Name		Register Number		Class	

DUNMAN HIGH SCHOOL
Mid-Year Examination
Year 4 DHP

MATHEMATICS 2

11 May 2021
2 hours 30 minutes

Additional Materials: Answer Sheet
Insert A

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.
Write your answers on the separate answer paper provided.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded on the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \tfrac{1}{2} ab \sin C$$

Answer **all** the questions

1 Find the range of values of m for which the line $4y=(m-2)x$ intersects the curve $y=\dfrac{2}{x+1}$. [4]

2 Given that the curve $y=ax^2+8x-c$ lies completely above the line $y=c$, find the condition(s) that must be applied to constants a and c. [4]

3 Find the x-coordinates of the points on the curve $y=\mathrm{e}^{x+\sin 2x}$, $0\le x\le\pi$, at which the gradient is zero. [4]

4 Express $\dfrac{2x^2+x-5}{(x-1)^2}$ in partial fractions. [4]

5 Solve $\log_5 x^2=3\log_{\sqrt{x}}5-1$. [5]

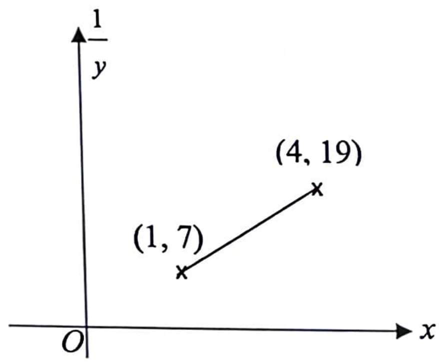
6 **(i)** Explain why $12x-3x^2-16<0$ for all real values of x. [3]

(ii) Hence, solve $\dfrac{(7x-3)(x+5)}{12x-3x^2-16}>0$. [2]

7 **(i)** Given that $y=\dfrac{5x}{\left(\sqrt{1-x^2}\right)^3}$, find the values of a and b such that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{5(ax^2+b)}{\left(\sqrt{1-x^2}\right)^5}$. [3]

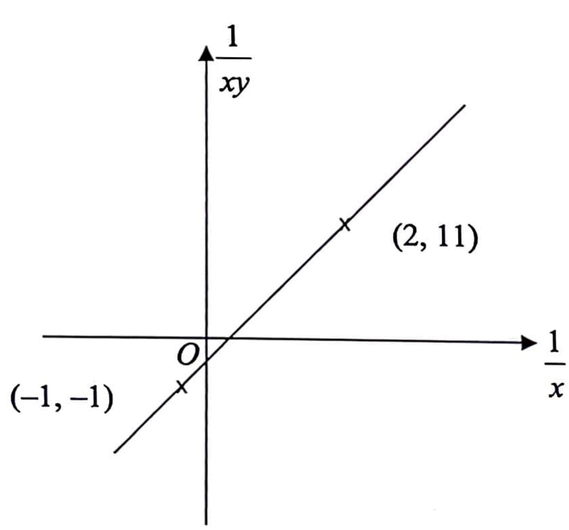
(ii) Hence, show that $(1-x^2)\dfrac{\mathrm{d}y}{\mathrm{d}x}-\dfrac{y}{x}=2xy$. [1]

8 Mary collected some data from a science experiment to verify an equation $1-by=axy$. She plotted a graph of $\frac{1}{y}$ against x and obtained the straight line graph below.



(i) Find the value of a and of b. [4]

(ii) John claimed that he used the same data set from Mary and plotted the graph of $\frac{1}{xy}$ against $\frac{1}{x}$ below. Assuming that Mary's graph is correct, comment if John's graph is correct. [2]



9 (i) Given that $\frac{\sin(2A+B)}{\sin(2A-B)}=\frac{3}{2}$, show that $\tan 2A = 5\tan B$. [3]

(ii) Hence, find $\tan A$ without evaluating the values of A and B, given that $\cos B = -\frac{15}{\sqrt{241}}$, where B is an obtuse angle. [3]

10 The temperature, $T^{\circ}C$, of a piece of meat left to thaw for t hours after removing from a chiller, is given by $T = 20 - Ae^{-kt}$, where A and k are constants.

(i) The temperature of the chiller is $-15^{\circ}C$.
Show that $A = 35$. [1]

(ii) The temperature of the meat is $1.5^{\circ}C$ after one hour.
Find the value of k. [3]

(iii) In order to reduce the risk of bacterial contamination in thawed food, the temperature of the food should not exceed $5^{\circ}C$ during the thawing process.
Determine, with working, whether the meat has a high risk of bacterial contamination, 3 hours after removing from the chiller. [2]

11 **(a)** Given that $1 \le y \le 6$, solve $2\cos\left(\frac{2(y+1)}{3}\right) + \sqrt{3} = 0$, leaving your answers in terms of π. [3]

(b) Find all the angles between -180° and 180° which satisfy the equation $\sec^2 x = 4\sec x - 3\tan^2 x$. [4]

12 **(a)** Given that $g(x)+3$ is a polynomial with factor $x-3$, find the remainder when $f(x) = (x^2 - 3x + 4)g(x)$ is divided by $x-3$. [3]

(b) Show that $\left(1 + \frac{1}{\cos x \cot x + \sin x}\right)(1 + \tan^2 x) = \frac{1}{1 - \sin x}$. [4]

13 $P(x) = 3x^3 - px^2 - qx - 8$ has a factor of $x^2 + 4$.

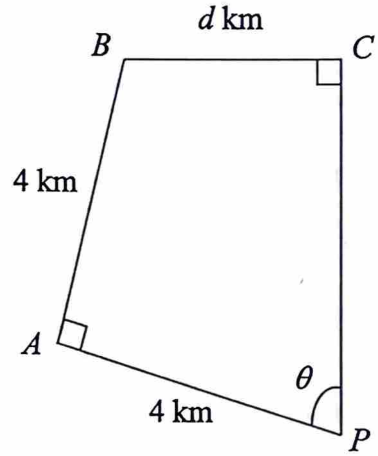
(i) Find the value of p and of q, and the other factor. [3]

(ii) $P(x)$ can also be expressed as $P(x) = (x^2 - 3)g(x) + h(x)$, where $g(x)$ and $h(x)$ are functions of x.

(a) Student Z claims that if $(x^2 - 3)$ is the divisor, the degree of $h(x)$ can only be 1. Explain whether his claim is true. [1]

(b) Find $g(x)$ and $h(x)$. [3]

14 The diagram shows the daily jogging path $PABC$ taken by Peter. He starts from point P and ends at point C. Angle $APC = \theta$, angle $PAB =$ angle $BCP = 90°$, $PA = AB = 4$ km and $BC = d$ km.



(i) Show that $d = 4\sin\theta - 4\cos\theta$, where $0° < \theta < 90°$. [3]

(ii) Express d in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [2]

(iii) By using **(ii)**, comment whether it is possible for Peter to complete a 12-km run using the track $PABC$ if θ varies. [2]

15 **Answer this question on Insert A.**

16 **Answer this question on Insert A.**

17 **Answer this question on Insert A.**

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MATHEMATICS 2 (Insert A)

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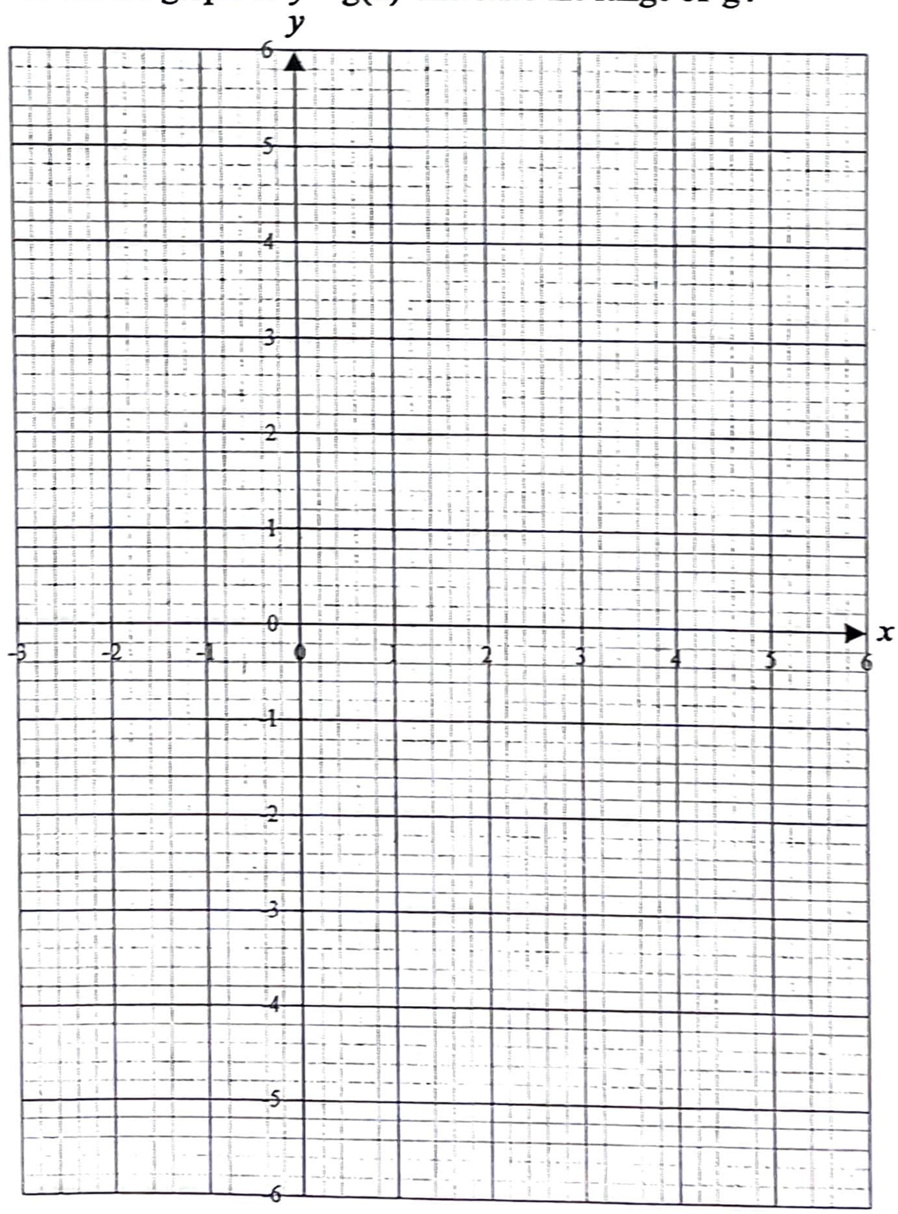
This insert consists of 5 printed pages.

15 Function g is defined by $\mathrm{g} : x \mapsto -\frac{1}{2}x^2 + x + 4,\ x \in \mathbb{R},\ -2 < x \leq 5.$

(i) Express $\mathrm{g}(x)$ in the form of $a(x+b)^2 + c$, where a, b and c are real numbers. [2]

(ii) Sketch the graph of $y = \mathrm{g}(x)$ and state the range of g. [4]



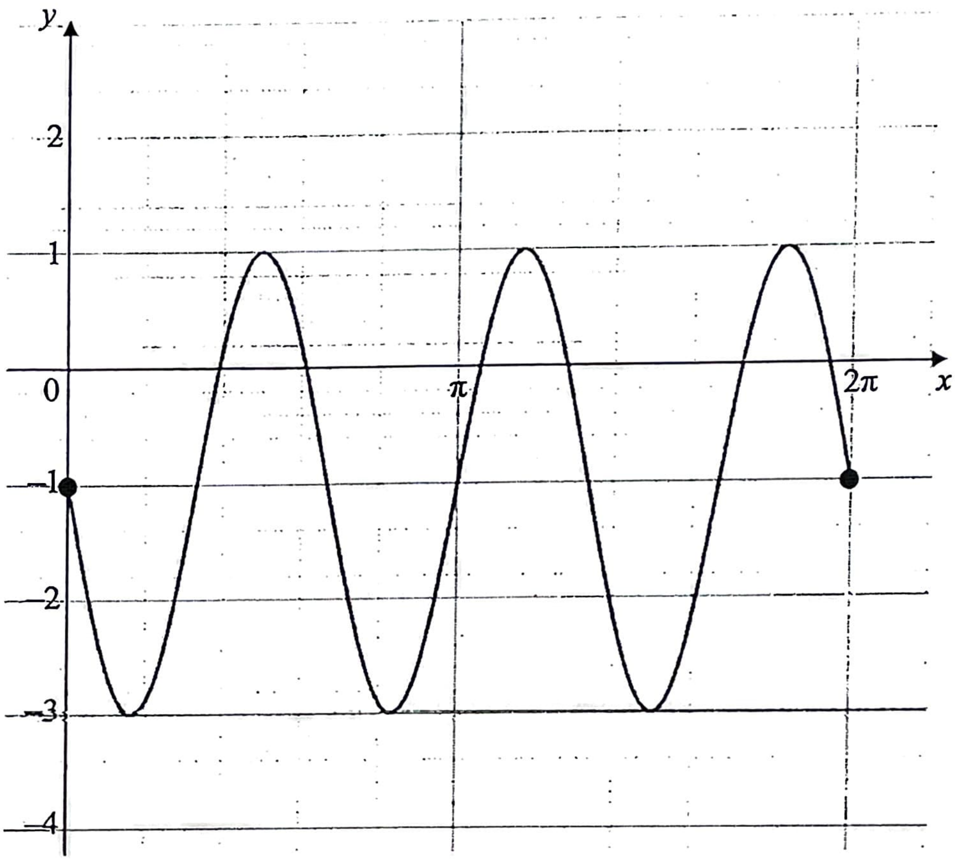
A function h is defined by $\mathrm{h} : x \mapsto -\frac{1}{2}x^2 + x + 4,\ x \in \mathbb{R}\ ,\ -2 < x \leq k$.

(iii) Find the largest value of k for which h has an inverse. [1]

(iv) For this value of k, find the function h^{-1} and express it in similar form. [3]

(v) Sketch the graph of $y = \mathrm{h}^{-1}(x)$ on the same axes as **(ii)**. [2]

16 The graph of function $y=\mathrm{f}(x)$ for $0 \le x \le 2\pi$ is shown below.



(i) State the period of $y=\mathrm{f}(x)$. [1]

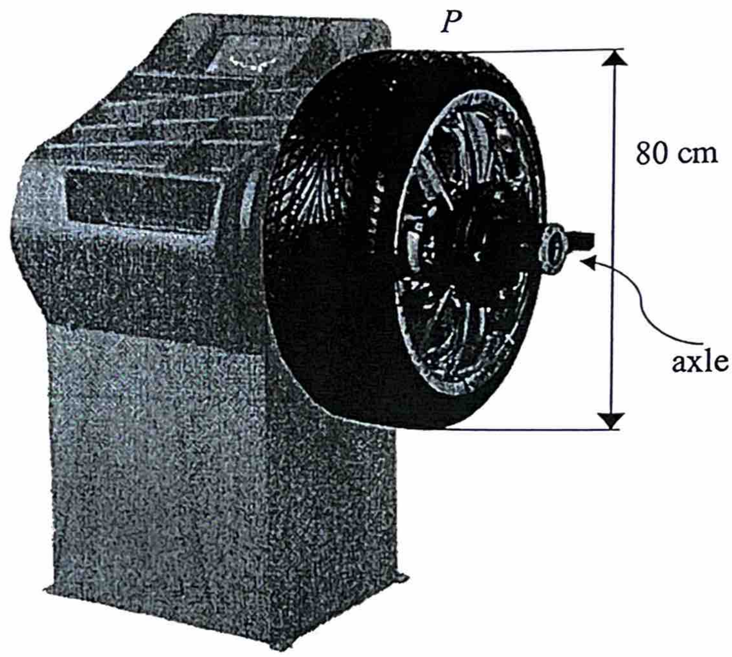
(ii) State the amplitude of $y=\mathrm{f}(x)$. [1]

(iii) Given that $\mathrm{f}(x)=a\sin bx+c$, write down the equation of the graph. [1]

(iv) On the same axes, sketch the graph of $y=1-\dfrac{2x}{\pi}$ for $0 \le x \le 2\pi$. [2]

(v) Hence state the number of solutions, for $0 \le x \le 2\pi$, of the equation $\pi(1-c-a\sin bx)=2x$. [2]

17

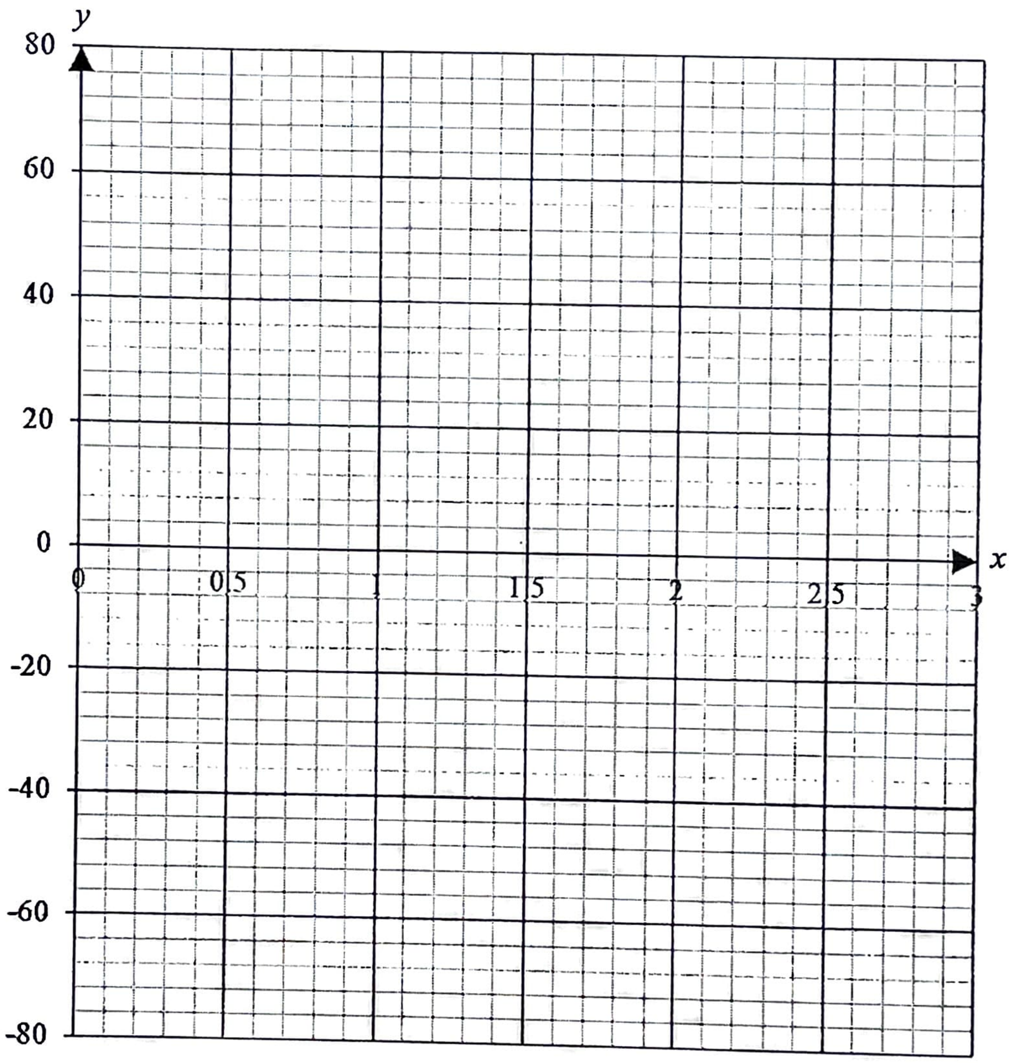


An important aspect to having a smooth car ride is to ensure the wheels are well balanced. To do that, the wheel is first mounted on the machine and rotated. The machine will then automatically calculate the weight and location of the balance counter weights to ensure that there are no heavy spots on the tyre.

The diagram shows a wheel of diameter 80 cm attached on the wheel-balancing machine. Initially, the point P is on the highest point of the circumference of the wheel as shown. The wheel is then rotated in the clockwise direction at 40 revolutions per minute about the axle.

(i) Find the time taken, in seconds, for the point P to complete one revolution. [1]

(ii) The vertical distance, y cm, of P above the axle is taken as positive. On the given axes, sketch the graph of P against time, t s, for $0 \leq t \leq 3$. You need not find the t-intercepts. [2]



(iii) Write down the equation of the graph in **(ii)**. [2]

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