Complex Numbers Tutorial 9B:
Polar and Exponential Forms
Solutions

Basic Mastery Questions

1i) $|1+\sqrt{3}i| = \sqrt{1+3} = 2$

$\arg(1+\sqrt{3}i) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$\therefore\ 1+\sqrt{3}i = 2\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$ #

ii) $|1-i| = \sqrt{1+1} = \sqrt{2}$

$\arg(1-i) = \tan^{-1}(-1) = -\frac{\pi}{4}$

$\therefore\ 1-i = \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$ #

iii) $\left|\frac{(1+\sqrt{3}i)^4}{(1-i)^6}\right| = \frac{|1+\sqrt{3}i|^4}{|1-i|^6} = \frac{2^4}{2^3} = 2$

$\arg\left[\frac{(1+\sqrt{3}i)^4}{(1-i)^6}\right] = 4\arg(1+\sqrt{3}i) - 6\arg(1-i)$

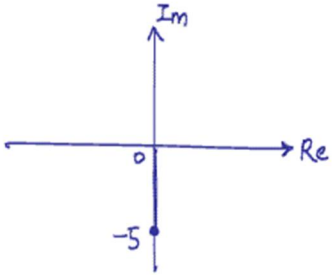
$= 4\left(\frac{\pi}{3}\right) - 6\left(-\frac{\pi}{4}\right) = \frac{17\pi}{6} \equiv \frac{5\pi}{6}$

$\therefore\ \frac{(1+\sqrt{3}i)^4}{(1-i)^6} = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$ #

2i) $5\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)^5 = 5(0-i)^5$

$= 5(-i)^5$

$= -5i$ #



ii)

$$\left(\sin\left(-\frac{\pi}{3}\right) + i\cos\left(-\frac{\pi}{3}\right)\right)^6$$

$$= \left(\cos\left(\frac{\pi}{2}+\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{2}+\frac{\pi}{3}\right)\right)^6$$

$$= \left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)^6$$

$$= \cos 5\pi + i\sin 5\pi = -1$$