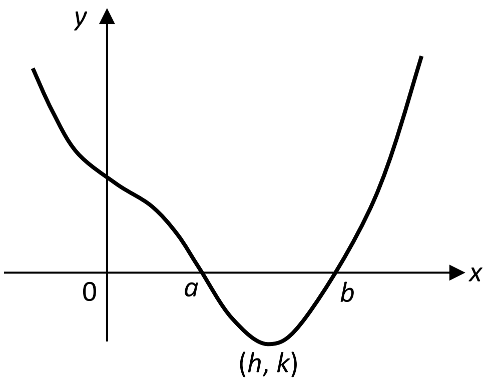
1.2 Graphs and Transformations 2

Transformation of Curves

1 The graph of $y = f'(x)$ below cuts the x-axis at $x = a$ and $x = b$. It has a turning point at (h, k).



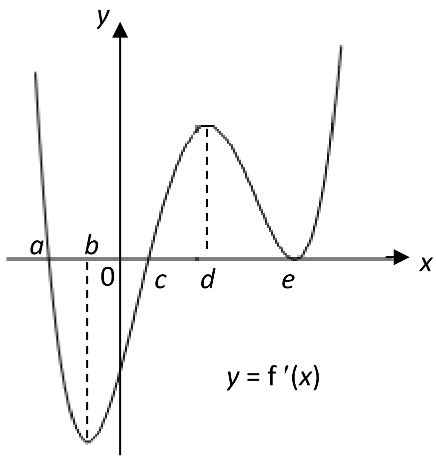
(i) State the range of values of x for which the graph of $f(x)$

(a) is strictly increasing,

(b) concaves upwards. [3]

(ii) State the value(s) of x for the stationary point(s) of $f(x)$, specifying clearly the nature of each stationary point. [2]

2 The diagram shows the graph of $y = f'(x)$ (not drawn to scale) with x-intercepts a, c, e and stationary points at $x = b$, $x = d$ and $x = e$.



(i) State the x-coordinates and nature of the stationary points of the graph of $y = f(x)$. [3]

(ii) Sketch a possible graph of $y = f(x)$ indicating clearly the x-coordinates of the stationary points. [2]

3 **ACJC/2010Promo/12**

The diagram below shows the graph of $y = f(x)$. It has a maximum point at $A(0, -2a)$, a minimum point at $B\left(-2a,\ -\frac{1}{6}a\right)$, x-intercept at $-a$, and asymptotes at $x = \pm\frac{a}{2}$.

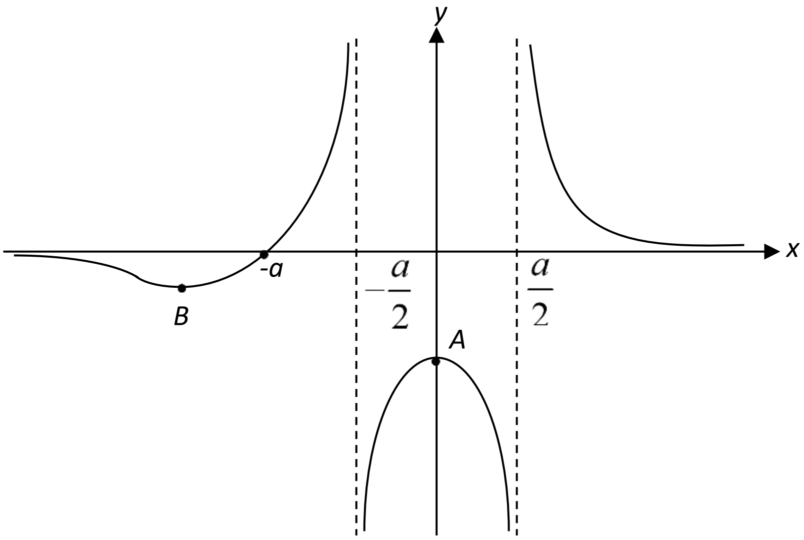
On separate diagrams, sketch the graphs of

(i) $y = |f(x+a)|$, [3]

(ii) $y = \dfrac{1}{f(x)}$, [3]

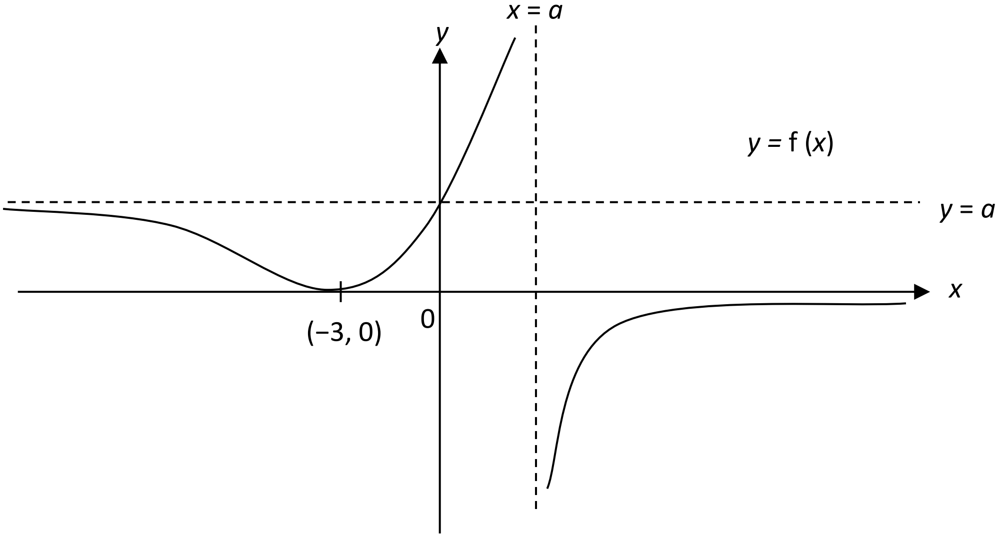
(iii) $y = f'(x)$ [3]

showing clearly the asymptotes and the coordinates of the turning points.



4 NYJC/2010Promo/11

The graph of $y = \mathrm{f}(x)$ is shown above, where the lines $y = a$ and $x = a$, $a > 1$, are asymptotes to the curve and (−3, 0) is a minimum point. The graph intersects the *y*-axis at (0, *a*).



On separate diagrams, sketch the graphs of

(i) $y = \mathrm{f}(a - x)$, [2]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

showing clearly the equations of asymptotes, coordinates of any points of intersection with the *x* and *y*-axes and the coordinates of the stationary point(s).

5 DHS/2013MYE JC1/12(a)

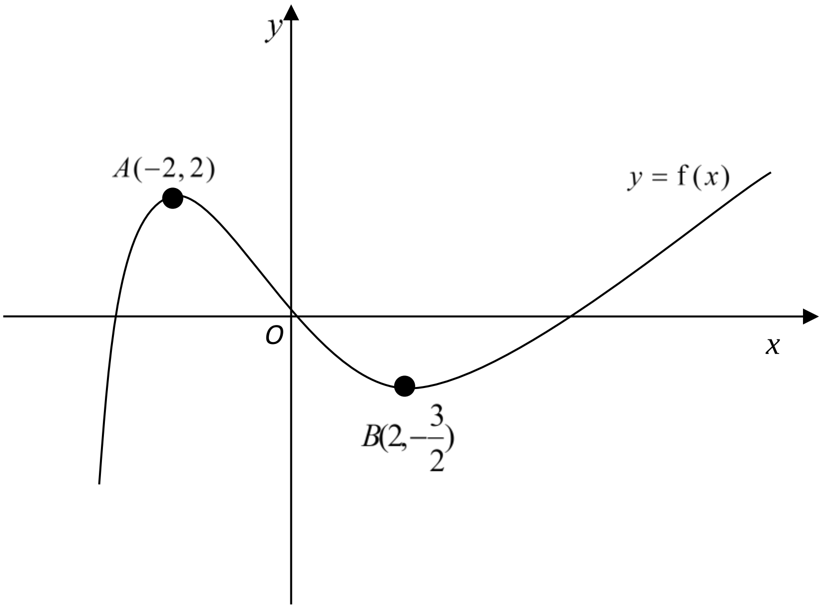
(a) The curves C_1 and C_2 are given by the equations $x + y^2 = 1$ and $x + (2y + 2k)^2 = 1$ respectively, where k is a positive constant.

(i) Describe clearly a sequence of transformations that maps C_1 to C_2 [2]

(ii) Given that $k = 3$, sketch the graph of C_2, indicating the coordinates of the intercepts and the vertex. [2]

6 RI/2010Promo/5

(a) The graph of $y = \mathrm{f}(x)$ passes through the origin, has a maximum turning point at $A(-2, 2)$ and a minimum turning point at $B\left(2, -\dfrac{3}{2}\right)$, as shown in the diagram on the right.



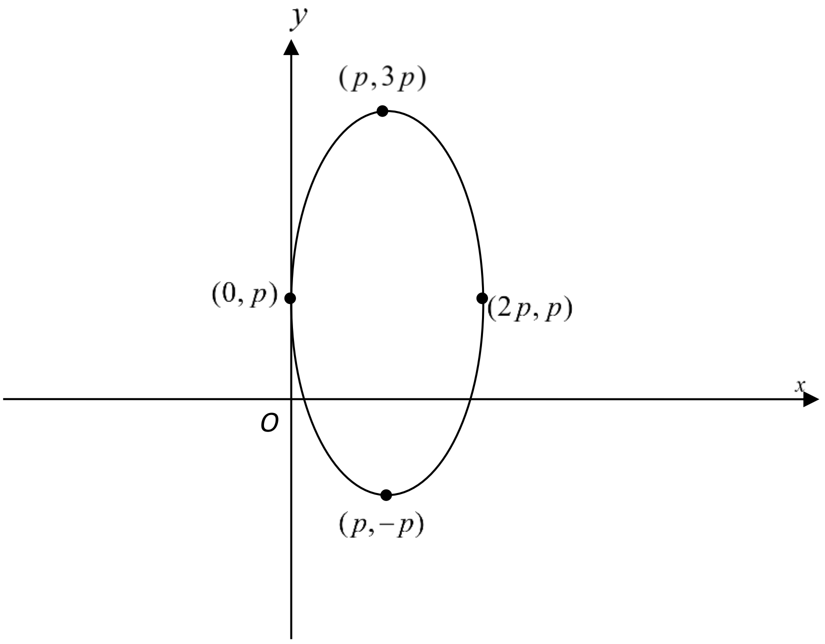
Sketch, on separate diagrams, the graphs of

(i) $y = \mathrm{f}(x+1)$, [2]

(ii) $y = \mathrm{f}(1-|x|)$, [3]

stating clearly the coordinates of the points corresponding to A and B (if any).

(b) The graph of $(x-p)^2+(y-p)^2 = p^2$, $p>0$ underwent exactly two simple transformations, involving either reflection, stretching or translation only, to become the following ellipse.

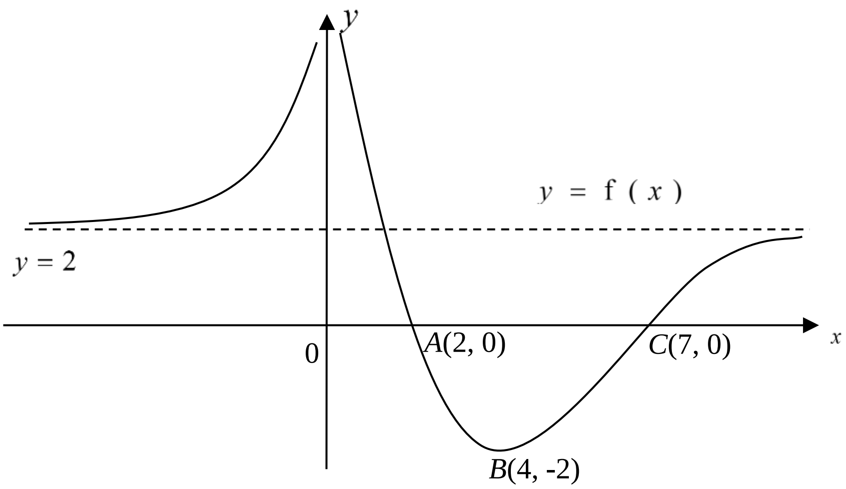


(i) Write down the equation of the ellipse. [1]

(ii) Describe clearly (in words) a sequence of the **two** transformations. [2]

7 **SRJC/2010Promo/8a(part)**

(a) The diagram shows the graph of $y = \mathrm{f}(x)$.

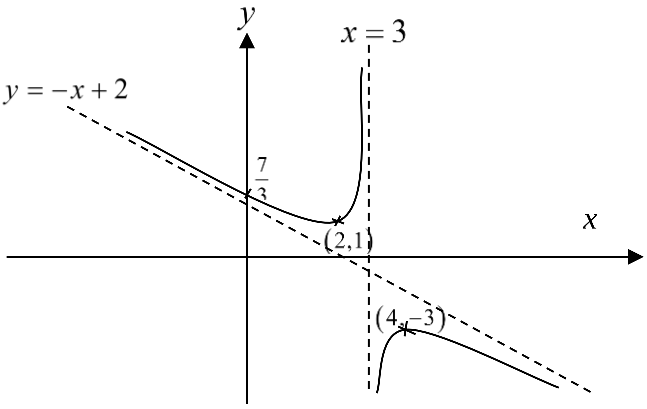


The points *A*, *B*, *C* have coordinates $(2,0)$, $(4,-2)$, $(7,0)$ respectively. The equations of the asymptotes are $x=0$ and $y=2$. Sketch the graph of $y=\dfrac{1}{\mathrm{f}(x)}$. [2]

State clearly the equations of all the asymptotes and show the coordinates of the points corresponding to *A*, *B* and *C*.

8 VJC/2010Promo/6

The diagram shows the graph of $y = \mathrm{f}(x)$.



On separate axes, sketch the curves of

(i) $y = \dfrac{1}{\mathrm{f}(x)}$ [3]

(ii) $y = \dfrac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{f}(x)\right]$ [3]

9 MJC/2010Promo/4

The curve C_1 has equation $k^2x^2 + y^2 = k^2$, where $k > 1$.

(i) Sketch C_1, indicating the axial intercepts, asymptotes and stationary points, if any. [2]

(ii) Hence, or otherwise, sketch $k^2x^2 + \frac{1}{4}y^2 = k^2$, where $k > 1$, indicating the axial intercepts, asymptotes and stationary points, if any. [1]

C_1 undergoes a single transformation to become C_2.

(iii) Given that C_2 has a line of symmetry $y = 2$ and passes through the origin, state the value of k. [1]

(iv) Describe a geometrical transformation by which C_2 may be obtained from C_1. [1]

(v) Write down the equation of C_2. [1]

10 DHS/2013Promo/9

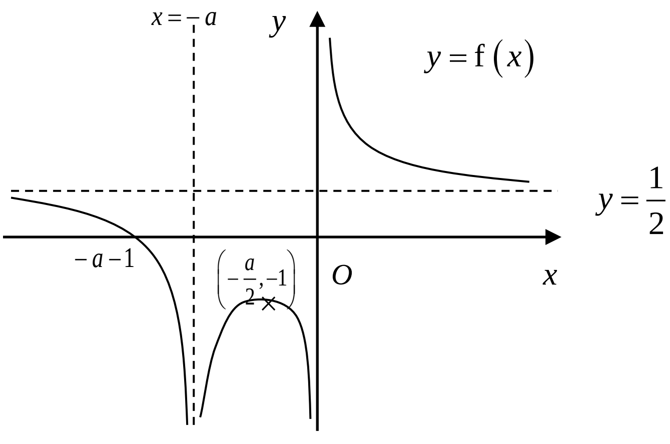
(a) Show that the equation $36y^2 - 36y - 4x^2 = 27$ can be expressed as $\dfrac{(2y-1)^2}{4} - \dfrac{x^2}{9} = 1$. [2]

Hence,

(i) state precisely a sequence of transformations by which the graph of $36y^2 - 36y - 4x^2 = 27$ may be obtained from the graph of $\dfrac{y^2}{4} - \dfrac{x^2}{9} = 1$. [2]

(ii) sketch the graph of $36y^2 - 36y - 4x^2 = 27$, indicating the coordinates of the intercepts and the equations of the asymptotes. [2]

(b) The diagram shows the graph of $y = \mathrm{f}(x)$. The curve passes through the x-axis at $(-a-1, 0)$ and has a maximum point at $\left(-\frac{a}{2}, -1\right)$, where a is a positive constant. The curve also has horizontal and vertical asymptotes $y = \frac{1}{2}$ and $x = -a$ respectively.



On separate diagrams, sketch the graphs of

(i) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

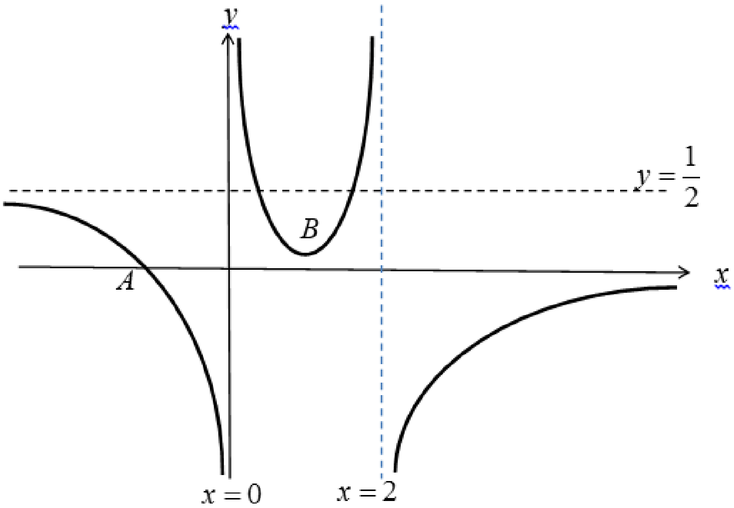
(ii) $y = -\mathrm{f}(x-a)$, [3]

indicating clearing the equations of any asymptotes, and the coordinates of turning points and axial intercepts.

11 PJC/2013Promo/9

(a) Describe a sequence of transformations which transforms the graph of $y=\frac{1}{x}$ to the graph of $y=\frac{1}{2x+3}$. [2]

(b)



The diagram above shows the graph of $y=\mathrm{f}(x)$. The curve passes through the point $A(-2,0)$ and has turning point at $B\left(1,\ \frac{1}{3}\right)$. The lines $x=0$, $x=2$, $y=0$, and $y=\frac{1}{2}$ are asymptotes to the curve. On separate diagrams, sketch the graphs of

(i) $y=\frac{1}{\mathrm{f}(x)}$, [2]

(ii) $y=\mathrm{f}(-|x|)$, [2]

stating, in each case, the axial intercepts, asymptotes and the coordinates of any turning points.

12 SAJC/2013Promo/9b

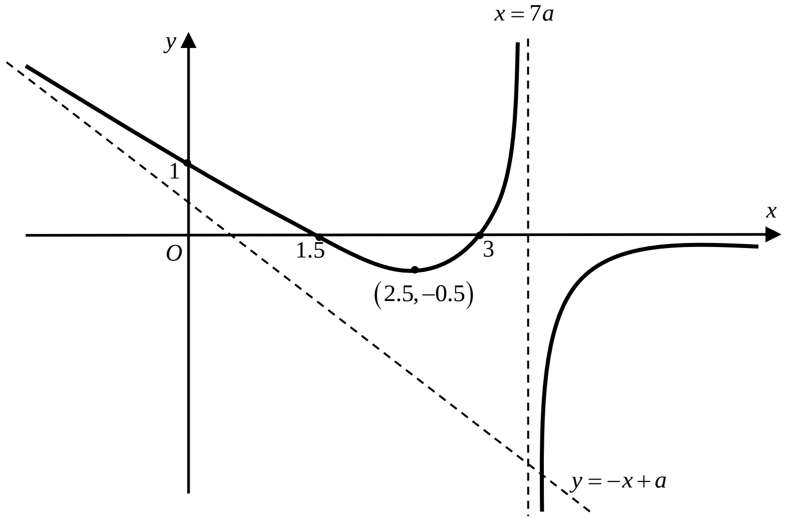
State the sequence of transformations which transform the graph of $y=\frac{1}{x}$ to the graph of $y=\frac{2x+7}{2x+5}$. [3]

Sketch the graph of $y=\frac{2x+7}{2x+5}$, giving the equations of any asymptotes and the coordinates of any points of intersection with the *x*- and *y*-axes. [3]

By adding an additional curve on the same sketch, determine the number of real roots of the equation $(2x+7)^2=9(2x+5)^2\left(1-\frac{x^2}{4}\right)$. [3]

13 HCI/2013Promo/4

The diagram below shows the graph of $y = \mathrm{f}(x)$. It cuts the axes at the points $(0,\ 1)$, $(1.5,\ 0)$ and $(3,\ 0)$. It has a minimum point at $(2.5, -0.5)$. The horizontal, vertical and oblique asymptotes are $y = 0$, $x = 7a$ and $y = -x + a$ respectively, where a is a positive constant.



On separate diagrams, sketch the graphs of

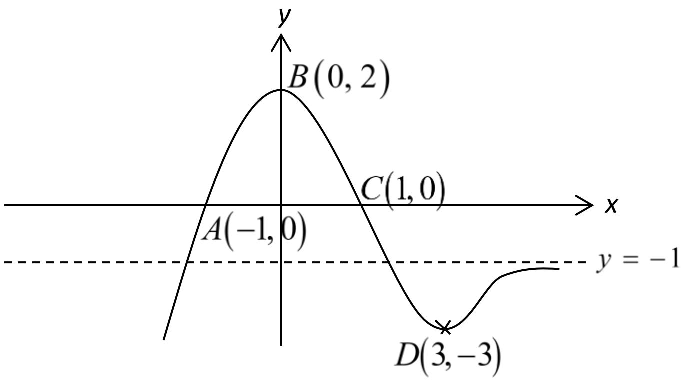
(i) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

(ii) $y = \mathrm{f}'(x)$, [3]

showing clearly the axial intercepts, the stationary points and the equations of the asymptotes where applicable.

14 VJC/2012Promo/4

The diagram shows the graph of $y = \mathrm{f}(2x)$. The curve passes through the points $A(-1, 0)$, $B(0, 2)$, $C(1, 0)$ and $D(3, -3)$.



Sketch, on separate clearly labelled diagrams, the graphs of

(i) $y = \mathrm{f}(-2x)$, [1]

(ii) $y = \mathrm{f}(2 - 2x)$, [2]

(iii) $y = \dfrac{1}{2}\mathrm{f}(2x)$. [2]

<u>Answers</u>

Transformation of Curves

1(i)(a) $x < a$ or $x > b$, (b) $x > h$ (ii) Maximum point at $x = a$, minimum point at $x = b$

2(i) $x = a$ (Max), $x = c$ (Min), $x = e$ (Stationary point of inflexion)