

RAFFLES INSTITUTION MATHEMATICS DEPARTMENT 2024 YEAR 3 RP MATHEMATICS TOPIC 9:EXPONENTIAL AND LOGARITHMIC EQUATIONS & FUNCTIONS (MATHS 1 & MATHS 2)

SUPPLEMENTARY WORKSHEET

Name:	Class: Sec 3 ()	Date:

1 <u>2022/Y3RP/M2/T2/Q1</u>

Solve the equation $\log_6 2(3x+4) - \log_6(x-4) = \log_{0.2} \frac{1}{25}$.	[3]
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2 <u>2022/Y3RP/M2/T2/Q2</u>

Solve the equation $3e^{x+1} - 2e^{\frac{1}{2}x+1} - 8e = 0$, giving your answer(s) correct to 3 significant figures. [4] [Ans: 1.39]

3 <u>2022/Y3RP/M2/T2/Q3</u>

Given that $\log_{a^2} x = m$ and $\log_{\sqrt{a}} y = n$, express $\log_x y^4$ in terms of m and n. [4]

[Ans: $\frac{n}{m}$]

[Ans: $\frac{76}{15}$]

4 <u>2022/Y3RP/M2/T2/Q4</u>

- (i) Sketch the graph of $y = \ln(2x+3)$, indicating clearly the asymptote and any intersection(s) with the axes. [2]
- (ii) Insert on your sketch the additional straight line graph required to obtain a graphical solution of the equation $e^x = \frac{3}{2x+3}$. [2]

[Ans: $y = -x + \ln 3$]

5 <u>2022/Y3RP/M2/T2/Q5</u>

Carbon dating is a method used to estimate the age of archaeological artefacts. The age, W years old, of an artefact with a carbon ratio of N is given by $W = -\frac{\ln N}{0.693} \times 5700.$

- (i) Find the age of an artefact, correct to the nearest 1000 years, with a carbon ratio of 0.48. [1]
- (ii) Express N in the form $N = e^{kW}$, where k is a constant correct to 3 significant figures. [2]
- (iii) Hence, sketch the graph of N against W.

[Ans: (i) 6000 years old (ii) $N = e^{-0.000122 W}$]

2021/Y3RP/M2/T2/O2

7

Solve $\lg(\ln x^2) = \frac{1}{2}$.

[Ans: ±4.86]

[2]

[2]

[4]

Solve
$$(3^{2x})(2-5^x) = 15^{2x+1}$$
.

8 <u>2021/Y3RP/M2/T2/Q3</u>

Sketch the graph of $y = \ln(5-2x)$. Insert in your sketch the additional straight line required to obtain a graphical solution to the equation $e^{x+3} = (5-2x)^3$. [4]

[Ans:
$$y = \frac{1}{3}x + 1$$
]

9 <u>2021/Y3RP/M2/T2/Q4</u>

Solve the simultaneous equations $\frac{27^{x}}{9} = 3^{y},$ $\log_{2} x - 1 + \log_{3} y = \log_{3}(x+4).$

[5] [Ans: x = 3, y = 7]

[3]

[3]

[2]

[Ans: $3^x = \sqrt[3]{p}$]

10 <u>2021/Y3RP/M2/T2/Q5</u>

Given that $\log_8 x = p$ and $\log_8 y = q$, find

- (i) $m \text{ in terms of } p \text{ if } 64x = 2^m$, [2]
- (ii) $\log_{\sqrt{x}} 4y$ in terms of p and q.

[Ans: (i)
$$m = 6 + 3p$$
 (ii) $\frac{4+6q}{3p}$]

11 <u>2020/Y3RP/M2/T2/Q1</u>

- (i) State the equation of the asymptote of the graph of $y = \ln(2-3x)$. [1]
- (ii) To solve the equation $\sqrt{e^{x+1}} + 3x = 2$ using the graph of $y = \ln(2-3x)$, a straight line graph needs to be inserted. Determine the equation of this straight line. [2]

[Ans: (i)
$$x = \frac{2}{3}$$
 (ii) $y = \frac{1}{2}x + \frac{1}{2}$]

12 <u>2020/Y3RP/M2/T2/Q4</u>

Without using a calculator, solve the equation

$$2\log_{16}(2x-1)^{2} + 6\log_{27}9 = \log_{2}(5-3x).$$
[5]
[Ans: $x = \frac{3}{5}$]

13 <u>2020/Y3RP/M2/T2/Q5</u>

- (i) Solve the equation $(2^x)^3 + 15 = 2^{6x+1}$.
- (ii) Hence, without using a calculator, express $\log_8 72$ in terms of x where x is the solution of the equation in (i). [2] [Ans: (i) 0.528 (ii) 2x + 1]

14 <u>2019/Y3RP/M2/T2/Q1</u>

15

Given that $\log_{27} p = x$, find 3^x in terms of p.

- <u>2019/Y3RP/M2/T2/Q5</u> Solve the following equations, leaving your answer correct to 3 significant figures where necessary.
- (a) $\ln(8-3^x) x \ln 3 = \ln 2$ [4]
- (b) $\log_3(x+3) 6\log_{x+3}3 = \log_x x$ [5]

[Ans: (a) 0.893 (b) 24]

16 <u>2019/Y3RP/M2/T2/Q6</u>

In a laboratory experiment to determine conditions suitable for amoeba, a researcher studied a sample which initially contained 3 500 000 amoebae. The observed size of the amoeba population, N, is given by $N = N_0 e^{at}$ where N_0 and a are constants and t is the time in days since the start of the experiment. Due to strongly unfavourable conditions, it was observed that 30% of the population died each day.

(i) Determine the values of N_0 and a.

(ii) Calculate the number of days that will pass before the population reduces to 2000, giving your answer to the nearest day. [2]

[Ans: (i) -0.357 (ii) 21 days]

17 <u>2015/Y3RP/T2/Q5</u>

Solve the equation:

- (a) $3^{x+1} = 5$, [2] (b) $\log_5(x-1) + \log_5(x-2) = 2\log_5\sqrt{6}$, [3] (c) $e^{1+x} - 30e^{1-x} = e$. [3] [Ans: (a) 0.465 (b) 4 (c) 1.79]
- **18 <u>2013/Y3RP/T2/Q1</u>** Solve $2^{2x} - 2^{x+1} = 8$.

[4] [Ans: x = 2]

19 <u>2012/Y3RP/T2/Q3</u>

Solve the equation $5^{4x-3} = 2^{2x+7}$, giving your answer correct to three significant figures. [3] [Ans: 1.92]

20 $\frac{2012/Y3RP/T2/Q4}{\text{Solve the equation } \log_5(3x+11) = \log_{\sqrt{5}}(1-x)+1.$ [4]

[-]

[Ans: $-\frac{2}{5}$]

21 <u>2012/Y3RP/T2/Q5</u>

Sketch the graph of $y = \ln(2x-3)$, $x > \frac{3}{2}$. Insert in your sketch the additional graph required to obtain a graphical solution to the equation $2x = e^{2-x} + 3$. [4] [Ans: y = 2 - x]

22 <u>2012/Y4RP/T1/Q1</u>

Given that $\log_3 x = a$ and $\log_3 y = b$, express $\log_3 \frac{(3x)^4}{\sqrt{y}}$ in terms of *a* and *b*. [3]

[Ans: $4 + 4a - \frac{1}{2}b$]

23 <u>2012/Y4RP/T1/Q2</u>

Solve the following equations

- (i) $3e^x + e^{-x} = 4$ leaving your answers in logarithmic form where necessary, [4]
- (ii) $3\log_6 y \log_y 6 = 2$, leaving your answers to 3 significant figures where necessary. [5]

[Ans: (i) -ln3 or 0 (ii) 0.550 or 6]

24 <u>2012/Y4RP/T1/Q3</u>

Sketch the graph of $y = \ln(x+3)$ for x > -3. Determine the equation of the additional straight line which would need to be drawn in order to obtain a graphical solution of the equation $e^{x+2} = (x+3)^4$. [4]

[Ans:
$$y = \frac{1}{4}x + \frac{1}{2}$$
]

25 <u>2009/Y3RP/T4/Q1</u>

Given that $\lg x = a$ and $\lg y = b$, express $\lg \left(\sqrt[3]{\frac{x^5}{100y^2}} \right)$ in terms of a and b. [3]

[Ans:
$$\frac{1}{3}(5a-2-2b)$$
]

26 <u>2009/Y3RP/T4/Q2</u>

Solve the equation

- (a) $(2^{x-1})(5^{x+2}) = 10^{2x}$, giving your answer to 3 significant figures, [4]
- **(b)** $\log_3 y^2 2 = 12 \log_y 3$. [4]

[Ans: (a) 1.10 (b) 27 or $\frac{1}{9}$]

27 <u>2009/Y3RP/T4/Q3</u>

(i) Given that $\log_5 x = a$ and $\log_{25} y = b$, express xy^2 and $\frac{x^2}{y}$ as powers of 5. [3]

(ii) Further given that $xy^2 = 625$ and $\frac{x^2}{y} = \frac{1}{25}$, find the values of *a* and *b*. [3]

[Ans: (i) 5^{a+4b} , 5^{2a-2b} (ii) a = 0, b = 1]

28 <u>2009/Y3RP/T4/Q4</u>

Sketch the graph of $y = \ln(x-2)$ for x > 2. On the same axes, insert the graph of the additional straight line which would need to be drawn in order to obtain a graphical solution of the equation $e^{2x}(x-2) = e^3$. [5]

[Ans: y = 3 - 2x]

29 <u>2008/Y3RP/T3/Q2</u>

Solve $5^{x+1} - 21\left(\frac{1}{5}\right)^x + 32 = 0$.

[Ans: -0.317]

[4]

30 <u>2008/Y3RP/T3/Q3</u> Civer that 21ag up 1 ag up 4 where up 10 and 11 annual up to the total of total of the total of the total of the total of the total of total o

Given that $2\log_x y + \log_{\frac{1}{x}} y = 4$, where x, y > 0 and $x, y \neq 1$, express y in terms of x in its simplest form. [3] [Ans: $y = x^4$]

31 2008/Y3RP/T3/Q5

Solve the simultaneous equations $3^{y} = \frac{81^{x}}{9}$, [5] $\log_{2}(3y-2)-3 = \log_{2} x$.

[Ans: x = 2, y = 6]

32 <u>2008/Y3RP/T3/Q6</u>

Solve
$$\ln(2^{x}+2) + \ln(2^{x+1}-1) = 2x \ln 2 + 2 \ln \sqrt{2}$$
. [5]
[Ans: $x = -0.585$]

33 <u>2007/Y3RP/T3/Q3</u>

Solve the equation $e^{x} - 1 = 6e^{-x}$, leaving your answers in the form $a \ln b$, where a and b are integers. [3]

[Ans: ln3]