2023 H3 Prelim Suggested Solutions

1 (a) It is a frame, which moves at a **constant velocity** and **Newton's laws are applicable** in such frames. B1 B1

Comments: Several students forgot to mention about Newton's laws in their answers.

(b) Let the identical mass be m. The kinetic energy of the system in this frame is

$$K = \frac{1}{2}m(2u)^2 + \frac{1}{2}m(3u)^2 = \frac{1}{2}mu^2(13)$$

A frame that moves towards cart A with a speed of u.
In this frame, the speed of cart A becomes $3u$ and the speed of cart B becomes $2u$. B1
Hence, the same kinetic energy K is obtained in this inertial frame. A1

Comments: Many students managed to express the correct frame of reference.

(c) Let the speed of the mass m be u. The collision must be a perfectly inelastic one.

By the principle of conservation of linear momentum, B1

$$mu + 0.5(0) = 1.5mv_{common}$$
$$v_{common} = 0.667u$$

The initial kinetic energy of the system before the collision is $\frac{1}{2}mu^2$.

The kinetic energy after the collision is $\frac{1}{2}(1.5m)(0.667u)^2 = 0.333mu^2$.

The increase in internal energy of the system is $(0.5 - 0.333)mu^2 = 0.167mu^2$ B1

The fraction of the initial KE that is convertible to internal energy of the system would be

$$\frac{0.167mu^2}{0.5mu^2} = \frac{1}{3}$$ A1

Comments: This part was a distinguishing factor. Weaker students could not properly set their thought process on the inelastic nature of collision, only in which, energy losses take place.

2 (a)(i)

$$\rho = \frac{2M}{\pi R^2 d + 2\pi (2R)^2 d}$$ A1

(a)(ii) Moment of inertia of a circular disc about an axis through its center is given by

$$I_{disc} = \int_0^R \left(\frac{m}{\pi R^2}\right)(2\pi r dr)\, r^2 = \frac{mR^2}{2}$$ B1

$$I_{reel} = \frac{(M_{inner} + M_{outer})r^2}{2} = \frac{\rho\left(V_{inner}R^2 + V_{outer}(2R)^2\right)}{2}$$

$$= \frac{\left(\dfrac{2M}{\pi R^2 d + 2\pi (2R)^2 d}\right)\left(\left(\pi R^2 d\right)R^2 + \left(2\pi (2R)^2 d\right)(2R)^2\right)}{2}$$ B1

$$I_{reel} = \frac{\left(\dfrac{2M}{\pi R^2 d(1+8)}\right)\left(\pi R^2 d\right)\left(R^2 + 32R^2\right)}{2}$$ B1

$$I_{reel} = \frac{2M}{2}\frac{33R^2}{9} = \frac{11}{3}MR^2$$

Comments: The proof for the moment of inertia of a disc should be shown in the working as the expression is not given in the formula list.

(b) Let the tension in the string be *T*.

Consider the **box** falling with acceleration *a*,

$$Mg - T = Ma$$ M1

Consider the translational motion of **reel**,

$$T - 2Mgsin30° - f = 2M(2R\alpha)$$ M1

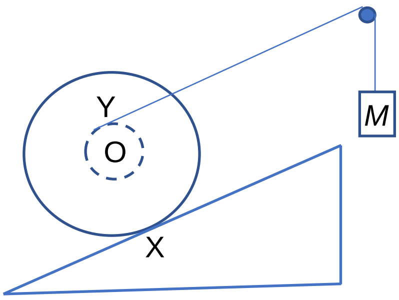
Consider the rotational motion of **reel**,

$$T(R) + f(2R) = \left(\frac{11MR^2}{3}\right)\alpha$$

Solving,

$$\frac{3T}{2} - Mg = 4MR\alpha + \frac{11MR\alpha}{6} = \frac{35MR\alpha}{6}$$ M1

Further, as the **reel** rotates,



$v_M = v_{OY} + v_O$

where v_M is velocity of box wrt earth
V_{OY} is velocity of Y wrt reel center of mass
V_O is velocity of reel wrt earth

differentiate the velocities with respect to time,

$$\frac{dv_M}{dt} = \frac{dv_{OY}}{dt} + \frac{dv_O}{dt}$$

$a = R\alpha + 2R\alpha = 3R\alpha$

M1

Solving,

$$\frac{3T}{2} - Mg = \frac{35Ma}{18}$$

$$\frac{1}{2}Mg - \frac{3}{2}Ma = \frac{35Ma}{18}$$

$$a = \frac{9g}{62}$$ A1

Comments: Quite a number of candidates have difficulty forming the equations of motion. As the system is going through acceleration, the tension in the string is not equal to the weight of the box. Also, many students often neglect friction when writing down the equation of motion for the reel, for both its translational as well as its rotational motion.

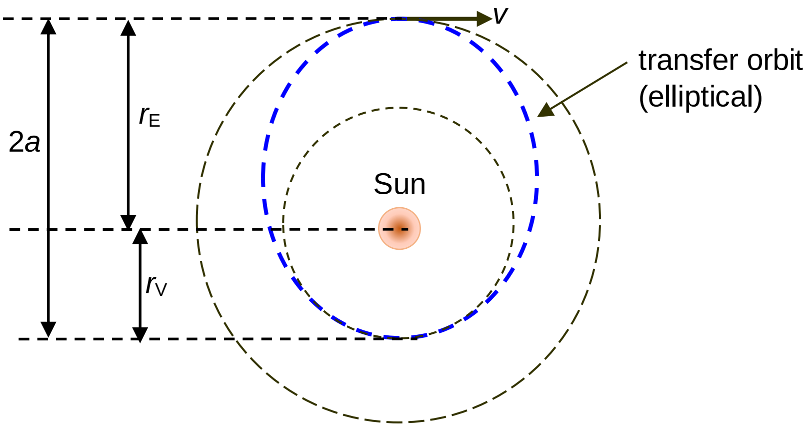
3 (a) Kepler's First Law states the planets move in elliptical orbits with the Sun at one focus of the ellipse. **B1**

Alternative answer: **B1**

Every planet (in the solar system) orbits the Sun in an elliptical orbit, with the Sun at one of the ellipse's foci.

Comments: Kepler's Laws are empirical laws about the planets in the solar system. It was only later found that they follow naturally from Newton's law of universal gravitation – and that other systems, such the moon's of Jupiter and extrasolar planets – follow similar laws. Students who did not get the mark often missed the second part, that the Sun is at one focus of the ellipse. There were also quite a few answers that were written so badly that the meaning of the answer distorted.

(b) (i)



Elliptical orbit **[B1]**

that is completely outside Venus' orbit and completely inside Earth's orbit **[B1]**

Comments: Apparently, it was difficult for students to draw elliptical orbits. As such, it is advisable to annotate the sketch as being elliptical.

(ii) Let v_E be the speed and r_E be the distance from the Sun of the rocket at launch point, and v_V the speed and r_V the distance from the Sun when it reaches Venus' orbit. Then, by the principle of conservation of energy,

$$E = \frac{1}{2}mv_E^2 - \frac{GMm}{r_E} = \frac{1}{2}mv_V^2 - \frac{GMm}{r_V} \quad \textbf{[M1]}$$

Given that, for an elliptical orbit, the total mechanical energy is

$$E = -\frac{GMm}{2a}$$

where a is the semi-major axis of the elliptical orbit.

Combining, we get

$$\frac{1}{2}mv_E^2 - \frac{GMm}{r_E} = -\frac{GMm}{2a} \quad \textbf{[M1]}$$

Rearranging, and using that $2a = r_E + r_V$, **[M1]**

we can solve for v_E,

$$v_E = \sqrt{\frac{2GM}{r_E} - \frac{2GM}{r_E + r_V}} = \sqrt{\frac{2(6.67\times10^{-11})(1.99\times10^{30})}{(1.50\times10^{11})} - \frac{2(6.67\times10^{-11})(1.99\times10^{30})}{(1.50\times10^{11}) + (1.08\times10^{11})}}$$ **[M1]**

$$v_E = 2.72 \times 10^4 \text{ m s}^{-1} = 27.2 \text{ km s}^{-1}$$ (shown) **[A0]**

Alternative answer:

The semi-major axis of the Hohmann transfer orbit would be

$$a = \frac{r_E + r_V}{2} = \frac{1.50\times10^{11} + 1.08\times10^{11}}{2} = 1.29 \times 10^{11} \text{ m}$$ **[M1]**

At the Earth's orbit, the gravitational potential energy is

$$\text{GPE} = -\frac{GMm}{r_E} = -\frac{(6.67\times10^{-11})(1.99\times10^{30})\left(\frac{m}{\text{kg}}\right)}{(1.50\times10^{11})} = -8.85 \times 10^8 \left(\frac{m}{\text{kg}}\right) \text{ J}$$ **[M1]**

The total energy in the Hohmann transfer orbit should be

$$E = -\frac{GMm}{2a} = \frac{(6.67\times10^{-11})(1.99\times10^{30})\left(\frac{m}{\text{kg}}\right)}{2a} = -5.14 \times 10^8 \left(\frac{m}{\text{kg}}\right) \text{ J}$$ **[M1]**

Hence, the kinetic energy upon entering the Hohmann transfer orbit should be

$$\text{KE} = E - \text{GPE} = \left(-5.14 \times 10^8 \left(\frac{m}{\text{kg}}\right)\right) - \left(-8.85 \times 10^8 \left(\frac{m}{\text{kg}}\right)\right) = 3.71 \times 10^8 \left(\frac{m}{\text{kg}}\right) \text{ J}$$ **[M1]**

$$\text{KE} = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2 \times (3.71 \times 10^8)} = 27222 \text{ m s}^{-1} = 27.2 \text{ km s}^{-1}$$ (shown) **[A0]**

Comments: Several students could do the required calculations. However, the presentation left a lot to be desired.

Students should explain what they are doing, not just include random statements. For instance, several of them mentioned the principle of conservation of energy. As this question is only about one pint in the rocket's orbit, the principle of conservation of energy is not relevant.

(iii) By the principle of conservation of angular momentum,

$$mr_E\, v_E = mr_V\, v_V$$ **[M1]**

Solving for v_V,

$$v_V = \frac{r_E}{r_V} v_E = \frac{1.50\times10^{11}}{1.08\times10^{11}} (27.2 \text{ km s}^{-1})$$

$$v_V = 37.8 \text{ km s}^{-1} = 3.78 \times 10^4 \text{ m s}^{-1}$$ **[A1]**

Alternative answer:

At Venus' orbit,

$$\text{GPE} = -\frac{(6.67\times10^{-11})(1.99\times10^{30})\left(\frac{m}{\text{kg}}\right)}{(1.08\times10^{11})} = -1.229 \times 10^9 \left(\frac{m}{\text{kg}}\right) \text{ J}$$ **[M1]**

$$E = -5.14 \times 10^8 \left(\frac{m}{\text{kg}}\right) \text{ J}$$

$$\text{KE} = 7.15 \times 10^8 \,(m/\text{kg}) \text{ J}$$

$$v = 3.78 \times 10^4 \text{ m s}^{-1}$$ **[A1]**

Comments: The principle of conservation of angular momentum should have made this one of the easiest questions in the paper. Still, several students left this part blank.

Part (ii) was a show question. In an exam, "show" questions suggest that the quoted value can be used in subsequent parts.

(iv) By Kepler's Third Law,

$$\frac{GM}{a^3} = \frac{4\pi^2}{T^2}.$$

Solving for the period of the Hohmann orbit (twice the time of flight),

$$T = \sqrt{\frac{4\pi^2 a^3}{GM}} = \sqrt{\frac{4\pi^2\left(\frac{1.50\times10^{11}+1.08\times10^{11}}{2}\right)^3}{(6.67\times10^{-11})(1.99\times10^{30})}} = 2.53\times10^7\ \text{s} \qquad \textbf{[M1]}$$

Hence, the time of flight is

$$\frac{T}{2} = 1.26\times10^7\ \text{s} \qquad \textbf{[A1]}$$

Alternative method, using Kepler's Third Law in solar-system units,

$$T = \left(\frac{\frac{1.50+1.08}{2}}{1.50}\right)^{3/2} = 0.798\ \text{years} \qquad \textbf{[M1]}$$

Hence, the time of flight is

$$\frac{T}{2} = 0.399\ \text{years} = 1.26\times10^7\ \text{s} \qquad \textbf{[A1]}$$

Comments: A surprising number of students apparently used different values for G, M or a, or made rounding errors to arrive at an incorrect answer. It is rather unfortunate to lose marks in this way, even if you know the correct method.

There were also several students who left this question blank, even though they drew a reasonable orbit in part (i). This suggests that they could not recall Kepler's Third Law. As any gravitation question for H3 Physics is quite likely to have a question related to Kepler's Third Law, students are strongly advised to memorise the associated equations.

4 **(a)** *The amplitude and intensity of the microwaves* *<u>from point source P</u>* *alone at point H are A_o and I_o, respectively.* [1]

Amplitude at point H due to source Q = $\frac{10.0}{7.0} A_o$

resultant amplitude at point H = $\frac{10.0}{7.0} A_o + A_o$ = $2.43\ A_o$ [1]

resultant intensity at point H = $(2.43^2)\ I_o$ = $5.90\ I_o$

Comments: This part of qn is void. Question should be "The amplitude and intensity of the microwaves from point source P (not Q) alone at point H are A_o and I_o, respectively."

(b) *The amplitude and intensity of the microwaves* *<u>from point source P</u>* *alone at point H are A_o and I_o, respectively.*

Since the path difference = 2.5 x wavelength, [1]

the distance between the point J and the point source P is 9.8 cm [1]

Amplitude at point J due to source Q = $\frac{10.0}{7.3} A_o$

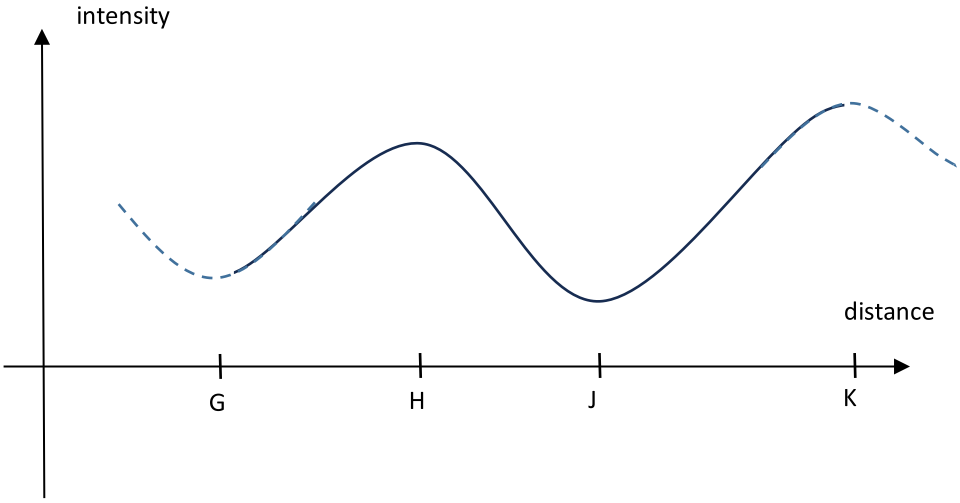
Amplitude at point J due to source P = $\frac{10.0}{9.8} A_o$

resultant amplitude at point J = $\frac{10.0}{9.8} A_o - \frac{10.0}{7.3} A_o$ = $0.349\ A_o$ [1]

resultant intensity at point J = $(0.349^2)\ I_o$ = $0.122\ I_o$ [1]

Comments: Many students can do it correctly. Some mistakenly calculated that the distance between J and P = 9.7 cm.

(c)

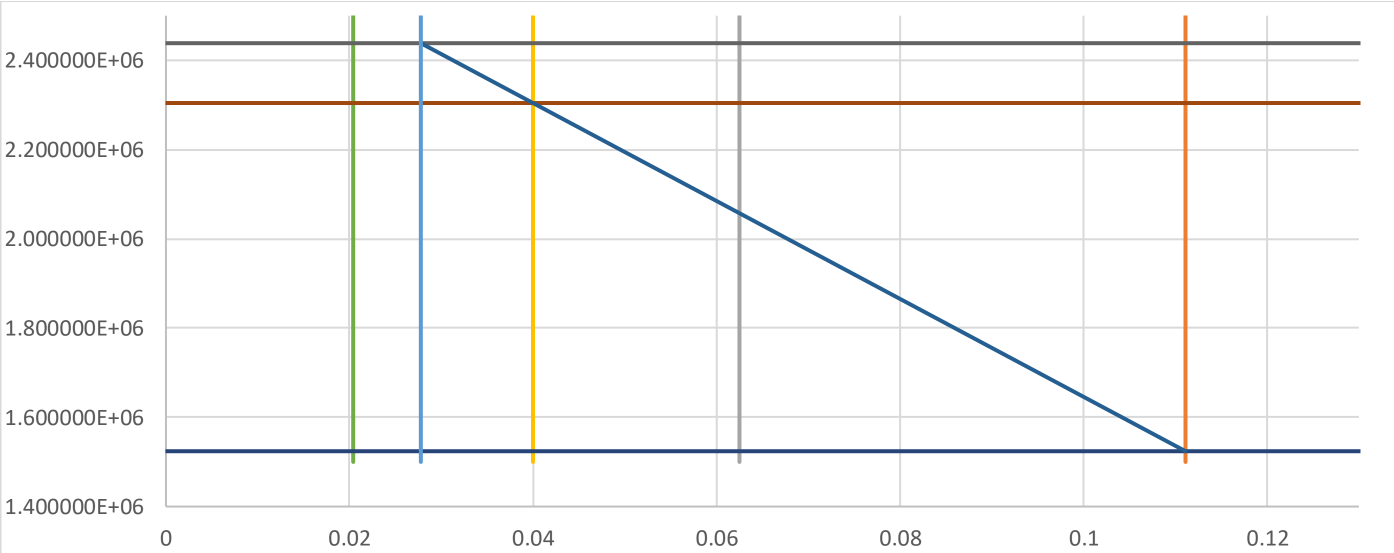


Comments: Some mistakenly drew the troughs and peaks of same heights.

5 **(a)(i) and (ii)** Correct vertical straight lines plotted. [1]

Correct horizontal straight lines plotted. [1]

Comments: Most can do it correctly.



(iii) Correct straight line with a negative gradient is plotted. [1]

Comments: Most can do it correctly.

b(i) Using the gradient of graph, R = 1.097 x 10^7 m^{-1} . [1]

Using the y-intercept of graph, y-int = $\frac{R}{n_1^2}$, we can get $n_1 = 2$. [1]

Comments: Most can do it correctly.

b(ii) From the graph, when $\left(\frac{1}{n_2^2}\right) = 0.0625$, we can read the wavelength, 486 nm. [1]

Comments: Most can do it correctly.

6 **(a)(i)** For both cases, the direction of magnetic field is along +*y*-axis. The mobile charge carriers in Fig. 6.1a are negative charged particles. The electrons must be moving towards -*x* direction because the current is in +*x* direction. By using the direction of current (+*x*) and *B* (+*y*) appropriately for the Fleming's left-hand rule, **the magnetic force on the electrons** would be upwards. B1

In Fig. 6.1b, the mobile charged carriers are positive charged particles. They move in the same direction as the current. By using FLHR, direction of current (+*x*) and *B* (+*y*), **the magnetic force on the positive charged carriers** would also be upwards. B1

Comments: A number of students did not talk about the direction of magnetic field, which was crucial in finding out the direction of magnetic force.

(a)(ii) To deflect the mobile charge carriers perpendicular to the direction of the current so that **the Hall voltage** (or **transverse electric field**) could be generated. This would subsequently help us determine the **sign of the mobile charge carriers** and charge density in the conductor. B1 B1

Comments: Some students treated this question as the velocity selector and talk about the undeflected nature of speeds.

(b)(i) A Voltmeter reads positive if the positive lead is connected to a higher potential than the negative lead. In this case, **the positive lead must have been connected to a lower potential**. The front of the material is at a higher potential and the back of the material is at a lower potential. B1

Comments: The connection points must have been expressed accurately.

(b)(ii) *The voltmeter reading indicates that the front is at a higher potential and back of the bar is at a lower potential.*
By FLHR, the direction of magnetic force on the charge carriers must be towards the back. Since the back of the bar is at a lower potential (more populated with negative charges), hence, the charge carriers must be negative. B1 B1
Comments: Weaker students could not conceptualize the Hall effect well in their explanations.

(b)(iii)
$$E_{\perp} = \frac{\Delta V}{d} = \frac{0.29\times10^{-3}}{0.02} = 1.45\times10^{-2}\ \text{V m}^{-1}$$
M1 A1

Comments: Some did not use the correct expression for the depth.

(b)(iv)
$$F_E = F_B$$
$$eE_{\perp} = eBv_d \rightarrow v_d = \frac{E_{\perp}}{B} = \frac{1.45\times10^{-2}}{1.7} = 8.5\times10^{-3}\ \text{m s}^{-1}$$
M1 A1

Comments: Some used the current relationship but since they had troubles in determining *n*, which is asked in the next part.

(b)(v)
$$I = nAqv_d \rightarrow n = \frac{I}{Aqv_d} = \frac{0.7}{(0.02)(0.04)(1.6\times10^{-19})(8.5\times10^{-3})} = 6.4\times10^{23}\ \text{m}^{-3}$$
M1 A1

Comments: Some used incorrect expression for the cross-sectional area. .

(b)(vi) *d*: depth of the conductor, *n*: number of mobile charge carriers per unit volume, *A*: the cross-sectional area of the conductor (length and height), *q*: elementary charge.

$F_B = F_E$ M1

$Bqv = qE = q\dfrac{\Delta V_{Hall}}{d}$ M1

$Bq\dfrac{I}{nAq} = q\dfrac{\Delta V_{Hall}}{d} \rightarrow \dfrac{\Delta V_{Hall}}{I} = B\dfrac{d}{nAq}$ A0

$R_{Hall} = B\dfrac{d}{nAq}$

Comments: Some ignored the Hall voltage in their workings.

(c)(i) Unit of $\dfrac{h}{e^2}$ is $\dfrac{\text{J.s}}{\text{C}^2} = \dfrac{\text{kg.m}^2\text{.s}^{-2}\text{.s}}{\text{A}^2\text{.s}^2} = \text{kg m}^2\ \text{s}^{-3}\ \text{A}^{-2}$ A1

Unit of ohms is $\Omega = \dfrac{V}{I} = \dfrac{\text{J C}^{-1}}{\text{A}} = \dfrac{\text{kg m}^2\ \text{s}^{-2}\ \text{A}^{-1}\ \text{s}^{-1}}{\text{A}} = \text{kg m}^2\ \text{s}^{-3}\ \text{A}^{-2}$ A1

Comments: A sizeable number of students did not express their answers in SI base units.

(c)(ii) At large magnetic fields, the Hall resistance appears to take discrete values but at smaller magnetic fields, it has a continuous slope or (directly proportional) to *B*. B1 B1

Comments: It was encouraging to see that plenty of students appreciated the discrete nature of Hall resistance at very large magnetic fields.

7ai	E = 0. The electric field inside a conductor at electrostatic equilibrium is zero. (If the electric field were not zero, charges would move until it is)	A1 A1

Comments:
Most were able to state that E = 0.

However, merely stating that a charge of equal magnitude and opposite polarity to the charge of the insulator is induced on the inner surface of the conductor does not guarantee that the electric field inside the entire conductor (the cylindrical shell) is zero. The idea that "enclosed charge = 0" also requires defining a Gaussian surface – credit was not given otherwise.

The electric field strength is not always zero inside a conductor either. For example, the wires in a closed electrical circuit are conductors, but there is a flow of charge because the electric field is non-zero.

The idea of electrostatic equilibrium encapsulates all these considerations: it refers to a conductor in which there is no net motion of charge. The electric field inside a conductor is not zero if the conductor is not in electrostatic equilibrium. (Actually, property of a conductor is enough to explain electrostatic equilibrium. Gauss' Law is used to find the magnitude and polarity of the charges.)

7aii	Draw a cylindrical Gaussian surface of radius $a \leq r \leq b$ and height h centred on the axis of the cylinder. $$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$ $$E(2\pi r h) = \frac{\rho_0(\pi a^2 h)}{\varepsilon_0}$$ $$E = \frac{\rho_0 a^2}{2\varepsilon_0 r}$$	M1: Appropriate Gaussian surface M1: Q_{enc} is $\rho_0 \times$ volume A1

Comments:
Many students did not bother stating or sketching the Gaussian surface they were using. Besides losing a method mark, this usually led to mistakes. Common mistakes were getting the area of the Gaussian surface wrong (e.g. using the area of a circle), or getting the volume of the insulator wrong (e.g. using the volume of a sphere).

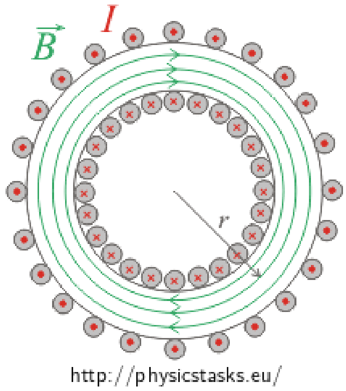
Some students used a spherical Gaussian surface rather than a cylindrical one. Just use the same shape as the distribution given (like in Appendix 2 of the lecture notes) – you'll usually get the right answer.

7aiii	Draw a cylindrical Gaussian surface of radius $b < r < c$ and height *h* centred on the axis of the cylinder (i.e. *inside* the outer cylinder) $$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$ $$E(2\pi rh) = \frac{\sigma(2\pi bh) + \rho_0(\pi a^2 h)}{\varepsilon_0}$$ But $E = 0$ (from ai) $$-2\sigma b = \rho_0 a^2$$ $$\sigma = -\frac{\rho_0 a^2}{2b}$$ OR Draw a cylindrical Gaussian surface of radius $b < r < c$ and height *h* centred on the axis of the cylinder (i.e. *inside* the outer cylinder) $$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$ Since $E = 0$ inside the outer conductor (from ai), $$0 = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow Q_{enc} = 0 \Rightarrow Q_{surface\ of\ outer\ cylinder} = -Q_{inner\ cylinder}$$ $$\therefore \sigma(2\pi bh) = \rho_0(\pi a^2 h)$$ $$\sigma = -\frac{\rho_0 a^2}{2b}$$	M1: Appropriate Gaussian surface M1: Charge on surface & inner cylinder M1: *E* = 0 M1: Appropriate Gaussian surface M1: *E* = 0 M1: Charge on surface & inner cylinder

Comments:

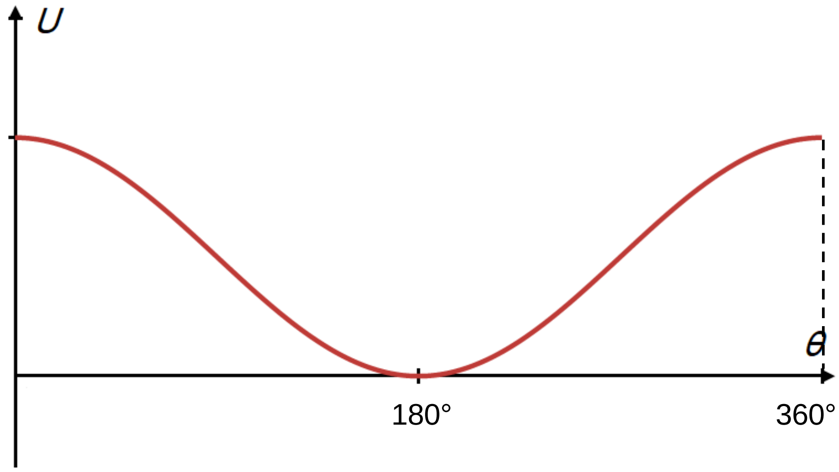
If $r = b$ it can be ambiguous whether the surface charge σ is included in the volume enclosed by the Gaussian surface, hence to avoid ambiguity this solution uses $b < r < c$.

Some students began by assuming that the charge on the inner surface of the conductor is equal in magnitude and opposite in polarity to the charge on the insulator. This must first be shown using Gauss' Law.

7bi	Draw a circular Amperian loop of radius *r* centred on the centre of the toroid, where $a \le r \le b$.  $$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$ $$B(2\pi r) = \mu_0 N I$$ $$B = \frac{\mu_0 N I}{2\pi r}$$	M1 Appropriate Amperian loop M1 I_{enc} A1
Comments: A number of students did not clearly state or sketch what Amperian loop they were using and thus received less credit. Some used a rectangular loop instead (as in the calculation of *B* for a straight solenoid) and managed to get the right answer – note that for this to work, the part parallel to the magnetic field must be very short so that $\vec{B} \parallel \vec{\ell}$, and so $\vec{B} \cdot \vec{\ell} \approx B\ell$. In general, you should use an Amperian loop that is the same shape as the magnetic field (in this case, circular).		
7bii	The current flowing around the circumference of the toroidal solenoid is roughly analogous to <u>current flowing in a circular loop</u>. This magnetic field is strongest at the centre of the loop, where it is given by $B_{out} = \frac{\mu_0 I}{2r}$ Taking the ratio of this with the answer in (b)(i) $\frac{B_{out}}{B} = \left(\frac{\mu_0 I}{2r}\right) \div \left(\frac{\mu_0 N I}{2\pi r}\right) = \frac{\pi}{N} \approx 0$ (N is a large number)	M1 M1
Comments: This was a tough question which saw some valiant attempts – some credit was given where appropriate. Of those who were on the right track, some students misquoted the formula for a loop of current, adding a factor of π in the denominator. (Note that the formula is given in the formula list behind the coverpage.)		
7ci	$$p = 2p_{one\ dipole,\ y} = 2p_{one\ dipole} \cos\left(\frac{104.5°}{2}\right)$$ $$= 2q\ d \cos\left(\frac{104.5°}{2}\right)$$ $$\approx 1.22qd$$	M1 $p = qd$ A1 (optional evaluation)
Comments: Some students forgot to resolve the p and only consider the vertical components (the horizontal components are equal and opposite).		

7cii	$\vec{\tau} = \vec{p} \times \vec{E}$ $\tau = pE\sin\theta$ $= \left(2q\,d\cos\left(\frac{104.5°}{2}\right)\right) E\sin\theta$ $\approx 1.22qEd\sin\theta$	A1 (optional evaluation), e.c.f. allowed

Comments:
Some students simply wrote $\vec{\tau} = \vec{p} \times \vec{E}$. On hindsight, we should have specified that the expressions be in terms of the given variables, so we let this go.

7ciii	$\Delta U = \int_{0°}^{\theta} \tau\,d\theta' = 1.22qEd \int_{0°}^{\theta} \sin\theta'\,d\theta' = 1.22qEd(-\cos\theta + \cos 0°)$ $= 1.22qEd(1 - \cos\theta)$  • While conventionally, U is taken to be 0 at the minimum value (i.e. we pick $U = 0$ when $\Delta U = 0$), this is an arbitrary choice – we are free to shift the graph up or down as we please. • Due to how θ is defined in the question, the lowest potential energy is when the positive charges are furthest to the left (i.e. $\theta = 180°$)	(no calculation required) A2 cosine curve with minimum at $\theta = 180°$ and maximum at $\theta = 0°, 360°$ 1 mark if drew negative cosine 0 mark if sine

Comments:
A number of students drew a sine curve, perhaps not realising that integration was necessary. Some students drew a negative cosine graph, not realising that the minimum potential energy is when the positive end of the dipole (the two hydrogen atoms) is as far to the left as possible (i.e. at 180°).

7iv	Parallel to the electric field / $\theta = 180°$ / when the dipole moment is in the same direction as the electric field. This is a point of stable equilibrium / lowest potential energy. A small displacement will result in a restoring torque back to this position.	B1 (allow e.c.f.) B1

Comments:
Saying that there is zero torque on the dipole at this position is not enough, as there is another point of unstable equilibrium ($\theta = 0°$) where the torque is also zero.

8 **(a)(i)** Due to the rotational motion of the sphere, the angular momentum is given by

$$I\omega = \frac{2}{5}m(2a)^2\omega$$ M1

$$= \frac{8}{5}ma^2\omega$$ A1

(a)(ii) Due to the translational motion of the sphere, the angular momentum about P is given by

$$mv(2a - a) = m(2a)\omega a$$ M1

$$= 2ma^2\omega$$ A1

Comments: The angular momentum of the force is given by *mvr* where *r* is the **perpendicular distance** from the pivot.

(b) Hence total angular momentum is the sum of (a)(i) and (a)(ii), which is $\frac{18}{5}ma^2\omega$ A1

(c)(i) Using parallel axis theorem, its moment of inertia (about P) is now given by

$$I = \frac{2}{5}m(2a)^2 + m(2a)^2$$ M1

$$= \frac{28}{5}ma^2$$ A1

(c)(ii) Angular momentum about P is given by

$$I\omega_1 = \frac{28}{5}ma^2\omega_1$$

Hence by conservation of angular momentum,

$$\frac{18}{5}ma^2\omega = \frac{28}{5}ma^2\omega_1$$ M1

$$\omega_1 = \frac{9}{14}\omega$$ A1

(c)(iii) Before the collision, kinetic energy of the sphere is given by

$$KE_{before} = \frac{1}{2}m(2a\omega)^2 + \frac{1}{2}\left(\frac{8}{5}ma^2\right)\omega^2$$ M1

$$= \frac{14}{5}ma^2\omega^2$$ A1

After the collision, the kinetic energy of the sphere due to its rotation is given by

$$KE_{after} = \frac{1}{2}\left(\frac{28}{5}ma^2\right)\omega_1^2 = \frac{1}{2}\left(\frac{28}{5}ma^2\right)\left(\frac{9}{14}\omega\right)^2 = \frac{81}{70}ma^2\omega^2$$ M1

Hence percentage energy loss

$$= \frac{\left(\frac{14}{5} - \frac{81}{70}\right)}{\left(\frac{14}{5}\right)} \times 100\% = 58.7\%$$ A1

Comments: The kinetic energy of the sphere before the collision is the sum of its kinetic energy due to its translational motion as well as its rotational motion.

(c)(iv) Since there is an increase in gravitational potential energy, as energy is conserved, there must be a corresponding decrease in the kinetic energy of the sphere. B1

As v is proportional to ω, the angular speed will decrease as well. B1

OR

The weight of the sphere generates an anti-clockwise torque which will lead to a reduction in the angular speed with which the sphere rotates about P.

(d) Total mechanical energy of the sphere at the bottom of step must be the same as when it is at the top, hence, M1

$$\frac{1}{2}\left(\frac{28}{5}ma^2\right)\omega_1^2 + mga = \frac{1}{2}\left(\frac{28}{5}ma^2\right)\omega_2^2 + mg2a$$ M1

$$\frac{14}{5}ma^2\omega_1^2 - mga = \frac{14}{5}ma^2\omega_2^2$$ M1

$$\omega_2^2 = \omega_1^2 - \frac{5g}{14a}$$ A0

(e) For the sphere to mount the step, ω_2 must have a real value,
Hence

$$\omega_1^2 - \frac{5g}{14a} > 0$$ M1

$$\left(\frac{9}{14}\omega\right)^2 - \frac{5g}{14a} > 0$$ M1

$$\omega > \frac{1}{9}\sqrt{\frac{70g}{a}}$$ A0

9 Comments: Only a handful of students attempted this question.

(a) (i) When the voltage is zero, the diode will be in reverse bias; i.e. operating like an open switch. B1

(ii) The capacitor which is charged, will discharge continuously through the resistor. B1

Thus, the voltage across resistor which is equal to voltage across the capacitor, will drop continuously. B1

(iii) p.d. across resistor = p.d. across capacitor M1

$i\,R = q/C$

Since $i = -\frac{dq}{dt}$ M1

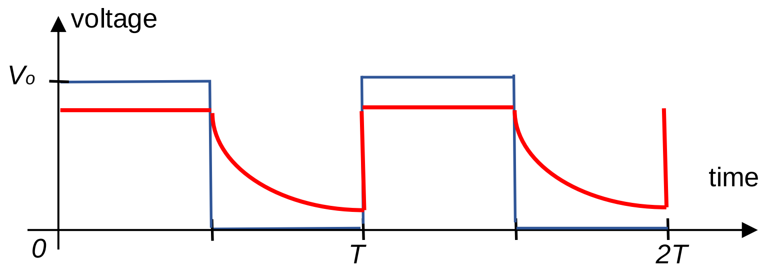
$$q = q_o[e^{-t/RC}]$$

$V = V_o[e^{-t/RC}]$

A1

(iv) Correct positive plateau for *t = 0* to *t = T/2* B1

Correct decreasing and negative gradient for *t = T/2* to *t = T* B1



(b) (i) When the voltage of source drops, the diode will be in forward bias; i.e. operating like a closed switch. B1

The inductor which is energised, will ensure the current flow continuously but with a decreasing magnitude through the diode and resistor. B1

Thus, the voltage across resistor will drop continuously. B1

(ii) p.d. across resistor = p.d. across inductor M1

$i\,R = -L\frac{di}{dt}$ M1

$i = i_o[e^{-tR/L}]$ A1

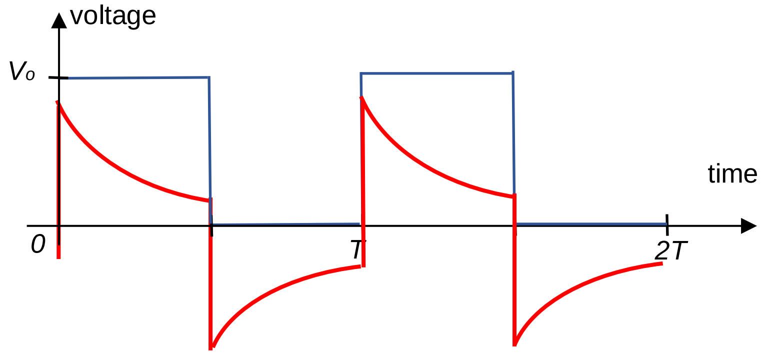
(iii) 1. Correct increasing and negative gradient for *t = 0* to *t = T/2* B1

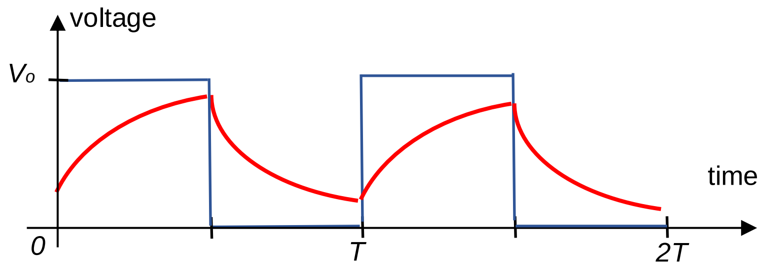
Correct decreasing and positive gradient for *t = T/2* to *t = T* B1

2. Correct decreasing and positive gradient for *t = 0* to *t = T/2* B1

Correct increasing and negative gradient for *t = T/2* to *t = T* B1

Concept : p.d. across voltage source is the sum of voltage across resistor and inductor.





(iv) $i_{min} = i_{max}[e^{-TR/2L}]$

$\ln(0.95) = -TR/2L$ M1

$T = 0.10L/R$ A1

Comments: Some forgot about the factor of 2 or half in above equation.