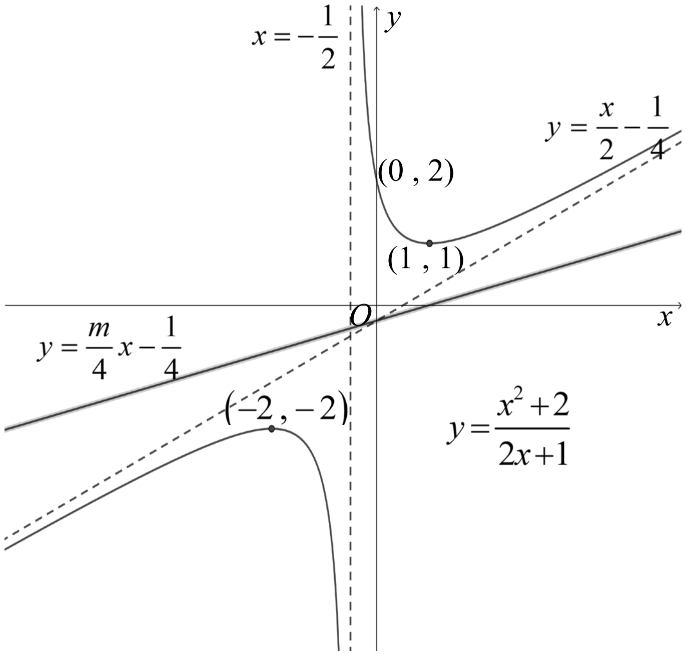
Section A: Pure Mathematics (40 marks)

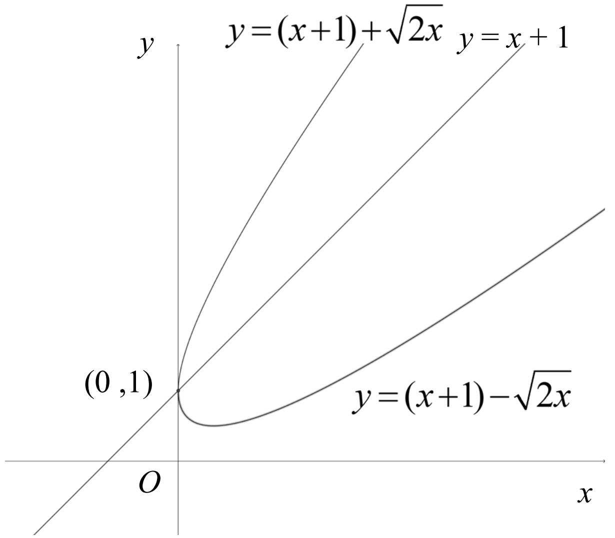
1(i)	$y=\dfrac{x^2+2}{2x+1}=\dfrac{x}{2}-\dfrac{1}{4}+\dfrac{9}{4(2x+1)}$ Vertical Asymptote: $x=-\dfrac{1}{2}$ Oblique asymptote: $y=\dfrac{x}{2}-\dfrac{1}{4}$ Intersection with axes: $x=0, y=2 \rightarrow (0, 2)$ No intersection with x-axis From G.C. Turning points: Min $(1, 1)$, Max $(-2, -2)$ 

(ii)	Substitute $x = 0$ into $y = \frac{x}{2} - \frac{1}{4}$ $y = -\frac{1}{4}$ $P\left(0, -\frac{1}{4}\right)$ For $4y = mx - 1$, $LHS = 4\left(\frac{-1}{4}\right)$ $= -1$ $= RHS$ when $x = 0$ Therefore, P lies on $4y = mx - 1$ for all real values of m.
(iii)	$4x^2 + 8 = (mx-1)(2x+1)$ $\frac{x^2+2}{2x+1} = \frac{m}{4}x - \frac{1}{4}$ Since $0 < m < 2$, $0 < \frac{m}{4} < \frac{1}{2}$, $y = \frac{m}{4}x - \frac{1}{4}$ will not intersect with the graph of $y = \frac{x^2+2}{2x+1}$. Hence, there is no real root for the equation $4x^2 + 8 = (mx-1)(2x+1)$.
(iv)	Since there are no real roots for the equation, $4x^2 + 8 = (mx-1)(2x+1)$, the two roots are complex roots. Since all the coefficients in the equation are real, the roots must occur in complex conjugate pairs, i.e. $x = a + b\text{i}$ or $x = a - b\text{i}$, $a, b \in \mathbb{R}$.

2(a)	Equation of the line : $\mathbf{r}=\mathbf{a}+\lambda\mathbf{u},\ \lambda\in\mathbb{R}$ $\mathbf{r}-\mathbf{a}=\lambda\mathbf{u},\lambda\in\mathbb{R}$ $\Rightarrow \mathbf{r}-\mathbf{a}$ is parallel to $\mathbf{u}$ $(\mathbf{r}-\mathbf{a})\times\mathbf{u}=\mathbf{0}$
(b)	$\lvert\mathbf{b}-\mathbf{a}\rvert^2=(\mathbf{b}-\mathbf{a})\cdot(\mathbf{b}-\mathbf{a})$ $=\mathbf{b}\cdot\mathbf{b}-2\mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{a}$ $\lvert\mathbf{b}-\mathbf{a}\rvert^2=\lvert\mathbf{b}\rvert^2-2\mathbf{a}\cdot\mathbf{b}+\lvert\mathbf{a}\rvert^2$ $\left(\sqrt{7}\lvert\mathbf{a}\rvert\right)^2=\left(\sqrt{3}\lvert\mathbf{a}\rvert\right)^2-2\mathbf{a}\cdot\mathbf{b}+\lvert\mathbf{a}\rvert^2$ $7\lvert\mathbf{a}\rvert^2=4\lvert\mathbf{a}\rvert^2-2\mathbf{a}\cdot\mathbf{b}$ $2\mathbf{a}\cdot\mathbf{b}=-3\lvert\mathbf{a}\rvert^2$ $\mathbf{a}\cdot\mathbf{b}=-\frac{3}{2}\lvert\mathbf{a}\rvert^2\neq 0$ $\Rightarrow \overrightarrow{OA}$ is not perpendicular to $\overrightarrow{OB}$ Hence, $\angle AOB$ is not a right angle. Therefore, points O, A and B are not points that lie on the circumference of a circle with diameter AB.
3(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x-y}{x-y+1}$ --- (1) Let $u=x-y+1$ Differentiating with respect to x: $\frac{\mathrm{d}u}{\mathrm{d}x}=1-\frac{\mathrm{d}y}{\mathrm{d}x}$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=1-\frac{\mathrm{d}u}{\mathrm{d}x}$

	From equation (1): $1-\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{u-1}{u}$ $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{u}$ --- (2) $\int u\,\mathrm{d}u=\int \mathrm{d}x$ $\frac{u^2}{2}=x+A$ $\frac{(x-y+1)^2}{2}=x+A$ $\left(x-y+1\right)^2=2x+2A$ $y=\left(x+1\right)\pm\sqrt{2x+C}$, where C=2A A and C are arbitrary constants.

(ii)



(iii)

When $y > x+1$. $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \dfrac{1}{(x-y+1)^3} < 0.$

The part of the solution graph above the line $y = x + 1$ is concave downwards.

4 (i)

$x^4 + 4 = (x^2+2)^2 - 4x^2$

$\quad\quad = (x^2+2+2x)(x^2+2-2x)$ (Shown)

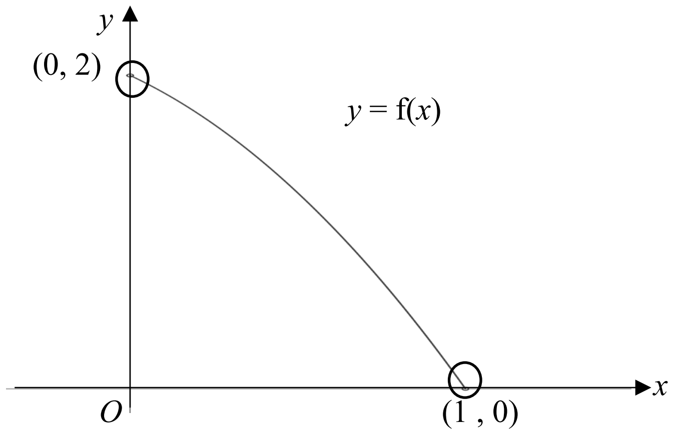
Using partial fractions

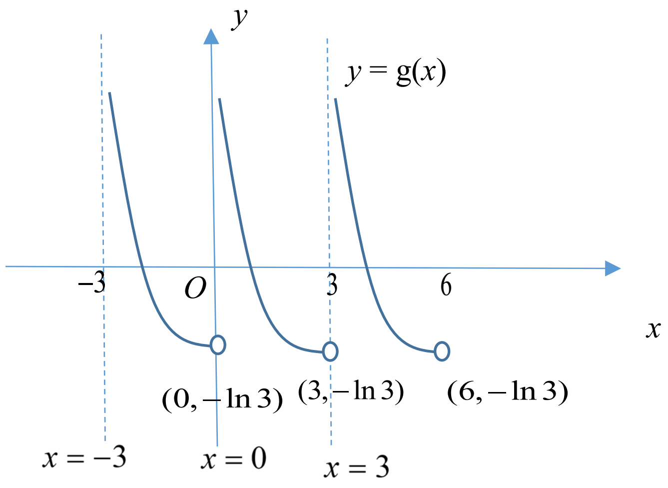
$$\frac{1}{(x^2+2x+2)(x^2-2x+2)} = \frac{ax+b}{x^2+2x+2} + \frac{cx+d}{x^2-2x+2}$$

Multiplying across by $(x^2+2+2x)(x^2+2-2x)$,

	$1=(ax+b)(x^2-2x+2)+(cx+d)(x^2+2x+2)$ Comparing coefficients of x^0, x^3, x^2 and x respectively in sequence, $2b+2d=1---(1)$ $a+c=0---(2)$ $-2a+b+2c+d=0---(3)$ $2a-2b+2c+2d=0---(4)$ Solving using GC, $a=\frac{1}{8}, b=\frac{1}{4}, c=-\frac{1}{8}, d=\frac{1}{4}$. $\frac{1}{x^4+4}=\frac{1}{(x^2+2x+2)(x^2-2x+2)}=\frac{x+2}{8(x^2+2x+2)}-\frac{x-2}{8(x^2-2x+2)}$
(ii)	$\therefore \int_0^1\left(\frac{1}{x^4+4}\right)\mathrm{d}x=\frac{1}{8}\int_0^1\left(\frac{x+2}{x^2+2x+2}\right)-\left(\frac{x-2}{x^2-2x+2}\right)\mathrm{d}x$ $\frac{1}{8}\int_0^1\left(\frac{x+2}{x^2+2x+2}\right)\mathrm{d}x=\frac{1}{8}\left(\frac{1}{2}\int_0^1\frac{2x+2+2}{x^2+2x+2}\right)\mathrm{d}x$ $=\frac{1}{16}\int_0^1\frac{2x+2}{x^2+2x+2}\mathrm{d}x+\frac{1}{16}\int_0^1\frac{2}{(x+1)^2+1}\mathrm{d}x$ $=\frac{1}{16}\left[\ln(x^2+2x+2)\right]_0^1+\frac{1}{8}\left[\tan^{-1}(x+1)\right]_0^1$ $=\frac{1}{16}\left(\ln 5-\ln 2\right)+\frac{1}{8}\tan^{-1}2-\frac{\pi}{32}$

	$\frac{1}{8}\int_0^1\left(\frac{x-2}{x^2-2x+2}\right)\mathrm{d}x=\frac{1}{8}\left(\frac{1}{2}\int_0^1\frac{2x-2-2}{x^2-2x+2}\right)\mathrm{d}x$ $=\frac{1}{16}\int_0^1\frac{2x-2}{x^2-2x+2}\,\mathrm{d}x-\frac{1}{16}\int_0^1\frac{2}{(x-1)^2+1}\,\mathrm{d}x$ $=\frac{1}{16}\left[\ln(x^2-2x+2)\right]_0^1-\frac{1}{8}\left[\tan^{-1}(x-1)\right]_0^1$ $=\frac{1}{16}(-\ln 2)-\frac{\pi}{32}$ $\frac{1}{8}\int_0^1\left(\frac{x+2}{(x+1)^2+1}\right)-\left(\frac{x-2}{x^2-2x+2}\right)\mathrm{d}x$ $=\left(\frac{1}{16}(\ln 5-\ln 2)+\frac{1}{8}\tan^{-1}2-\frac{\pi}{32}\right)-\left(\frac{1}{16}(-\ln 2)-\frac{\pi}{32}\right)$ $=\frac{1}{16}\ln(5)+\frac{1}{8}\tan^{-1}(2)$ (Ans)

5 (i)	Let $y=\left\|x^2+x-2\right\|$  Since $x^2+x-2\le 0$ when $0<x<1$, $y=-\left(x^2+x-2\right)$ $=-\left(x+\frac{1}{2}\right)^2+\frac{9}{4}$ $\left(x+\frac{1}{2}\right)^2=\frac{9}{4}-y$ $x+\frac{1}{2}=\pm\sqrt{\frac{9}{4}-y}$ $x=\pm\sqrt{\frac{9}{4}-y}-\frac{1}{2}$

	$x=\sqrt{\frac{9}{4}-y}-\frac{1}{2}$ since $0<x<1$ $\therefore \mathrm{f}^{-1}(x)=\sqrt{\frac{9}{4}-x}-\frac{1}{2}$
(ii)	 $\mathrm{D}_{\mathrm{f}^{-1}}=\mathrm{R}_{\mathrm{f}} \xrightarrow{\mathrm{f}^{-1}} \mathrm{R}_{\mathrm{f}^{-1}}=\mathrm{D}_{\mathrm{f}} \xrightarrow{\mathrm{g}} \mathrm{R}_{\mathrm{gf}^{-1}}$ $(0,2) \xrightarrow{\mathrm{f}^{-1}} (0,1) \xrightarrow{\mathrm{g}} (0,\infty)$ $\mathrm{R}_{\mathrm{gf}^{-1}}=(0,\infty)$.

(iii)	Area required $= -\int_4^{5.5} [g(x)] \, dx$ $= -\int_1^{2.5} (-\ln x) \, dx$ $= 0.79073$ $= 0.791$ (to 3 sf, by GC)

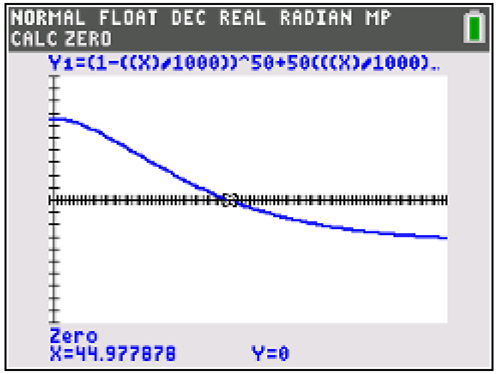
Section B: Statistics (60 marks)

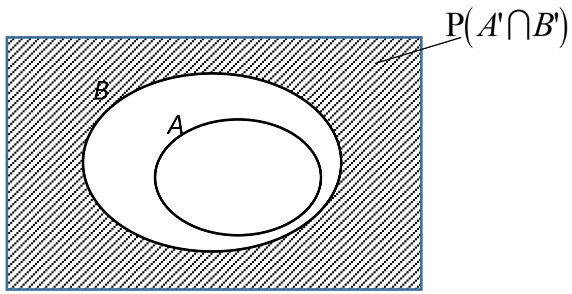
6 (a)	The researchers obtained the *sampling frame* from the 30 000 patients who volunteered for the study. **Step 1**: Number the members of the volunteers, say from 1 to *30,000.* **Step 2**: *Randomly* select 500 of these members by generating *500 distinct* random numbers and then selecting the *corresponding* members to form the required sample
(b)	Random selection means that the subjects in the sample were assigned to a treatment at random, without bias. Or The selection of patients to assign treatment to is free of bias.
7(i)	1. The condition of one semi-conductor chip is independent of the condition of any other semi-conductor chips.

	2. The probability that a semi-conductor chip is faulty is constant at $\dfrac{p}{1000}$ for every semi-conductor chip.
(ii)	Let X be the random variable denoting the number of faulty semiconductor chips out of 50 chips in a box. $X \sim \mathrm{B}\left(50, \dfrac{p}{1000}\right)$ $\mathrm{P}(X \le 1) = 0.336$ $\mathrm{P}(X = 0 \text{ or } 1) = 0.336$ $\mathrm{P}(X = 0) + \mathrm{P}(X = 1) = 0.336$ $\left(1-\dfrac{p}{1000}\right)^{50} + {}^{50}C_1\left(\dfrac{p}{1000}\right)\left(1-\dfrac{p}{1000}\right)^{49} = 0.336$ $\left(1-\dfrac{p}{1000}\right)^{50} + 50\left(\dfrac{p}{1000}\right)\left(1-\dfrac{p}{1000}\right)^{49} = 0.336$ Using G.C table.

p	$\left(1-\dfrac{p}{1000}\right)^{50} + 50\left(\dfrac{p}{1000}\right)\left(1-\dfrac{p}{1000}\right)^{49}$
44	0.348
45	0.3357≈0.336
46	0.3238

$p = 45$

	Using GC Graph: consider $\left(1-\frac{p}{1000}\right)^{50}+50\left(\frac{p}{1000}\right)\left(1-\frac{p}{1000}\right)^{49}-0.336=0$  $\frac{p}{1000}=0.044978$ $p=44.978\approx 45$ (nearest integer)
(iii)	Required probability $=\mathrm{P}(X_1\le 3)\mathrm{P}(X_2=5)+\mathrm{P}(X_1=5)\mathrm{P}(X_2\le 3)$ $=2\times\left[\mathrm{P}(X\le 3)\right]\left[\mathrm{P}(X=5)\right]$ $=0.080057$ $=0.0801$ (to 3 sf)
8 (i)	For the greatest possible value of $\mathrm{P}\left(A'\cap B'\right)$, $A\subseteq B$.

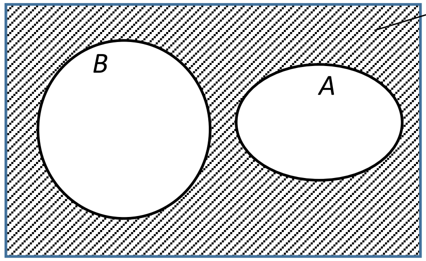


This means $\mathrm{P}(A \cap B) = \mathrm{P}(A)$

The largest possible value of

$$\begin{aligned}
&\mathrm{P}(A' \cap B') \\
&= 1 - \mathrm{P}(B) \\
&= 1 - \frac{1}{3} = \frac{2}{3}
\end{aligned}$$

For the least value of $\mathrm{P}(A' \cap B')$, A and B are mutually exclusive events.

	 $P(A' \cap B')$ Hence, Least possible value of $P(A' \cap B')$ $= 1 - P(A \cup B)$ $= 1 - [P(A) + P(B)]$ $= 1 - \left[\frac{1}{6} + \frac{1}{3}\right]$ $= 1 - \frac{1}{2} = \frac{1}{2}$

(ii)	$\mathrm{P}\left[(A\cup B)'\cap C\right]$ $=\mathrm{P}\left[A'\cap B'\cap C\right]$ $=\mathrm{P}(C)-\mathrm{P}\left[(A\cup B)\cap C\right]$ $=\dfrac{4}{9}-\dfrac{7}{36}$ $=\dfrac{1}{4}$ $\mathrm{P}(A\cup B)'\times\mathrm{P}(C)$ $=\mathrm{P}(A'\cap B')\mathrm{P}(C)$ $=\mathrm{P}(A')\times\mathrm{P}(B')\times\mathrm{P}(C)$, since A and B are independent events $=\left(1-\dfrac{1}{6}\right)\times\left(1-\dfrac{1}{3}\right)\times\dfrac{4}{9}$ $=\dfrac{20}{81}\neq\dfrac{1}{4}$ *Alternatively*

	$\mathrm{P}\left[(A\cup B)'\right]\times P(C)$ $=\{1-P(A\cup B)\}\times P(C)$ $=\{1-[P(A)+P(B)-P(A)P(B)]\}\times P(C)$ $=\left\{1-\left[\frac{1}{6}+\frac{1}{3}-\left(\frac{1}{6}\right)\left(\frac{1}{3}\right)\right]\right\}\times\frac{4}{9}$ $=\frac{20}{81}\neq\frac{1}{4}=\mathrm{P}\left[(A\cup B)'\cap C\right]=\mathrm{P}[A'\cap B'\cap C]$ Hence $(A\cup B)'$ and C are not independent.
9 (i)	Angle of sector 5 = 36^0. Let d be the common difference. AP: $36°,\ (36+d)°,\ (36+2d)°,\ (36+3d)°,\ (36+4d)°$ Since the sum of angles for a circle is $360°$, $\frac{5}{2}(2(36°)+(5-1)d)=360°$ $d=18°$ Hence the angles of the 5 sectors are Sector 5 : 36^0

Sector 4: 54^0

Sector 3: 72^0

Sector 2: 90^0

Sector 1: 108^0.

(ii) Let X be the random variable denoting the number obtained in a spin, where $x = 1, 2, 3, 4$ or 5.

Probability distribution of X

x	1	2	3	4	5
$\mathrm{P}(X = x)$	$\frac{108}{360} = \frac{3}{10}$	$\frac{90}{360} = \frac{1}{4}$	$\frac{72}{360} = \frac{1}{5}$	$\frac{54}{360} = \frac{3}{20}$	$\frac{36}{360} = \frac{1}{10}$

(iii) $C = \left|X_1 - X_2\right|$:

X_1 \ X_2	1	2	3	4	5
1	0	1	2	3	4
2	1	0	1	2	3
3	2	1	0	1	2
4	3	2	1	0	1
5	4	3	2	1	0

$\mathrm{P}(C = 0)$

=P[(x_1, x_2)]= P[{(1,1), (2,2), (3,3), (4,4), (5,5)}]

$$= \frac{3}{10} \times \frac{3}{10} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{5} + \frac{3}{20} \times \frac{3}{20} + \frac{1}{10} \times \frac{1}{10}$$

$$= 0.225$$

P($C = 1$)

=P[(x_1, x_2)]=P[{(1,2), (2,1), (2,3), (3,2), (4,3), (3,4), (4,5), (5,4)}]

$$= \frac{3}{10} \times \frac{1}{4} \times 2 + \frac{1}{4} \times \frac{1}{5} \times 2 + \frac{1}{5} \times \frac{3}{20} \times 2 + \frac{3}{20} \times \frac{1}{10} \times 2$$

$$= 0.34$$

P(C= 2)

=P (x_1, x_2) =P[{(1,3), (3,1), (2,4), (4,2), (3,5), (5,4)}]

$$= \frac{3}{10} \times \frac{1}{5} \times 2 + \frac{1}{4} \times \frac{3}{20} \times 2 + \frac{1}{5} \times \frac{1}{10} \times 2$$

$$= 0.235$$

P(C= 3)

=P (x_1, x_2) =P[{(1,4), (4,1), (2,5), (5,2)}]

$$= \frac{3}{10} \times \frac{3}{20} \times 2 + \frac{1}{4} \times \frac{1}{10} \times 2$$

$$= 0.14$$

P(C= 4)

=P (x_1, x_2) =P[{(1,5), (5,1)}]

$$= \frac{3}{10} \times \frac{1}{10} \times 2$$

$$= 0.06$$

c	0	1	2	3	4
$P(C=c)$	0.225	0.34	0.235	0.14	0.06

$\therefore$ Expectation of cash received by Shayna = $E(C)$

$$= \sum_{\text{all } c} cP(C=c)$$

$$= 0\times 0.225 + 1\times 0.34 + 2\times 0.235 + 3\times 0.14 + 4\times 0.06$$

= 1.47

Variance of cash received by Shayna = $Var(C)$

$$= E(C^2) - \left(E(C)\right)^2$$

$$= \sum_{\text{all } c} c^2 P(C=c) - \left(E(C)\right)^2$$

$$= \left(0^2\times 0.225 + 1^2\times 0.34 + 2^2\times 0.235 + 3^2\times 0.14 + 4^2\times 0.06\right) - (1.47)^2$$

= 1.3391 (exact) = 1.34 (2 d.p.)

Last part

Shayna plays 30 games.

Let the total cash Shayna received, $T = C_1 + C_2 + \ldots + C_{30}$

Since sample size n = 30 is sufficiently large, by Central Limit Theorem, $T \sim N(1.47\times 30, 1.3391\times 30)$ approximately.

$T \sim N(44.1, 40.173)$

$P(45 < T < 110) = 0.444$ (3 sf) (using GC).

10 (i)

No. of ways in which C and D sit together = $5!\times 2!$

No. of ways in which A and B sit together and C and D also sit together = $4!\times 2!\times 2!$

$$\underbrace{\overset{2!}{\boxed{AB}}\ \overset{2!}{\boxed{CD}}\ E\ \ F}_{4!}$$

	No. of ways C and D sit next to each other and A and B do not sit next to each other $=5!\times 2!-4!\times 2!\times 2!=144$ <u>Alternative method</u> No. of ways to sit C, D , E and F such that C and D are considered as 1 unit $= 3!\times 2!$ $\uparrow \boxed{CD} \uparrow E \uparrow F \uparrow$ No. of ways to slot in A and B = ${}^4C_2\times 2!$ Total numbers of ways required $={}^4C_2\times 2!\times 3!\times 2!$ =144
(ii)	Total no. of possible ways without restriction =6!=720 Consider if students A, B and C are seated together by grouping them as 1 ‘unit’ No. of ways = $3!\times 4!=144$ No. of ways required such that A, B and C are not all seated together =720 – 144 = 576 <u>Alternative method</u> Case 1 : A, B and C are separated No. of ways to arrange D, E, F = 3! No. of ways to slot in and arrange A, B, C such that they are separated = ${}^4C_3\times 3!$ No. of ways for Case 1 = $3!\times {}^4C_3\times 3!$

	Case 2 : Two of A, B, C are together but separated from the third one (e.g. A and B together but separated from C) No. of ways to choose two of A, B, C to arrange them to be together = ${}^{3}C_{2} \times 2!$ No. of ways to arrange D, E, F = 3! No. of ways to slot in and arrange the unit of 2 students together(e.g. 'AB') and the third student(e.g. C) = ${}^{4}C_{2} \times 2!$ No. of ways for Case 2 = $\left({}^{3}C_{2} \times 2!\right) \times \left({}^{4}C_{2} \times 2!\right) \times 3!$ Total no. of required ways = $3! \times \left({}^{4}C_{3} \times 3!\right) + \left({}^{3}C_{2} \times 2!\right) \times \left({}^{4}C_{2} \times 2!\right) \times 3! = 576$
(iii)	There are 2 cases for the two pairs, A and B, and ,C and D ,to be seated at the two tables: Case 1: A and B seated at the 6-seater and C and D at the 5-seater No. of ways to arrange the people at the 6-seater $= {}^{7}C_{4} \times (6-1)! = {}^{7}C_{4} \times 5!$ No. of ways to arrange the people at the 5-seater $= {}^{3}C_{3} \times (5-1)! = 4!$ No. of ways = ${}^{7}C_{4} \times 5! \times 4!$ Case 2: C and D seated at the 6-seater and A and B at the 5-seater No. of ways to arrange the people at the 6-seater $= {}^{7}C_{4} \times (6-1)! = {}^{7}C_{4} \times 5!$ No. of ways to arrange the people at the 5-seater $= {}^{3}C_{3} \times (5-1)! = 4!$ No. of ways = ${}^{7}C_{4} \times 5! \times 4!$

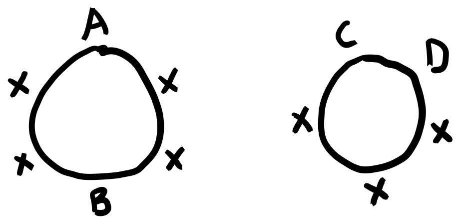
Total no. of ways = $^7C_4 \times 5! \times 4! \times 2 = 201600$

Alternatively

No. of ways in which A and B are seated at the same table and C and D are seated at the other table

$= \left(^7C_4 \times 5! \times {^3C_3} \times 4!\right) \times 2 = 201600$

For A and B to be opposite each other at the same table, A and B must be seated at the 6-seater only.



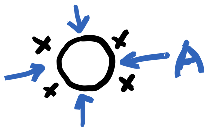
To arrange A and B at the 6-seater, we first fix A and B opposite each other, this can be done in 1 way only. Then, we arrange choose 4 others to sit at the table in 4! ways.

To arrange C, D and the 3 others left at the 5 seater, no. of ways = $^3C_3 \times (5-1)!$

No. of ways in which A and B are opposite each other at the 6-seater and C and D are seated at the 5-seater

$= \left(^7C_4 \times 4!\right) \times \left(^3C_3 \times (5-1)!\right) = 20160$

Alternatively, we can first choose 4 others to be at the 6 seater and arrange them in 3! ways since it is a circular permutation. Then we slot in A in 4 ways and B in 1 way(since B can only be opposite A):



	No. of ways in which A and B are opposite each other at the 6-seater and C and D are seated at the 5-seater $= \left({}^7C_4 \times (3-1)!\right) \times 4 \times \left({}^3C_3 \times 4!\right) = 20160$ P(A and B are opposite each other |A and B are seated at the same table and C and D are seated at other table) $= \dfrac{\mathrm{P}\begin{pmatrix}\text{A and B are opposite each other on 6-seater} \\ \text{and C and D are seated on 5-seater}\end{pmatrix}}{\mathrm{P}\left(\text{A and B are on different tables from C and D}\right)}$ $= \dfrac{\dfrac{\left({}^7C_4 \times 4!\right) \times \left({}^3C_3 \times 4!\right)}{\left({}^{11}C_6 \times 5! \times {}^5C_5 \times 4!\right)}}{\dfrac{\left({}^7C_4 \times 5! \times 4!\right) + \left({}^7C_3 \times 4! \times 5!\right)}{\left({}^{11}C_6 \times 5! \times {}^5C_5 \times 4!\right)}}$ $= \dfrac{20160}{201600}$ $= \dfrac{1}{10}$

11(i)	Let F be the random variable denoting the mass of a packet of fettucine in grams. $F \sim \mathrm{N}\left(450, \sigma^2\right)$ Given that	

	$\mathrm{P}\left(F \leq 437\right) = 0.1$ $\mathrm{P}\left(\frac{F-450}{\sigma} \leq \frac{437-450}{\sigma}\right) = 0.1$ $\mathrm{P}\left(Z \leq \frac{-13}{\sigma}\right) = 0.1$, where $Z \sim \mathrm{N}(0,1)$ $-\frac{13}{\sigma} = -1.2816$ $\sigma = 10.144 = 10.1$ (3 sig fig)	

(ii)	Let S be the random variable denoting the mass of a packet of spaghetti produced in g. $S \sim \mathrm{N}(500\,, 9.2^2)$ $F \sim \mathrm{N}\left(450\,, 10.144^2\right)$ Required probability $=\mathrm{P}\left(\left\|S_1+S_2+S_3+S_4+S_5+S_6-2\left(F_1+F_2\right)\right\| \geq 1250\right)$ Let $A=S_1+S_2+S_3+S_4+S_5+S_6-2\left(F_1+F_2\right)$ $\mathrm{E}(A)$ $=\mathrm{E}\left[S_1+S_2+S_3+S_4+S_5+S_6-2\left(F_1+F_2\right)\right]$ $=\mathrm{E}\left(S_1+S_2+S_3+S_4+S_5+S_6\right)-2\mathrm{E}\left(F_1+F_2\right)$ $=6\mathrm{E}\left(S\right)-2(2)\mathrm{E}\left(F\right)$ $=6\left(500\right)-4(450)$ $=1200$	

	Var(A) $= \text{Var}\left[S_1+S_2+S_3+S_4+S_5+S_6-2\left(F_1+F_2\right)\right]$ $= \text{Var}\left(S_1+S_2+S_3+S_4+S_5+S_6\right)+4\text{Var}\left(F_1+F_2\right)$ $=6\text{Var}\left(S\right)+4(2)\text{Var}\left(F\right)$ $=6\left(9.2^2\right)+8(10.144^2)$ $=1331.045$ $A \sim \text{N}(1200\ ,\ 1331.045)$ $\text{P}\left(\lvert A\rvert \geq 1250\right)$ $=1-\text{P}\left(-1250<A<1250\right)$ $=0.085268$ $=0.0853$ (to 3 sf)	

(iii)	The mass of each packet of pasta (for both spaghetti and fettuccine) must be independent of each other. [Reject if did not consider intermingling or within the pasta group]	

(iv)	P(495 < S < 505) = 0.41320	

	Let X be the random variable denoting the number of packets of spaghetti out of n purchased that has a mass between 495 g and 505 g each. $X \sim \mathrm{B}(n\ ,\ 0.41320)$ Given $\mathrm{P}(X > 7) \geq 0.85 \Rightarrow \mathrm{P}(X \geq 8) \geq 0.85$ $1 - \mathrm{P}(X \leq 7) \geq 0.85$ $\mathrm{P}(X \leq 7) \leq 0.15$ Using GC <table><tr><td>n</td><td>$\mathrm{P}(X \leq 7)$</td></tr><tr><td>24</td><td>0.1582>0.15</td></tr><tr><td>25</td><td>0.1241<0.15</td></tr><tr><td>26</td><td>0.0963<0.15</td></tr></table> Hence, the least value of n is 25.	

(v)	From (iv), $X \sim \mathrm{B}(25\ ,\ 0.41320)$ $\mathrm{E}(X) = np$ $= 25(0.41320)$ $= 10.33$ $\mathrm{Var}(X) = np(1-p)$ $= 25(0.41320)(1-0.41320)$ $= 6.0616$	

	Since the sample size = 24 is sufficiently large, by Central Limit Theorem, $\overline{X} \sim N\left(10.33\,,\dfrac{6.0616}{24}\right)$ approximately $\mathrm{P}(\overline{X} \le 11) = 0.90876 = 0.909$ (to 3 sf)	

12(i)	No, this sample is not a random sample because <u>employees who do not travel by bus do not have a chance of being chosen as part of the sample</u>, hence not all employees have an equal probability of being selected.	

(ii)	Let T be the random variable denoting the number of hours in a week an employee works and μ be the population mean hours. Let $y = t - 30$ $t = y + 30$	

	Unbiased estimate of population mean, $\bar{t}$ $=\dfrac{\sum y}{n}+30$ $=\dfrac{\sum(t-30)}{n}+30$ $=\dfrac{944}{80}+30$ $=41.8$ (exact) Unbiased estimate of population variance, s^2 $=\dfrac{1}{n-1}\left[\sum y^2-\dfrac{\left(\sum y\right)^2}{n}\right]$ $=\dfrac{1}{n-1}\left[\sum(t-30)^2-\dfrac{\left(\sum(t-30)\right)^2}{n}\right]$ $=\dfrac{1}{79}\left[11825-\dfrac{(944)^2}{80}\right]$ $=\dfrac{3429}{395}$ or 8.6810 $=8.68$ (to 3 sf)	

(iii)	Test $H_0: \mu=42.5$ against $H_1: \mu\neq 42.5$ at 5% level of significance.	

	Under H_0, since sample size 80 is sufficiently large, by Central Limit Theorem, $\overline{T} \sim \mathrm{N}\left(42.5\,,\dfrac{8.6810}{80}\right)$ approximately Using a 2-tailed z-test, Test statistic value, $\bar{t} = 41.8$ gives rise to $z_{calc} = -2.1250$ and $p-\text{value} = 0.0336 \le 0.05$ We reject H_0 and conclude that at the 5% level of significance, there is sufficient evidence that the mean number of hours in a week each employee works is not 42.5 hours.	

(iv)	5% level of significance is the probability of 0.05 of concluding that the mean number of hours in a week an employee works is not 42.5 hours when the mean number of hours in a week an employee works is actually 42.5 hours.	

(v)	Unbiased estimate of population variance, s^2	

	$= \frac{n}{n-1}\left[\text{sample variance}\right]$ $= \frac{50}{49}\left(k^2\right)$ Test $H_0: \mu = 42.5$ against $H_1: \mu < 42.5$ at 5% level of significance. Under H_0, since sample size 50 is sufficiently large, by Central Limit Theorem, $\overline{T} \sim N\left(42.5, \frac{\frac{50}{49}k^2}{50}\right)$ approximately $\Rightarrow \overline{T} \sim N\left(42.5, \frac{k^2}{49}\right)$ approximately Using a 1-tailed z-test, To reject H_0, $z_{calc} \leq -1.6449$ $\Rightarrow z_{calc} = \frac{41.2 - 42.5}{\sqrt{\frac{k}{49}}} \leq -1.6449, \quad Z \sim N(0,1)$ Since we do not reject H_0,	

	$\dfrac{41.2-42.5}{\sqrt{\dfrac{k^2}{49}}} > -1.6449$ $\sqrt{\dfrac{k^2}{49}} > 0.79032$ $\dfrac{k}{7} > 0.79032$ $k > 5.53224$ $k > 5.53$ (to 3 sf) $\left\{k \in \mathbb{R} : k > 5.53\right\}$	