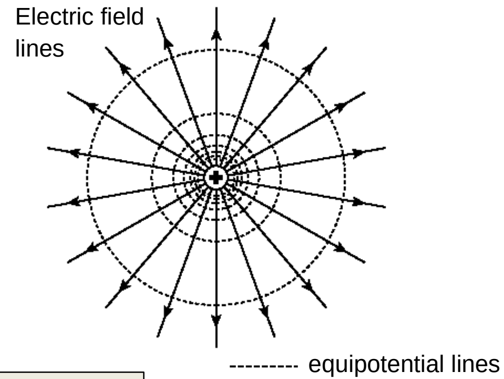




- $a = \frac{F_{net}}{m} = \frac{F_E}{m} = \frac{qE}{m} = \frac{q\Delta V}{md}$
- charged particles with velocity component at right angles to the field will follow a **<u>parabolic path</u>**
- apply **Kinematics** (projectile motion) or/and **Conservation of Energy** (total $E_K + U_E$ = constant, where $U_E = Vq$)

Electric Field Lines

- go from high to low potential (start on a positive charge, end on a negative charge)
- are perpendicular to surface of charge conductor
- are smooth curves or lines that never touch or intersect
- closer lines indicate a strong electric field
- the tangent to the field lines at a point indicates the direction of the electric field vector at that point.



Relationship between *E* and

The electric field strength at a point is numerically equal to the potential gradient at that point.

$$E = -\frac{dV}{dr}$$

direction of the electric field points from high potential to low

Electric Potential, *V*

- The electric potential at a point in an electric field is defined as <u>the work done per unit positive charge</u> by an external agent in bringing <u>a small test charge</u> <u>from infinity to that point</u> without a change in the kinetic energy of the charge.
- $V = 0$ at infinity

Point Charges

<u>Electric Field Strength, *E* (vector)</u>

Unit: N C^{-1} or V m^{-1}

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$ substitute Q without signs

E points away from positive charge, towards negative charge

$$E = -\frac{dV}{dr}$$

<u>Electric Potential, *V* (scalar)</u>

Unit: V

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$ **substitute signs of Q**

$F = Eq$

$U = Vq$

<u>Electric Force, F_E (vector)</u>

Unit: N

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2}$$ substitute Q,*q* without signs

Unlike charges attract. Like charges repel.

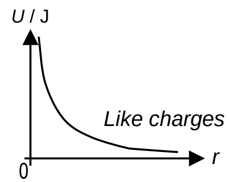
Coulomb's law: The magnitude of the force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between

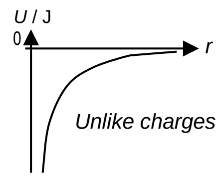
$$F_E = -\frac{dU}{dr}$$

<u>Electric Potential Energy, U_E (scalar)</u>

Unit: J

$$U = \frac{Qq}{4\pi\varepsilon_0 r}$$ **substitute signs of Q and *q***





Electromagnetism

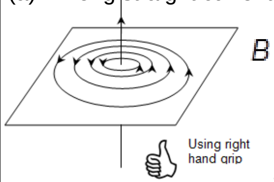
Field

A **magnetic field** is a region of space where a <u>permanent magnet</u> or a <u>moving charge</u> or a <u>current-carrying conductor</u> will experience a magnetic force.

The **magnetic flux density** of a magnetic field is defined as the <u>force per unit current per unit length</u> (of conductor) acting on a straight current-carrying conductor placed at right angles to the magnetic field.

Magnetic field lines (use RHGR)

(a) Long straight current-carrying conductor



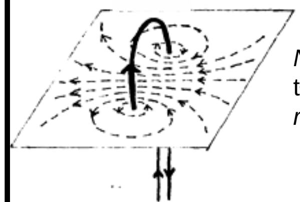
$$B = \frac{\mu_o I}{2\pi d}$$

d: distance perpendicular to the current to a point.

(b) Circular flat coils


using right hand grip rule

$$B = \frac{\mu_o N I}{2r}$$

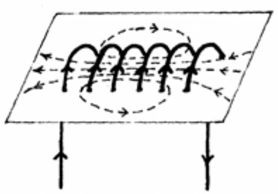


N: number of turns in the coil

r : radius of coil

(c) Solenoid


using right hand grip rule

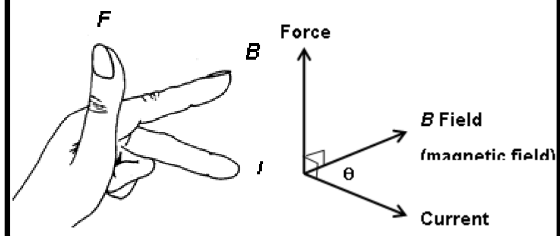


$$B = \mu_o n I$$

n: number of turns per unit length

Force

Force on Current-carrying conductor



Magnitude of the force F_B:

$$F_B = BIL\sin\theta$$

where L: length of conductor

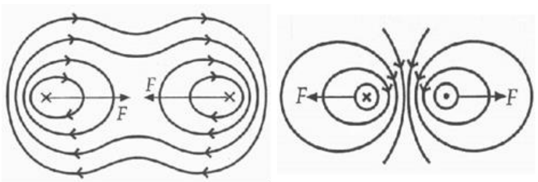
θ : angle between B and I

Note F_B is zero when current is along direction of the external B.

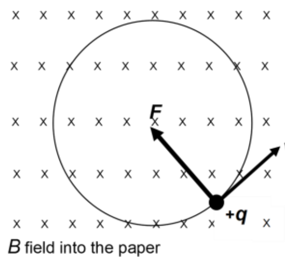
Direction of F_B: Use Fleming's Left Hand Rule

Parallel current-carrying conductors

- Like currents <u>attract</u>
- Unlike currents <u>repel</u>
- By Newton's 3rd law, the forces on both currents are equal in magnitude and opposite in direction.



Force on Moving Charge:



Magnitude of the force F_B:

$$F_B = Bqv\sin\theta$$

where q: charge v : speed

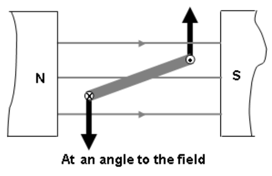
θ : angle between B and v

Note F_B is zero when the direction of v is along B or when $v = 0$.

Direction of F_B : Use FLHR

Direction of I in FLHR will be given by the direction of flow of positive charge. For negative charges, the direction of I will be opposite to direction of v .

Torque in DC motor



Torque = force × distance perpendicular to the forces between the forces

Velocity selector:

An E-field & B-field exist in the same region of space and are directed perpendicular to each other. Consider a positive ion entering the region:

For E-field: $F_E = qE$ and the force is in direction of the E-field.

For B-field: $F_E = Bqv$ and direction is given by FLHR.

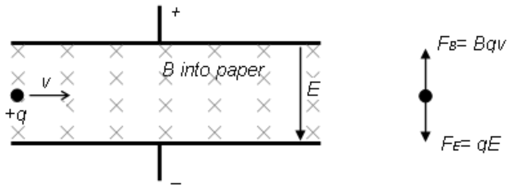
To pass through undeflected:

$F_B = F_E$

$Bqv = qE$

$v=E/B$

- If $v > E/B$, $F_B > F_E$: deflect in direction of F_B.
- If $v < E/B$, $F_B < F_E$: deflect in direction of F_E.



From FLHR, we conclude that the direction of F_B is always perpendicular to the direction of v. Hence the moving charge moves in a uniform circular path where <u>centripetal force is provided by the magnetic force</u>.

$$F_B = ma_C$$

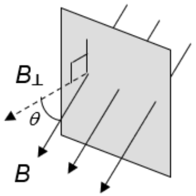
$$Bqv = \frac{mv^2}{r}$$

$$r = \frac{mv}{Bq}$$

ELECTROMAGNETIC INDUCTION

Magnetic flux ϕ **:** Magnetic flux through a plane surface is the product of the flux density normal to the surface and the area of the surface.

$\phi = B_{\perp} A$; $[\phi]$ = weber Wb = T m^2
where
$B_{\perp}$ = component of B perpendicular to the surface plane,
A = area of the plane



Magnetic flux linkage $N\phi$: Magnetic flux linkage through a coil of N turns is the product of the number of turns N of the coil and the magnetic flux ϕ linking each turn.

$N\phi = N B_{\perp} A$

Magnetic flux density $\boldsymbol{B}$ is a **vector**;
Magnetic flux $\boldsymbol{\phi}$ is a **scalar**.

Faraday's Law of Electromagnetic Induction states that the induced e.m.f. ε is directly proportional to the rate of change of magnetic flux linkage.

$$\varepsilon = -\frac{d(N\phi)}{dt}$$

-ve sign in the expression is due to Lenz's law

$$\varepsilon = -\frac{Nd\Phi}{dt} = -\frac{NAdB}{dt} = -\frac{NAkdI}{dt}$$

The ε vs t graph is thus the <u>negative</u> of of the <u>gradient</u> of ϕ vs t, B vs t, or I vs t graph (if N and A are constant).

For constant or average induced e.m.f.:
$|\varepsilon| = (\Delta N\phi) / (\Delta t)$

e.m.f. can be induced even when there is no induced current (eg. an isolated conductor not in a complete circuit).

Lenz's law states that the direction of the induced e.m.f. is such as to cause effects to oppose the change producing it.

Lenz's law is a statement of the **conservation of energy** where mechanical energy is converted to electrical energy.

Lenz's law allows the polarity of induced e.m.f. and direction of induced current to be determined.

3-step for closed loop

1) Direction of flux linkage? Change? Due to?

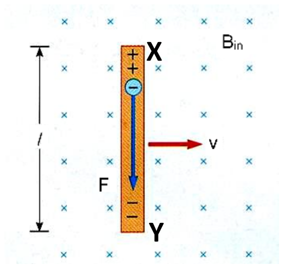
a. What is the direction of the magnetic flux linkage through the loop?
b. Magnetic flux linkage increasing or decreasing?
c. Cause of the change in magnetic flux linkage?
   - Flux density B increase/decrease?
   - Area A increase/decrease?
   - Angle θ of plane of loop to magnetic field changing?

2) Apply Faraday's Law → how magnetic flux linkage changes → e.m.f. induced.
If the loop is a closed circuit, the induced e.m.f. causes an induced current to flow.

3) Apply Lenz's Law to determine the direction of induced current -> The induced current flows in a (direction) so as to produce the (effect) to oppose the (change) in magnetic flux linkage.

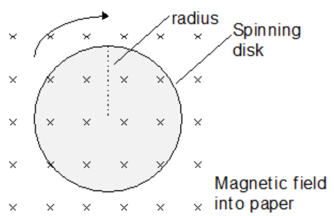
Examples of induced e.m.f.

1) Moving rod: $|\varepsilon| = B\ell v$

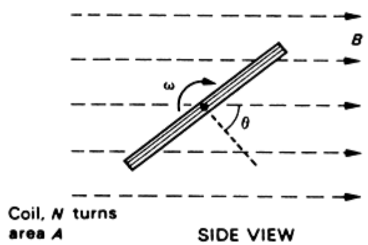


X has a HIGHER potential,
Y has a LOWER potential

2) Spinning disc: $|\varepsilon| = B\pi r^2 f$



3) Rotating coil: $|\varepsilon| = NBA\omega \sin(\omega t)$



Eddy (induced) currents, generated within thick/broad piece of conductor, dissipate energy and create magnetic fields that tend to oppose the changes in the magnetic field.

ALTERNATING CURRENT

Instantaneous power: $P = IV = \frac{V^2}{R} = I^2 R$

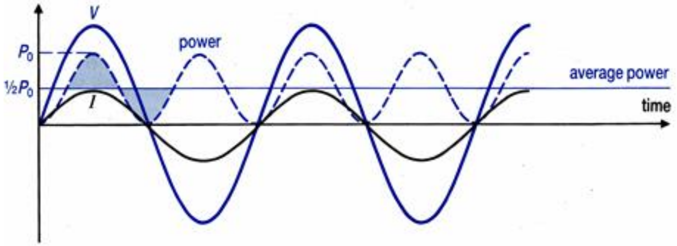
Maximum power: $P_{max} = I_o V_o = \frac{V_o^{\,2}}{R} = I_o^{\,2} R$

Mean power: $\langle P \rangle = I_{rms} V_{rms} = \frac{V_{rms}^{\,2}}{R} = I_{rms}^{\,2} R$

R.M.S. values: Different graphs will have different r.m.s functions. General rule to follow:
i) take the square of the instantaneous graph,
ii) find the mean, by find the area under graph divided by time considered,
iii) square-root the answer.

r.m.s. current I_{rms} (and voltage V_{rms}) of the a.c. is same as that of the steady d.c. I_{dc} (and V_{rms}).

Sinusoidal AC (e.g. $I = I_0 \sin \omega t$, $V = V_0 \sin \omega t$)



$I_{rms} = \frac{I_o}{\sqrt{2}}$ and $V_{rms} = \frac{V_o}{\sqrt{2}}$

$\Rightarrow \langle P \rangle = V_{rms} I_{rms} = \frac{V_o I_o}{2} = \frac{1}{2} P_{max}$

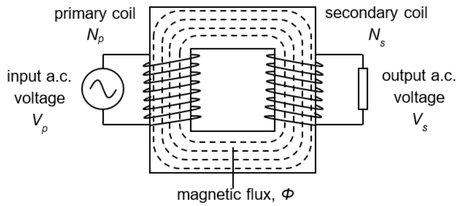
! The above formulae can be used **ONLY for sinusoidal** waveform.

A.C.: Charge carriers periodically reverse their direction of motion.

Transformers (To know how they work based on principles of EMI.)
If no power loss and both coils have the same magnetic flux through them,

Voltage to turns ratio

$$\frac{N_S}{N_P} = \frac{V_S}{V_P}$$



$$I_P V_P = I_S V_S$$

$$\frac{V_P}{V_S} = \frac{I_S}{I_P}$$

Combining the above equations,
For **ideal** transformers:

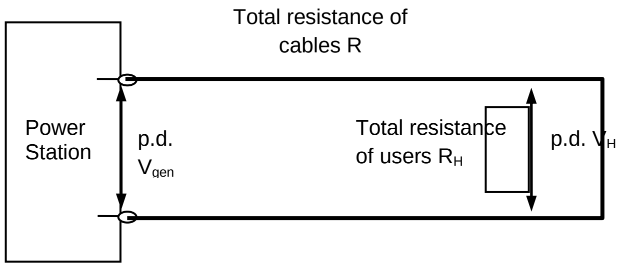
$$\frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{I_S}{I_P}$$

For step-up transformer, $N_s > N_p \Rightarrow V_s > V_p$.

For step-down transformer, $N_s < N_p \Rightarrow V_s < V_p$

In real life, power loss due to:
i) heating in coil due to resistance and in iron core due to eddy currents
ii) Hysteresis effect due to repeated change in magnetization and demagnetization of core

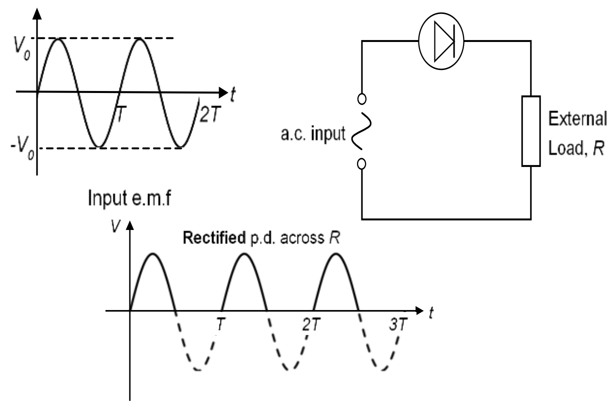
Power losses in transmissions
Power losses in line is mostly due to I^2R losses
For lower power loss, use higher voltage lines.



$I = (P_{gen} / V_{gen})$
$P_{gen} = P_H + P_{loss}$; $V_{gen} = V_H + V_{loss}$
$P_{loss} = I^2 R = (P_{gen} / V_{gen})^2 R$

Rectification is conversion of a.c. to d.c.
e.g. using diodes

Half-wave rectification



Output waveform follows the waveform of supply input.

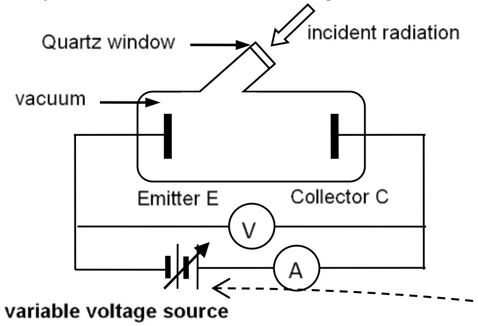
Quantum Physics I

PHOTOELECTRIC EFFECT

- Photoelectric effect is the phenomenon where electrons are emitted from a metal surface when electromagnetic radiation of sufficiently high frequency is incident on the surface.

- The photoelectric effect provides **evidence for the particulate nature** of electromagnetic radiation:
  (1) Existence of a threshold frequency below which no photoelectrons are emitted proves that electromagnetic radiation (EM) consists of discrete quanta of energy given by *hf*.
  (2) Instantaneous emission of photoelectrons when all the photon energy is delivered immediately to the electron in a single collision.
  (3) Maximum kinetic energy of the photoelectrons (existence of a stopping potential) being dependent only on the discrete energy of photon and independent on the intensity of radiation.

- Set-up for the **Photoelectric Experiment**:

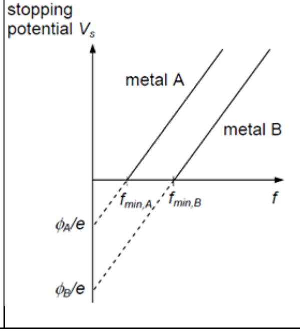


- When radiation of sufficiently high frequency (greater than the threshold frequency) is incident on the emitter plate E, electrons near the surface of the metal plate will gain sufficient energy to escape.
- The variable voltage source maintains electrodes at different known potentials.
- The emitted electrons that have sufficient energy will travel to the collector C and a current will be detected by the ammeter.

- To **determine Stopping Potential / max KE of photoelectrons**:
  - Illuminate metal E with an electromagnetic radiation of sufficient frequency. Adjust the potential difference between the emitter E and the collector C such that potential of C is held negative with respect to E, ***by reversing the polarity of the voltage source***.
  - Adjust the variable voltage source slowly such that the negative potential is made more negative just until no electron can reach C which is indicated as zero photocurrent by the ammeter. This is the stopping potential where this minimum negative potential will stop even the most energetic electron from reaching C.
  - In this situation, all the kinetic energy ($\frac{1}{2}mv^2$) of the fastest electrons will be converted into electric potential energy ($U = q\Delta V$) just before reaching C.

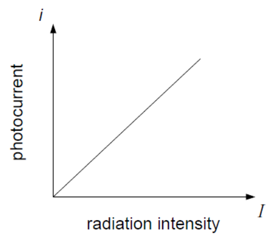
- **Photoelectric Electric Graphs**:

1.



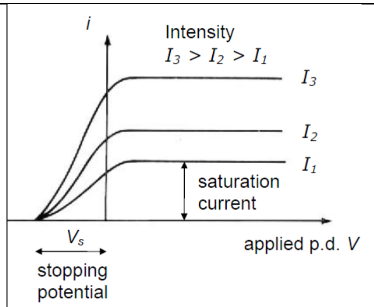
- Respective threshold frequencies are different as the metals have different work functions.
- The slope of the lines is the same since the slope is given by (h / e)

2.



- The photocurrent I (rate of emission of photoelectrons) is proportional to the intensity of EM radiation (rate of photons incident) and a straight line graph is obtained. (assumes constant frequency f and $f > f_o$)

3.



- Retarding potential V is increased negatively, the stopping potential V_S is the same for a beam of low intensity and one of high intensity.
- When V is positive all the photoelectrons are collected so that the current is constant.
- A beam of high intensity produces more electrons than one of low intensity.

- Photoelectric equation:

Photon energy	=	Work function energy	+	max KE of photoelectron
hf	=	Φ	+	$E_{k\ max}$
$\frac{hc}{\lambda}$	=	hf_o	+	$\frac{1}{2}m(v_{max})^2$
	=	$\frac{hc}{\lambda_o}$	+	eV_s

- A ***photon*** is a discrete bundle (or quantum) of electromagnetic energy.
  Energy of a single photon, $E = hf = \frac{hc}{\lambda}$
  Intensity of a beam of EM radiation, $\text{intensity} = \frac{P}{A} = \frac{E_{total}}{tA} = \frac{NE_{1photon}}{tA} = \frac{Nhf}{tA}$

- The **work function energy** Φ of a material is defined as *minimum* amount of the energy necessary to remove an electron from the *surface* of the material energy
  It is constant for a given metal.

- The **threshold frequency**, f_o is the minimum frequency of the incident radiation for the electron to escape.
  For photoelectric emission to occur, $f > f_o$ or $\lambda < \lambda_o$

- **Stopping potential**, V_s is the minimum retarding potential to stop all the electrons from reaching the collector plate.

- Electrons are emitted with a range of KE. Those most loosely-bound electrons will be emitted with more KE while the more tightly-bound ones will be emitted with smaller KE.

WAVE-PARTICLE DUALITY

- Waves can exhibit particle-like characteristics and particles can exhibit wave-like characteristics.

- **de Broglie wavelength** of a particle:
$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

- Packets of EM radiation of wavelength λ would therefore possess a momentum $p = \frac{h}{\lambda}$. When photons are incident on a surface, they therefore exert a force on the surface, resulting in a pressure on the surface. This pressure is known as "radiation pressure".

- Using $KE = \frac{p^2}{2m}$, the wavelength of a particle can be related to its *KE* by $\lambda = \frac{h}{\sqrt{2m(KE)}}$

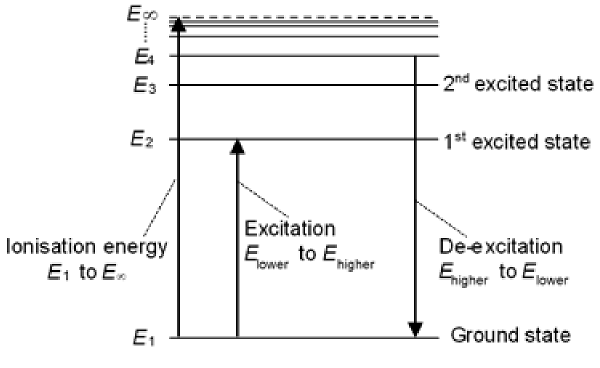
-

Observation	Evidence
Light as a wave	Double slit interference fringes are observed. Light through single slit undergoes diffraction. Light can be polarized.
Light as particles	Observations of **photoelectric effect** can only be explained if light is quantized.
Electrons as particles	Electrons undergo collision, has mass and charge
Electrons as a wave	**Electron diffraction** where electron beam produces a diffraction pattern when passed through a thin carbon film.

ENERGY DIAGRAMS & LINE SPECTRA

- Line Spectra provides evidence for the existence of **discrete energy levels** in the atom.

- Electrons can revolve round the nucleus only in *certain allowed orbits.*

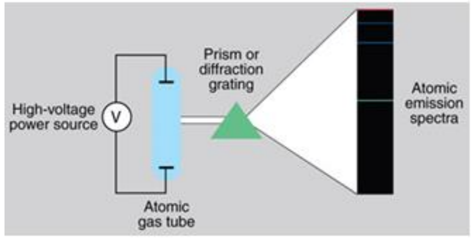


- An atom can only absorb or emit energy when an electron transits from one state to another.
$$\Delta E = E_{higher} - E_{lower} = hf = \frac{hc}{\lambda}$$

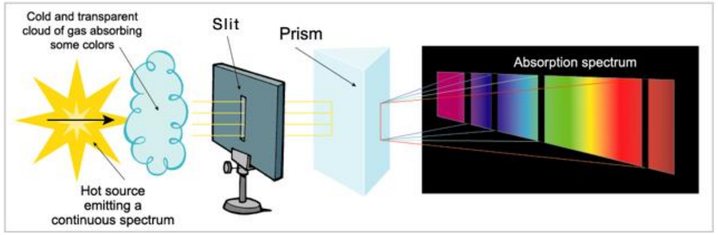
- Excitation by absorption of photons or high speed collision by another particle.

- An atom in excited states are very unstable and the electron would almost immediately de-excites to a lower energy level, emitting a photon corresponding to the energy difference in the process.

- **Emission line spectrum** consists of discrete bright coloured lines in a dark background.
It is produced when
(1) Gases are placed in a discharge tube at low pressure. A voltage (several kilo-volts) is applied between metal electrodes in the tube which is large enough to produce an electric current in the gas.
(2) The gas becomes excited by the collisions with the electrons passing through the tube, from cathode to anode of the discharge tube.
(3) The excited gas atoms are unstable. When the gas atoms transits to a lower energy level, the excess energy is emitted as electromagnetic radiation (photon) with a specific frequency.
(4) The frequency *f* of the emission line is dependent on the difference between the high and low energy levels, $\Delta E = hf$. Due to the discrete energy levels, only certain high-to-low energy level transitions are possible within the atom, therefore only certain frequency lines are present in the spectrum.

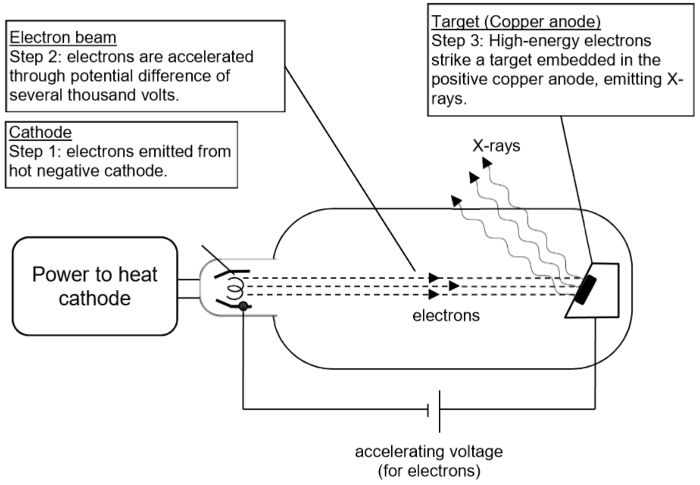


- **Absorption line spectrum** consists of dark lines against a continuous spectrum of the white light.
It is produced when
(1) It is produced when white light containing all frequencies passes through a cold gas.
(2) Those incident photons whose energies are exactly equal to the difference between the atom's energy levels are being absorbed. Since the energy levels are discrete, only photons of certain frequencies are absorbed.
(3) When the atoms transit back to the ground state, the photons of the same frequencies are then re-radiates but *in ALL directions*.
(4) Consequently, the parts of the spectrum corresponding to these wavelengths appear dark (or "missing") in comparison with the other wavelengths.



Quantum Physics II (X-Rays & Uncertainty Principle)

X-ray Production



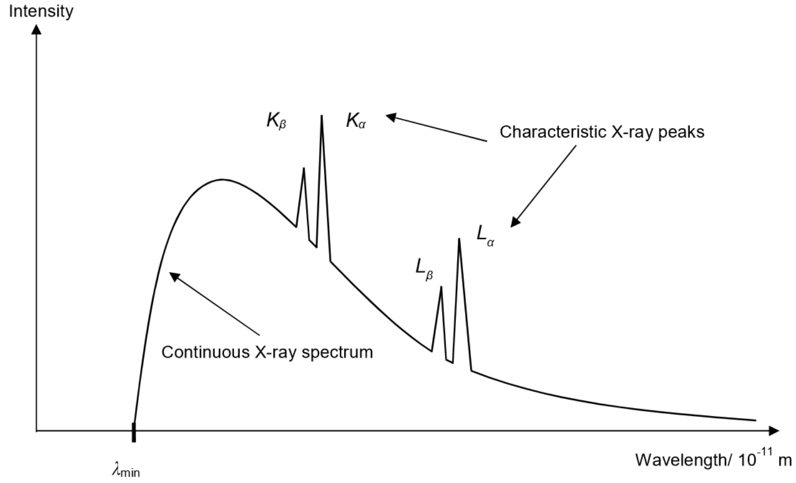
Heisenberg Uncertainty Principle

$$\Delta p \Delta x \gtrsim h$$

where h = Planck's constant 6.63×10^{-34} J s

It tells us that *simultaneous measurement of both position and momentum of an object precisely is not possible*. The more accurately we attempt to measure the position so that Δx is small, the greater will be the uncertainty in momentum Δp, and vice-versa.

X-ray Spectrum



Characteristic X-ray peaks

- An accelerated electron from the cathode collides into an orbiting electron of a target atom that is orbiting in the *K-shell*. If sufficient energy is transferred by the accelerated electron to the orbiting electron, the latter electron can be ejected from the target atom, leaving a *vacancy* in the K-shell.
- When the vacancy in the *K*-shell (n = 1) is filled by an electron from the *L*-shell (n = 2), an X-ray photon of the $\boldsymbol{K_\alpha}$ characteristic X-ray is emitted
- If the vacancy in the K-shell is filled by an electron dropping from the *M*-shell (n = 3), an X-ray photon of the $\boldsymbol{K_\beta}$ characteristic X-ray is emitted
- The wavelengths of these X-rays produced can be determined by the following equation: $hf = \frac{hc}{\lambda} = E_n - E_1$; $n = 2,3,...$
- Since the energy differences between the discrete energy levels are characteristics of the target atom, the wavelengths of the K_α and K_β characteristic X-rays are unique for each element.

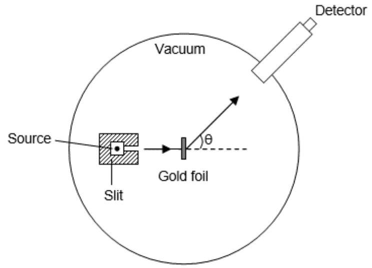
Continuous X-ray spectrum

- An electron with an initially high kinetic energy $E_{k,initial}$ collides with a target atom
- As the electron approaches a nucleus in the target atom, it deflects due to the attractive force between the nucleus and the electron and emits electromagnetic energy in the form of a photon (X-ray)
- Hence it loses kinetic energy
- The energy of the photon released depends on the magnitude of the acceleration. The closer an electron approaches the nucleus, the larger the deflecting force, the higher the energy of the photon.
- As different electrons approach the nucleus with different proximity, there will be a distribution of photon energies, and hence a wide range of wavelengths.
- There is a sharply defined minimum wavelength λ_{min} that corresponds to the highest energy x-ray photon, resulting from a collision in which an incident highly energetic electron stops abruptly in a single collision and all the kinetic energy of the electron is completely converted into a single X-ray photon.

Nuclear Physics

Model of the Atom

Rutherford's Alpha-particle Scattering Experiment



Results	Interpretations
Most particles passed through undeflected	Atom consists of mostly empty space, nucleus is small.
Small fraction of α-particles are deflected through large angles	Nucleus is massive and positively charged.

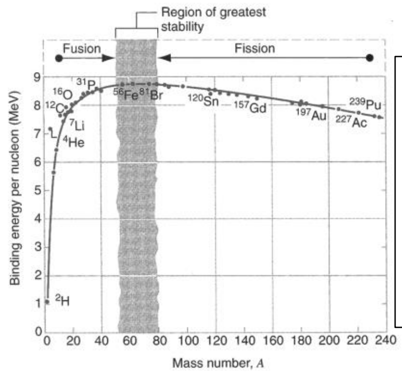
- The nucleus contains protons and neutrons – collectively referred to as nucleons.
- A nuclide is a particular species with a unique pair of values of A and Z.
- The notation to represent a nucleus, X, with atomic number Z and mass number A is ${}^{A}_{Z}X$
- **Isotopes are atoms that have the same number of protons but different number of neutrons.**
- **One unified atomic mass unit is one-twelfth the mass of the carbon-12 atom.**

Effects of Radiation on Living Organisms

- Radiation can cause immediate severe damage to body tissue such as radiation burns.
- Delayed effects such as cancer and eye cataracts.
- Hereditary defects may also occur in succeeding generations.
- When radiation passes through living tissues, it can damage the structure of molecules.
- Ionising radiation can result in genetic mutations which can lead to birth defects.

Mass-Energy Equivalence, BE & Nuclear Reactions

- Mass-Energy equivalence: $E = mc^2$
- **Mass defect of a nucleus is defined as the difference between the mass of the separated nucleons and the combined mass of the nucleus.**
  $\Delta m = \sum m_{nucleons} - m_{nucleus}$
- **Nuclear binding energy of a nucleus is defined as minimum energy required to completely separate a nucleus into its constituent neutrons and protons.**
  $BE = (\Delta m)c^2$
- Nuclear stability - B.E. per nucleon is a measure of the stability of the nucleus – the larger the binding energy per nucleon, the more stable the nucleus.



Iron-56 nucleus one of the most stable nuclides, located close to the peak with a B.E. per nucleon value of about 8.8 MeV.

- **Nuclear fission is the disintegration of a heavy nucleus into two lighter nuclei of approximately equal masses.**
- **Nuclear fusion is the combining of two light nuclei to produce a heavier nucleus.**
- Calculating energy released from nuclear reactions:
  - Energy released = $(\sum m_{\text{reactants}} - \sum m_{\text{products}})\, c^2$
  - Energy released = $BE_{\text{products}} - BE_{\text{reactants}}$
- In all nuclear processes, the following quantities are conserved:
  - nucleon number
  - proton number (charge)
  - linear momentum
  - mass-energy

Radioactivity

- **Radioactive decay is the spontaneous emission of particles (α or β particles) and/or radiation (γ ray) from an unstable nucleus so that it becomes more stable.**
- Radioactive decay is
  - spontaneous - not affected by external conditions and;
  - random - impossible to predict which nucleus will decay next.
- The random nature of radioactive decay can be demonstrated by observing the fluctuations in count rate
- Types of radioactive decay: alpha, beta and gamma.

Property	α-particles (${}^{4}_{2}\mathrm{He}$)	β-particles (${}^{0}_{-1}\mathrm{e}$)	γ-rays (γ)
Nature	Helium-4 nucleus	Electrons	Electromagnetic waves of short wavelength
Charge	+2e	-e	0
Mass	$6.6464835 \times 10^{-27}$ kg	$9.1093897 \times 10^{-31}$ kg	0
Deflection by E- & B-fields	Deflected by strong fields	Deflected by weak fields	Undeflected
Energy	Constant for a given source	From zero up to a maximum which depends on the source	Depends on frequency
Speed	$\approx 10^7$ m s^{-1}	$\approx 10^8$ m s^{-1}	3.0×10^8 m s^{-1}
Range in air	A few cm	A few metres	Follows the inverse square law
Ionising Power	most ← least; ionizing power is the ability of the radiation to ionize other atoms (remove electrons) and damage those atoms		
Penetrative power	least → most; alpha is more massive and has more charge therefore it is most ionizing and least penetrative		
Stopped by	10^{-2} mm aluminium or a few cm of air	5 mm aluminum	100 mm lead

- **The decay constant of a nucleus is defined as its probability of decay per unit time.**
- **The activity of a radioactive source is defined as the number of nuclear decays per unit time occurring in the source.**
- **Half-life is defined as the time taken for half the original number of radioactive nuclei to decay.**
- Important formulae:

$$A = \lambda N \qquad t_{1/2} = \frac{\ln 2}{\lambda}$$

$$N = N_0 e^{-\lambda t} \qquad A = A_0 e^{-\lambda t} \qquad C = C_0 e^{-\lambda t}$$