

### ST JOSEPH'S INSTITUTION PRELIMINARY EXAMINATION 2024 (YEAR 4)

	6		4052/04
CLASS		INDEX NUMBER	
CANDIDATE NAME			

## MATHEMATICS

### 4052/01

Paper 1

12 August 2024

Candidates answer on the Question Paper.

2 hours 15 minutes (09:50 – 12:05)

## READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in. Write in dark blue or black pen on both sides of the paper. You may use an HB pencil for any diagrams or graphs. Do not use paper clips, glue or correction fluid.

Answer all questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is 90.

### Mathematical Formulae

Compound Interest

Total amount = 
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = 
$$\pi rl$$
  
Surface area of a sphere =  $4\pi r^2$   
Volume of a cone =  $\frac{1}{3}\pi r^2 h$   
Volume of a sphere =  $\frac{4}{3}\pi r^3$   
Area of triangle  $ABC = \frac{1}{2}ab\sin C$   
Arc length =  $r\theta$ , where  $\theta$  is in radians  
Sector area =  $\frac{1}{2}r^2\theta$ , where  $\theta$  is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

**Statistics** 

Mean = 
$$\frac{\sum fx}{\sum f}$$
  
Standard deviation =  $\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$ 

1 Solve the inequality 
$$\frac{23-6x}{3} \le \frac{2+x}{6}$$
.

*Answer* ..... [2]

2 (a) Write 0.0001087 in standard form.

Answer ..... [1]

**(b)** 

City	Population	Land area, km <sup>2</sup>
Bangkok	$1.07 \times 10^{7}$	7700.0
Ho Chi Minh City	$2.12 \times 10^{7}$	30 595.0
Manila	$1.35 \times 10^{7}$	24 317.7
Kuala Lumpur	$8.46 \times 10^{6}$	8347.1

The table shows the population in some cities in ASEAN and their land areas. Write down the city in the table that has the smallest population per  $km^2$ .

Answer ..... [1]

3 Vikas takes a study loan of \$12 000 that charges an interest rate of *r* % per year compound interest for 4 years.
The total amount that he would have paid for the loan at the end of 4 years is \$12 688.
Find the value of *r*.

4



**(b)** Factorise  $4x^2 + 4 - y^2 - x^2y^2$  completely.

Answer ..... [3]

5 *m* and *n* are positive integers, where  $m \neq n$ .

Explain why  $(m+n)^2 - (m-n)^2$  is not a prime number for all positive integer values of *m* and *n*.

[2]

6 The marked price of a smart watch at a shop is 25% above the cost price. The shop gives a p% discount during a sale.

Calculate the value of p such that the shop makes neither a loss nor a profit from the sale of the smart watch.

*Answer* p = ..... [2]



7 The graph shows the average cost for a dozen of Grade A eggs in the year 2022.

(https://www.dailymail.co.uk/news/article-11671431/Egg-prices-soar-60-2022-group-blaming-collusive-scheme-producers.html)

State one misleading feature of the graph and explain how this feature affects the reader's interpretation of the cost for a dozen of Grade A eggs.

.....[2]

Source: News Daily Mail UK

- 9 The Mandai Wildlife Reserve conserves and promotes South-East Asian biodiversity.
  - (a) The Mandai Wildlife Bridge is 140 m long. It is represented by a length of 3.5 cm on a map. Express the scale of the map in the form 1 : *r*.

*Answer* ..... [1]

 (b) The Bird Paradise covers a land area of 17 hectares.
 Find, in cm<sup>2</sup>, the area representing the Bird Paradise on the same map. (1 hectare = 10000 m<sup>2</sup>)

*Answer* ......cm<sup>2</sup> [2]

**[TURN OVER** 

The sketch shows the graph of  $y = ka^{-x}$ . 10 ) and Q P(-2, 96)  $Q(0, \frac{3}{2})$  OThe points P(-2, 96) and  $Q(0, \frac{3}{2})$  lie on the graph. **→** *x* 

Find the value of *k* and of *a*.

Answer  $k = \dots$ 

# $11 \qquad \frac{v}{w} = \frac{1}{v} + \frac{3v}{u}$

Rearrange the formula to make v the subject.

*Answer* v = ..... [3]

12 On a coordinate plane, *P* is the point (0,12) and  $\overrightarrow{PR} = \begin{pmatrix} 8 \\ -10 \end{pmatrix}$ . (a) Calculate  $|\overrightarrow{PR}|$ .

Answer ..... [2]

(b) Find the equation of the line *PR*.

13 (a) Write down 252 as product of its prime factors in index notation.

Answer ..... [1]

(b) The highest common factor (HCF) of two numbers is 6. The lowest common multiple (LCM) of the two numbers is 252. One of the numbers is a perfect square.

Find the two numbers.

*Answer* ...... and ...... [2]



A container is a prism with a triangular cross-section. The container has a height of 3h cm and uniform width. Water is poured into the container at a constant rate. It takes 18 seconds to fill the container.

(a) Find the time taken for the water level to reach a depth of 2*h*.

Answer ..... seconds [2]

(b) On the axes in the answer space, sketch the graph of the height of the water level against time.



15 (a) The expression  $x^2 + 3x - 7$  can be written in the form  $\left(x + \frac{3}{2}\right)^2 + k$ . Find the value of k.

11

*Answer* k = ..... [2]

(b) Hence, explain why the equation  $x^2 + 3x - 7 = p$  does not have solutions for some values of *p*.

16 The sum of all reflex angles of a regular *n* sided polygon is 3060°.Find the number of sides of the regular polygon.

*Answer* ..... [2]

17 (a) Which of the following diagrams represents a directly proportional relationship between two variables.



- Answer ..... [1]
- (b) The frequency, F hertz (Hz), of a sound wave is directly proportional to the square root of the tension, T newton (N), in a guitar string.

Calculate percentage *F* increases by when the tension of the guitar string is increased 9 times.

Answer .....% [2]

18 Zara has only 2-dollar notes and 5-dollar notes in her wallet. The number of 2-dollar notes to 5-dollar notes is in the ratio 4:5. After Zara spends \$9, the number of 2-dollar notes to 5-dollar notes in her wallet becomes 2:3.

Calculate number of 2-dollar notes in Zara's wallet initially.

19 11.8 is the mean of five positive integers. The sum of their squares is 875. Each of the positive integer is now multiplied by 2.

Find the new standard deviation.

*Answer* ..... [3]

20 (a)  $\xi$  is the universal set and *P*, *Q* and *R* are three sets.  $\xi = \{j, k, l, m, n\}$  $P = \{j, k, l, n\}$ 

(i) It is given that 
$$P \cap Q = \{l\}$$
 and  $P' \cap Q \neq \emptyset$ .  
List the elements of  $Q$ .

Answer ..... [1]

(ii)  $R \subset P$ . Xena claim's that one possible set is  $R = \{j, k, l, n\}$ . Explain why Xena's claim is not true.

- (b) In a class of 23 students, 11 students speak French, and 7 students speak Spanish.
  - (i) Draw a Venn diagram to illustrate the case that there are some students who speak both languages.

Answer

[1]

(ii) Calculate the maximum possible number of students who do not speak both languages.

Answer ..... [1]



In the scale drawing, A, B and C define a boundary between coastline and the sea. B is a port and S is a ship.

(a) L is a lighthouse that is located

21

- (i) equidistant from A and B,
- (ii) equidistant from AB and BC,

Locate the lighthouse, L, on the scale drawing.

[2]

(b) Ships must not sail within 2 km of the lighthouse nor the port. Ship, *S*, sailed on a bearing of  $\theta^{\circ}$ . It passed between *B* and *L*.

Estimate the range of  $\theta^{\circ}$ .

Answer  $\ldots < \theta^{\circ} < \ldots$  [2]

In a science examination, a class of 25 students took two papers, paper A and paper B. Both papers were out of 50 marks.The dot diagram shows the distribution of the marks for paper A.



(a) Explain why mean is not an appropriate measure of central tendency to represent the distribution of marks for paper A.

.....[1]

(b) Calculate the interquartile range for paper A.

*Answer* ..... [2]

(c) The table below summaries the distribution of the class for paper B.

Median	41
Interquartile Range	7

Make two comments comparing the distribution of marks for both papers. Use figures to support your answer.

 **23** Han and Peng are training to take part in a triathlon.

Each training session involves training different segments, continuously in order. For each session Han plans to swim 15 minutes, cycle 20 minutes and run 22 minutes. For each session Peng plans to swim 20 minutes, cycle 25 minutes and run x minutes.

	Han	Peng	
	(15	20	swimming
This information can be represented by the matrix $\mathbf{E} =$	20	25	cycling
	22	x )	running

Han plans to train 3 sessions on the weekdays and 1 session on weekends. Peng plans to train 1 session on weekdays and 2 sessions on weekends.

This information can be represented by the matrix  $\mathbf{N} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$ .

(a) Find, in terms of x, the matrix  $\mathbf{T} = \mathbf{EN}$ .

Answer 
$$\mathbf{T} = [2]$$

(b) State what each element in the second column of matrix **T** represents.

(c) The total time ran by Han and Peng on the weekdays is equal to total time ran on the weekends.

Find the value of *x*.

**[TURN OVER** 

(d) It is given that swimming burns 8 calories per minute, cycling burns 10 calories per minute and running burns 12 calories per minute. This information can be represented by the matrix C. Another Matrix A is such that all the elements are 1. The elements of the matrix B, where B = CTA, represents the total number of calories burnt by Han and Peng for a week.

Write down the matrix **C** and state the order of matrix **A**.

Answer  $\mathbf{C} = [1]$ 

Answer order of  $\mathbf{A} = \dots \dots [1]$ 

**24** A ball is thrown vertically upwards. Its height, *h* m, above ground at time *t* seconds after being thrown is given by the formula  $h = -1.44t^2 + 1.6t + 1.5$ .

Find the time taken for the ball to return to the ground.

Answer ..... seconds [3]





The diagram shows a circle which passes through D, E and F. *AEB*, *BFC* and *ADC* are tangents to the circle, with centre O.

(a) Show that triangle *ABF* is congruent to triangle *ACF*. Give a reason for each statement you make.

......[3]

(b) AB = 25 cm and BC = 14 cm. Find the radius of the circle.

Answer ......cm [3]



The diagram shows two circles that intersect at *A* and *B*. *A*, *B* and *D* are three points on a circle I. *A*, *E*, *B* and *C* are four points on a circle II. *AD* is the diameter of the circle I. *BC* is the diameter of the circle II. *CA* is parallel to *BD*. Angle  $CAD = 109^{\circ}$  and angle  $ACB = 38^{\circ}$ .

(a) Find angle *DEB*. Give a reason for each step of your working.

(b) Show that *E* is the centre of circle I. Give a reason for each step of your working.

Answer

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### Y2024 Y4 Prelim MA4052 Mark Scheme

1	23-6x > 2+x
	$\boxed{\frac{3}{3}} \leq \frac{6}{6}$
	$\frac{46-12x}{46-12x} < \frac{2+x}{2}$
	$\overline{6}$ $\overline{6}$
	$-13x \le 2 - 46$
	$13x \ge 44$
	$x \ge 3\frac{5}{2}$
	<u>    13    </u>

2(a)	$0.0001087 = \underline{1.087 \times 10^{-4}}$
2(b)	Manila



4(a)	$\left(\frac{81u^{20}}{v^8}\right)^{-\frac{3}{4}} = \left(\frac{v^8}{81u^{20}}\right)^{\frac{3}{4}}$
	$=\frac{v^6}{27u^{15}}$
4(b)	$4x^{2} + 4 - y^{2} - x^{2}y^{2}$ = 4(x^{2} + 1) - y^{2}(x^{2} + 1)
	= -(x + 1) - y - (x + 1) = $(4 - y^2)(x^2 + 1)$
	$=(2+y)(2-y)(x^{2}+1)$

5 
$$(m+n)^{2} - (m-n)^{2}$$

$$= m^{2} + 2mn + n^{2} - m^{2} + 2mn - n^{2}$$

$$= 4mn$$
4 is a factor of  $(m+n)^{2} - (m-n)^{2}$ , 4 is not a prime number.  
OR  
Any prime number has exactly 2 factors, 1 and itself.  
Given,  $m \neq n$ , 4mn has more than 2 factors, therefore cannot be prime number.

6	Let <i>x</i> be the cost price.
	$1.25x\left(1-\frac{p}{100}\right) = x$
	$1 - \frac{p}{100} = \frac{1}{1.25}$
	$\frac{p}{100} = \frac{1}{5}$
	p = 20

7	The vertical axis does not start from \$0.
	Reader might <u>interpret</u> that price of eggs has <u>quadrupled/increased 4 times</u> between Jan/Feb to Dec, <u>when</u> in value the <u>price</u> of eggs has <u>risen by about \$2</u> <u>/doubled only.</u>

8	By Sine Rule
	$\frac{\sin(\angle YXZ)}{\sin(\angle XZY)} - \frac{\sin(\angle XZY)}{\sin(\angle XZY)}$
	13 20
	$\sin\left(\angle XZY\right) = \frac{20\sin\left(35^\circ\right)}{13}$
	$\angle XZY = \sin^{-1}(0.882425)$
	$\angle XZY = 61.936^{\circ}$

$\angle XZY = 61.9^{\circ}$
Or
$\angle XZY = 180^{\circ} - 61.936^{\circ}$ (By obtuse angle ratio)
$\angle XZY = 118.1^{\circ}$ (By obtuse angle ratio)

9(a)	Map scale = $\frac{140 \times 100}{3.5} = \frac{4000}{1}$
	Scale of map = $1:4000$
9(b)	Scale of map = $1:4000$
	Area scale of map = $1 : (4000)^2$
	Area on map $= \frac{17 \times 10000 \times 100 \times 100}{(4000)^2}$ $= 106.25 \text{ cm}^2$
	OR
	Area = $\frac{17 \times 10000}{(40)^2}$ = 106.25 cm <sup>2</sup>

10	$y = ka^{-x}$
	At $Q(0, \frac{3}{2})$ ,
	$\frac{3}{2} = ka^{-0}$
	$\frac{3}{2} = k(1)$
	$k = \frac{3}{2}$
	At P (-2, 96)
	$96 = \frac{3}{2}a^{-(-2)}$
	$a^2 = 64$
	$a = \pm \sqrt{64}$
	As, $a \neq 8$ ( $a > 1$ to be defined), $\underline{a = 8}$ ,

11	v = 1 = 3v
	+
	$y = u + 3v^2$
	$\frac{v}{2} = \frac{u+3v}{2}$
	$\frac{w}{2}$ $\frac{u}{2}$
	$uv^2 = uw + 3v^2w$
	$uv^2 - 3v^2w = uw$
	$v^2 \left( u - 3w \right) = uw$
	$v^2 = \frac{uw}{u}$
	u-3w
	$\dots \perp \boxed{uw}$
	$V \equiv \pm \sqrt{\frac{1}{u - 3w}}$
12(a)	
	$\left  \overline{PR} \right  = \sqrt{8^2 + \left( -10 \right)^2}$
	$ \overrightarrow{PR}  = 2\sqrt{41}$ units
	OR
	PR  = 12.8 units
12(b)	$\rightarrow$ (8)
	Given $PR = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$ ,
	-10
	Gradient = $\frac{-1}{8}$
	5
	$=-\frac{3}{4}$
	Equation of $PR$ $v = mr + c$
	$\frac{1}{5}$
	$y = -\frac{3}{4}x + 12$

13(a)	$252 = 2^2 \times 3^2 \times 7$
13(b)	$HCF = 2 \times 3$
	$LCM = 2^2 \times 3^2 \times 7$
	First number = $2^2 \times 3^2$ (perfect square)
	= 36
	Second number = $2 \times 3 \times 7$
	= 42



16	Sum of all exterior angles = $360^{\circ}$ Hence sum of all reflex angles $180^{\circ}(n) + 360^{\circ} = 3060^{\circ}$ $n = \frac{3060^{\circ} - 360^{\circ}}{180^{\circ}}$
	n = 15
	OR Sum of all interior angles = $180^{\circ}(n-2)$ Hence sum of all reflex angles $360^{\circ}(n) - 180^{\circ}(n-2) = 3060^{\circ}$

$$180^{\circ}(n) + 360^{\circ} = 3060^{\circ}$$
$$n = \frac{3060^{\circ} - 360^{\circ}}{180^{\circ}}$$
$$n = 15$$

17(a)	Diagram 1
17(b)	$F = k\sqrt{T}$
	$F_{new} = k\sqrt{T_{new}}$
	$F_{new} = k\sqrt{9T}$
	$F_{new} = 3k\sqrt{T}$
	$F_{New} = 3F$
	F increases by 200%

18	\$9 made up of 2, 2-dollars notes and 1, 5-dollars note.
	$\frac{4x-2}{5x-1} = \frac{2}{3}$ $12x-6 = 10x-2$ $x = 2$ $4x = 8$ Number of 2-dollar notes = 8

19  
S.D. = 
$$\sqrt{\frac{\sum x^2}{N} - (\bar{x})^2}$$
  
When all values are multiplied by 2,  
S.D.=  $\sqrt{\frac{\sum (2x)^2}{N} - (2\bar{x})^2}$   
 $= \sqrt{\frac{4\sum (x)^2}{N} - (2\bar{x})^2}$   
 $= \sqrt{\frac{4(875)}{5} - (2 \times 11.8)^2}$   
 $= \sqrt{\frac{4(875)}{5} - (2 \times 11.8)^2}$   
 $= 11.9599$   
 $= 12.0 (3 sf)$ 

20(a)(i)	$Q = \{l, m\}  \text{or } l, m$
20(a)(ii)	<i>R</i> is a proper subset of <i>P</i> . There is at least one element
	<b>IN P that is not an element of R.</b> If all elements of R also belong to P, then R is not a proper subset of P.
20(b)(i)	
	E Spanish French
20(b)(ii)	Max no. of students $= 23 - 11$ = 12



22(a)	Mean in this case is <b>distorted/skewed/affected by the</b>
	outlier 49 marks (out of 50)
22(b)	Lower quartile = 36 marks
	Upper quartile =40 marks
	Interquartile range = $40 - 36$
	= 4  marks
22(c)	Median for Paper B, 41 marks is higher than Paper A,
	37 marks. Hence on average, students did better for
	Paper B than Paper A.
	Interquartile range for Paper A is 4 while interquartile range for Paper B is 7. Hence students' marks in Paper B has a greater spread than students' marks in Paper A.

23(a)	T = EN
	$\mathbf{T} = \begin{pmatrix} 15 & 20\\ 20 & 25\\ 22 & x \end{pmatrix} \begin{pmatrix} 3 & 1\\ 1 & 2 \end{pmatrix}$
	$\mathbf{T} = \begin{pmatrix} 15 \times 3 + 20 \times 1 & 15 \times 1 + 20 \times 2 \\ 20 \times 3 + 25 \times 1 & 20 \times 1 + 25 \times 2 \\ 22 \times 3 + x \times 1 & 22 \times 1 + x \times 2 \end{pmatrix}$
	$\mathbf{T} = \begin{pmatrix} 65 & 55 \\ 85 & 70 \\ 66+x & 22+2x \end{pmatrix}$
23(b)	Elements in the second column of matrix T represent the total number of minutes both Han and Peng spend swimming, cycling and running respectively on weekends.
23(c)	66 + x = 22 + 2x $x = 44$
23(d)	$\mathbf{B} = \mathbf{CTA}$
	$\mathbf{B} = \begin{pmatrix} 8 & 10 & 12 \end{pmatrix} \begin{pmatrix} 65 & 55 \\ 85 & 70 \\ 66+x & 22+2x \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 8 & 10 & 12 \end{pmatrix}$ Order of $\mathbf{A} = -2 \times 1$

<i>t</i> the time reaches the ground= time ball was above ground.
$-1.44t^{2} + 1.6t + 1.5 = 0$ $t = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$
$t = \frac{-1.6 \pm \sqrt{(1.6)^2 - 4(-1.44)(1.5)}}{2(-1.44)}$
$t = \frac{-1.6 \pm \sqrt{11.2}}{2(-1.44)}$
t = -0.60647 or $t = 1.7175as t > 0, t = 1.72 seconds$

25(a)	$AF$ is common lengthAngle $BFA$ = Angle $CFA$ =90° (tangent $\perp$ radius)Angle $EAO$ = Angle $DAO$ (tangents from ext.pt)Hence triangle $ABF$ is congruent to triangle $ACF$ byASA congruency test.
25(b)	By congruency and tangents from ext.pt, BE = BF = FC = DC. Considering triangle $ABF$ , Angle $BFA = \sin^{-1}\left(\frac{7}{25}\right) = 16.260^{\circ}$ Considering triangle $AEO$ , $\tan(16.260^{\circ}) = \left(\frac{EO}{25-7}\right)$
	$EO = 18 \tan (16.260^{\circ})$ EO = 5.2499  cm EO = 5.25  cm

26(a)	Angle $AEB = 180^{\circ} - 38^{\circ}$ (angles in opp.seg) = 142°
	Angle $DEB = 180^{\circ} - 142^{\circ}$ (adj angles on st line) = 38°
26(b)	Angle $EAB = 109^{\circ} - 90^{\circ}$ (right angle in a semi-circle)
	= 19°
	Angle $DEB = 38^{\circ}$
	$\frac{38^{\circ}}{2} = 19^{\circ}$
	Angle $DEB = 2$ Angle $EAB$
	Hence, by angle at centre is twice the angle at
	circumference, E is the centre of circle I.
	OR
	Angle $AEB = 180^{\circ} - 38^{\circ}$ (angles in opp.seg) = 142°
	Angle $EAB = 109^{\circ} - 90^{\circ}$ (right angle in a semi-circle) = $19^{\circ}$
	Angle $ABE = 180^{\circ} - 142^{\circ} - 19^{\circ}$ (angles sum of triangle) = 19°
	Angle $EAB$ = Angle $ABE$
	<i>Hence,</i> $EA = EB$ , by radius of circle, <i>E</i> is the centre of circle I
	OR
	Angle $CAB = 90^{\circ}$ (right angle in a semi-circle) Angle $ABD =$ Angle $CAB = 90^{\circ}$ (alt. angles of parallel lines)
	Angle $EAB = 109^{\circ} - 90^{\circ}$ (right angle in a semi-circle) = 19°
	Consider triangle <i>ADB</i> Angle $ADB = 180 - 90^{\circ} - 19^{\circ}$ (angles sum of triangle)
	= 71°

Consider triangle <i>DEB</i>
Angle $ADB = 180 - 71^{\circ} - 38^{\circ}$ (angles sum of triangle)
= 71°
Angle $EDB =$ Angle $EBD$
<i>Hence</i> , $ED = EB$ , by radius of circle, E is the centre of
circle I