SWISS COTTAGE SECONDARY SCHOOL
SECONDARY FOUR
WEIGHTED ASSESSMENT 2

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Name: _____ ( ) Class: _____

MATHEMATICS 4052

Wednesday 10 May 2023

45 Minutes

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 30.

Qn	Mark	Concept	Application	Careless	Student Analysis/Remarks
1	/6	❑	❑	❑	
2	/4	❑	❑	❑	
3	/2	❑	❑	❑	
4	/8	❑	❑	❑	
5	/3	❑	❑	❑	
6	/7	❑	❑	❑	
Total Mark	/30	Parent's Signature			

This document consists of **9** printed pages.

Setter: Mr Ngoh Kia Joon, Ms Leung Yan Ru, Mr Ang Hanping

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

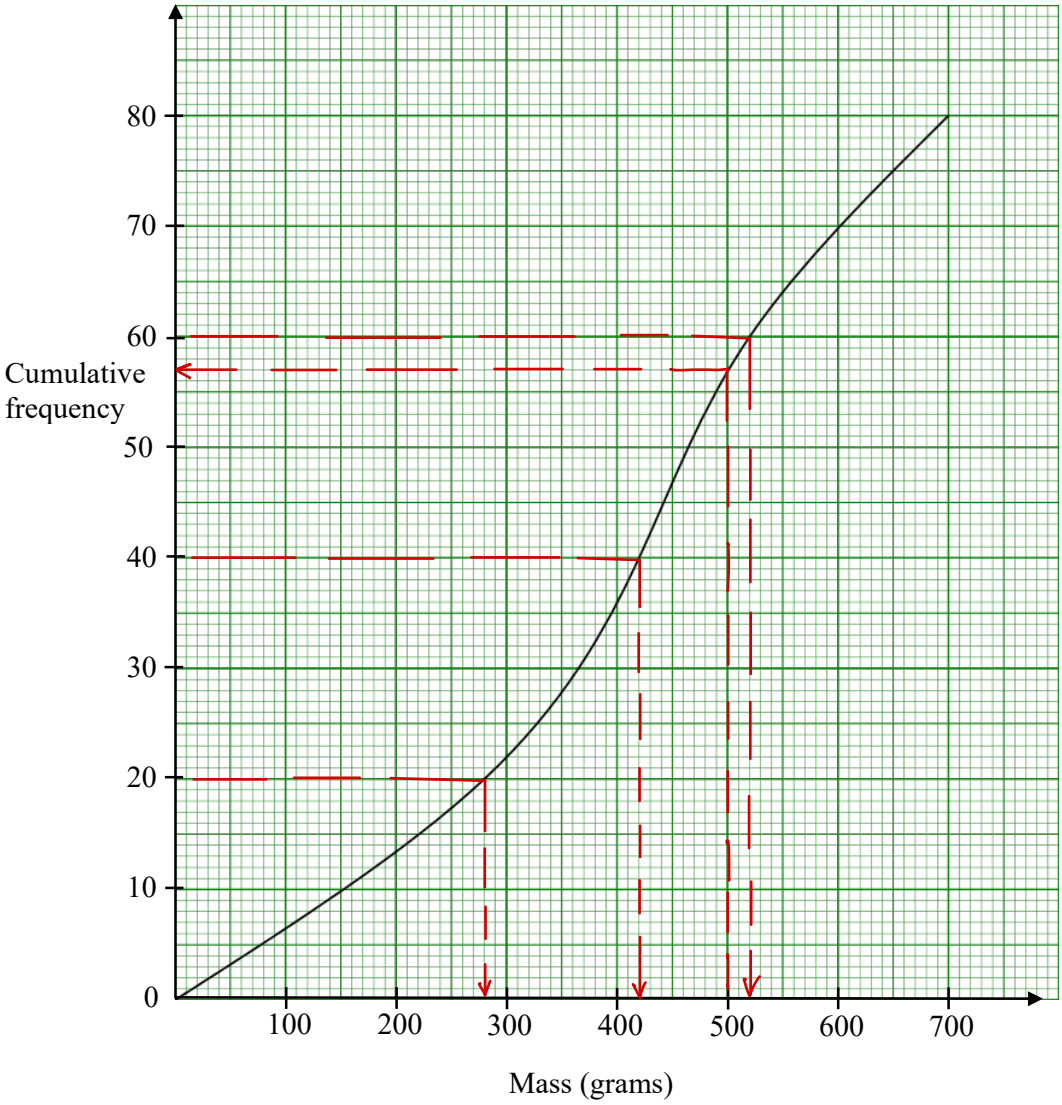
$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 A group of students participated in a competition to estimate the mass of a jar of marbles. The cumulative frequency diagram below shows the estimation by the students.



(a) Use the graph to estimate

(i) median,

$\frac{50}{100} \times 80 = 40$ students

Answer 420 g [1]

(ii) interquartile range for the mass of the jar of marbles.

$\frac{25}{100} \times 80 = 20$ students $\Rightarrow$ LQ = 280

$\frac{75}{100} \times 80 = 60$ students $\Rightarrow$ UQ = 520

$\therefore$ IQR = 520 − 280 = 240

Answer 240 g [2]

(b) Estimate the number of students who answered that the mass of the jar of marbles is at least 500 g. 500 or more

No of students (≤ 500g) = 57
" " " (>500g) = 80 − 57
= 23

Answer 23 [2]

(c) To increase the difficulty of the competition, the organiser announced that a jar with greater mass of marbles will be used next week.

higher median

Describe how the cumulative frequency curve for the estimation by students next week will differ from the current competition.

The curve will be shifted to the right.

[1]

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2 The table below shows the distance, in km, ran by each of the members of a club, in a week.

mid-values	Distance (km)	Number of members
2·5	$0 \leq x < 5$	2
7·5	$5 \leq x < 10$	4
12·5	$10 \leq x < 15$	11
17·5	$15 \leq x < 20$	d
22·5	$20 \leq x < 25$	4
27·5	$25 \leq x < 30$	3

(a) **(i)** Complete the following cumulative frequency table for the data given. [1]

Answer

Distance (km)	Number of members
$x < 5$	2
$x < 10$	6
$x < 15$	17
$x < 20$	23
$x < 25$	27
$x < 30$	30

(ii) Find the value of d.

$d = 23 - 17$
$= 6$

Answer $d =$ 6 [1]

(b) Find an estimate of the

(i) mean of the distance ran,

$\text{mean} = \frac{2\times 2.5 + 4\times 7.5 + \cdots + 3\times 27.5}{2+4+\cdots+3}$

$= \frac{450}{30}$

$= 15$

Answer 15 km [1]

(ii) standard deviation of the distance ran.

$\Sigma fx^2 = 2\times 2.5^2 + 4\times 7.5^2 + \cdots + 3\times 27.5$

$= 8087.5$

$SD = \sqrt{\frac{8087.5}{30} - (15)^2}$

$= 6.6771 = 6.68 \text{ (3sf)}$

Answer 6.68 km [1]

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3 $\mathbf{A} = \begin{pmatrix} 6 & 3.1 \\ 1 & \frac{5}{2} \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 3 \\ 4 & 1.8 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 2 & 3 \\ 4 & \frac{5y}{6} \end{pmatrix}$.

(a) Evaluate $\mathbf{A} + \mathbf{B}$.

$\begin{pmatrix} 6 & 3.1 \\ 1 & 2.5 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 4 & 1.8 \end{pmatrix}$

$= \begin{pmatrix} 8 & 6.1 \\ 5 & 4.3 \end{pmatrix}$

Answer $\begin{pmatrix} 8 & 6.1 \\ 5 & 4.3 \end{pmatrix}$ [1]

(b) It is given that $\mathbf{B} = \mathbf{C}$, find the value of y.

$\begin{pmatrix} 2 & 3 \\ 4 & 1.8 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 4 & \frac{5y}{6} \end{pmatrix}$

$1.8 = \frac{5y}{6}$

$5y = 10.8$

$y = 2.16$

Answer 2.16 [1]

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4 A dance school offers dance sessions for cultural and international dance students on weekdays and weekends.
Each student has a 10-week block of lessons with one session per week.
The matrix **D** shows the number of students attending the dance lessons each week in one 10-week block.

$$\mathbf{D} = \begin{array}{c} \begin{matrix} \text{Cultural} & \text{International} \end{matrix} \\ \begin{pmatrix} 3 & 10 \\ 5 & 8 \end{pmatrix} \end{array} \begin{matrix} \\ \text{Weekday} \\ \text{Weekend} \end{matrix}$$

(a) Evaluate the matric $\mathbf{L} = 10\mathbf{D}$.

$= 10\begin{pmatrix} 3 & 10 \\ 5 & 8 \end{pmatrix}$

$= \begin{pmatrix} 30 & 100 \\ 50 & 80 \end{pmatrix}$

Answer $\begin{pmatrix} 30 & 100 \\ 50 & 80 \end{pmatrix}$ [1]

(b) The dance school charges \$80 for each cultural dance session and \$90 for each international dance session.

Represent the session charges in a 2×1 column matrix **C**.

$C = \begin{pmatrix} 80 \\ 90 \end{pmatrix}$

Answer $\begin{pmatrix} 80 \\ 90 \end{pmatrix}$ [1]

(c) Evaluate the matrix $\mathbf{G} = \mathbf{LC}$.

\$

$= \begin{matrix} WD \\ WE \end{matrix}\begin{pmatrix} 30 & 100 \\ 50 & 80 \end{pmatrix}\begin{pmatrix} 80 \\ 90 \end{pmatrix}$

\$ 2×2 2×1

$= \begin{pmatrix} 11400 \\ 11200 \end{pmatrix}\begin{matrix} WD \\ WE \end{matrix}$

Answer $\begin{pmatrix} 11400 \\ 11200 \end{pmatrix}$ [1]

(d) State what the elements of **G** represents.

Answer It represents the total amount collected by the dance studio for weekday and weekend are \$11400 and \$11200 respectively. [1]

(e) The dance school wants to attract more students, so in the next 10-week block, the dance school reduces the price for cultural and international dance session by 10%.
For this block of sessions, on weekdays, there are 5 cultural dance students and 12 international dance students.
On weekends, there are 8 cultural dance students and 15 international dance students.

By using matrix multiplication, calculate the total amount of money the dance school earns for this 10-week block of sessions.

Total students for 10 week block $= 10\begin{pmatrix} 5 & 12 \\ 8 & 15 \end{pmatrix}$

$= \begin{pmatrix} 50 & 120 \\ 80 & 150 \end{pmatrix}$

charges after discount $= 0.9\begin{pmatrix} 80 \\ 90 \end{pmatrix}$

$= \begin{pmatrix} 72 \\ 81 \end{pmatrix}$

Total charges $= \begin{pmatrix} 50 & 120 \\ 80 & 150 \end{pmatrix}\begin{pmatrix} 72 \\ 81 \end{pmatrix}$

$= \begin{pmatrix} 13320 \\ 17910 \end{pmatrix}$

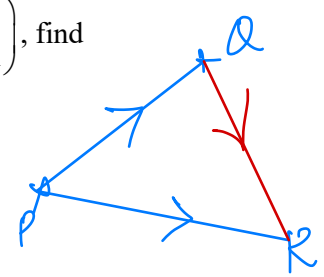
total amount $= \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 13320 \\ 17910 \end{pmatrix}$

$= (31230)$

∴ total amount is \$31230

Answer \$31230 ..... [4]

5 Given that P is the point (1, 1), $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ and $\overrightarrow{PR} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, find



(a) $\overrightarrow{QR}$,

$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

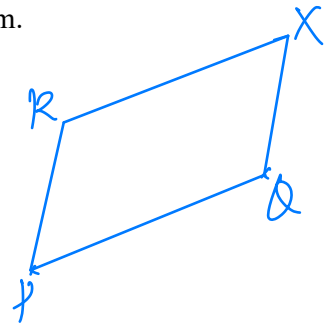
$\overrightarrow{QR} = \overrightarrow{PR} - \overrightarrow{PQ}$

$= \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

Answer $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ [1]

(b) the coordinates of the point X such that $PQXR$ is a parallelogram.



$\overrightarrow{PQ} = \overrightarrow{RX}$

$\begin{pmatrix} -3 \\ 1 \end{pmatrix} = \overrightarrow{OX} - \overrightarrow{OR}$

$\overrightarrow{OX} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\therefore X(0, 1)$

$\overrightarrow{PR} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

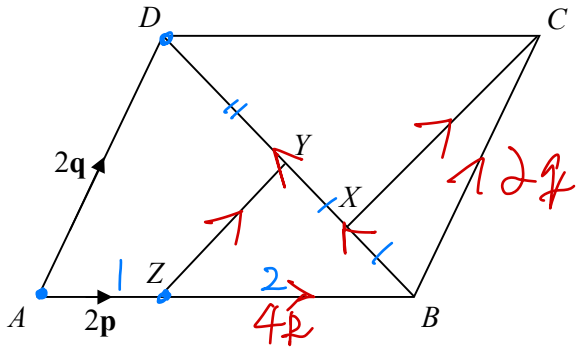
$\overrightarrow{OR} - \overrightarrow{OP} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\overrightarrow{OR} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

Answer $(0, 1)$ [2]

6



In the diagram, $ABCD$ is a parallelogram. Y is the midpoint of BD, X is the midpoint of BY and Z is the point on AB such that $AB = 3AZ$.

(a) Given that $\overrightarrow{AZ} = 2\mathbf{p}$ and $\overrightarrow{AD} = 2\mathbf{q}$, express as simply as possible, in terms of $\mathbf{p}$ and/or $\mathbf{q}$

(i) $\overrightarrow{BD}$,

ΔABD, $\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$

$6\mathbf{p} + \overrightarrow{BD} = 2\mathbf{q}$

$\overrightarrow{BD} = 2\mathbf{q} - 6\mathbf{p}$

Answer $2\mathbf{q} - 6\mathbf{p}$ [1]

(ii) $\overrightarrow{ZY}$,

$\overrightarrow{BY} = \frac{1}{2}\overrightarrow{BD}$

$= \frac{1}{2}(2\mathbf{q} - 6\mathbf{p})$

$= \mathbf{q} - 3\mathbf{p}$

ΔBZY, $\overrightarrow{ZB} + \overrightarrow{BY} = \overrightarrow{ZY}$

$4\mathbf{p} + \mathbf{q} - 3\mathbf{p} = \overrightarrow{ZY}$

$\therefore \overrightarrow{ZY} = \mathbf{p} + \mathbf{q}$

Answer $\mathbf{p} + \mathbf{q}$ [2]

(iii) $\overrightarrow{XC}$,

$\overrightarrow{BX} = \frac{1}{2}\overrightarrow{BY}$

$= \frac{1}{2}(\mathbf{q} - 3\mathbf{p})$

ΔBXC, $\overrightarrow{BX} + \overrightarrow{XC} = \overrightarrow{BC}$

$\overrightarrow{XC} = 2\mathbf{q} - \frac{1}{2}(\mathbf{q} - 3\mathbf{p})$

$\overrightarrow{XC} = \frac{3}{2}\mathbf{q} + \frac{3}{2}\mathbf{p}$

Answer ..... [2]

(b) Using your answers in **(a)(ii)** and **(a)(iii)**, deduce two facts about XC and ZY.

$\overrightarrow{ZY} = \mathbf{p} + \mathbf{q}$, $\overrightarrow{XC} = \frac{3}{2}(\mathbf{p} + \mathbf{q})$

$\therefore \overrightarrow{XC} = \frac{3}{2}\overrightarrow{ZY}$

Answer XC is parallel to ZY [1]

Answer $XC = \frac{3}{2}ZY$ [1]