National Junior College
2016 – 2017 H2 Further Mathematics
Topic F3: Further Differential Equations (Assignment)

Name: _____

Class: 1fm2__

Subject Tutor: _____

Time Spent

Suggested Duration: 55min

1 The normal at any point on a certain curve always passes through the point $(2,3)$. Form a differential equation to express this property and hence find the equation of the family of curves that possess this property. Sketch a typical member of this family. [5]

2 Spruce budworm is a serious pest in eastern Canada and northern Minnesota. It consumes the leaves of coniferous tree and excess consumption can damage and kill the tree. In the absence of predators, the worm's population, P (in millions), satisfies the logistic growth,

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP\left(1-\frac{P}{N}\right).$$

One scientist proposed that the worm's predators eat the worm at a rate proportional to the worm's population.

(i) Explain the significance of the constant N in this model. [1]

(ii) Write down the differential equation of the population growth of the worm in the presence of predators. [1]

(iii) Given that the population of worm increases towards an equilibrium value in the long run, draw the phase line diagram and comment on the stability of the equilibrium values. [3]

(iv) Under what condition will the population of the worms decrease and become extinct eventually. [1]

[2014 H3 Prelim/HCI/Modified]

3 The current in a particular electrical circuit is described by the equation

$$\frac{\mathrm{d}^2I}{\mathrm{d}t^2} + 25\frac{\mathrm{d}I}{\mathrm{d}t} + 100I = -170\sin 20t,$$

where I is the current in amperes and t is the time in seconds after the power source is switched on.

Find the solution for which $\frac{\mathrm{d}I}{\mathrm{d}t} = I = 0$ when $t = 0$. [8]

Given that $I \to I_1$ as $t \to \infty$, find the maximum value of I_1. [2]

4 A 2 kg object stretches a vertical spring 80cm beyond its natural length to reach the equilibrium position. Now the spring is stretched further by 5cm and released from that position with zero velocity.

(a) Assume that the air resistance is negligible. Write down the differential equation that models the behaviour of the spring. Define all the variables used in the equation. [2]

(b) It is known that air resistance is proportional to the velocity of the object with the proportionality constant being k.

(i) Take $k = 1$. Find the exact position of the spring at any time *t*. [4]

(ii) Describe the damping effect of the air resistance. [2]

(c) In order for the system to be critically damped, it is now placed in a viscous liquid with damping coefficient λ. What should be the value of λ? You may assume the buoyancy is negligible. [2]