4E EMath Prelim P2 2024 Mark Scheme

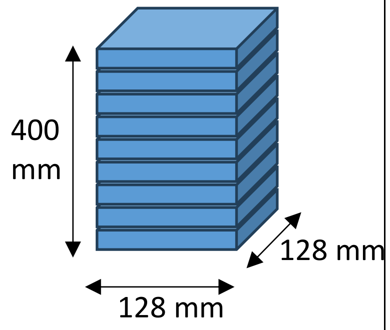
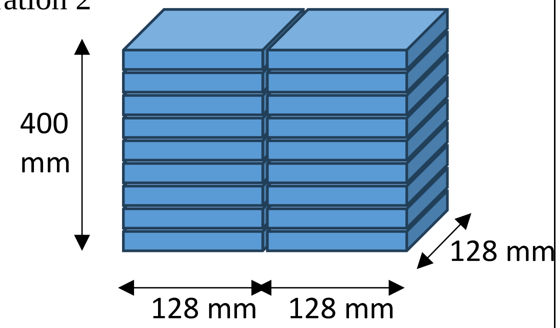
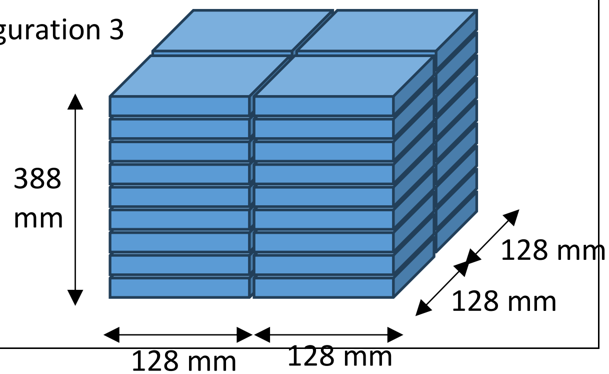
1a)	Since area $= 1536\pi$, $1536\pi = \pi(24)^2 + \pi(24)l$ $l = 40\,\text{cm}$ $\text{height} = \sqrt{40^2 - 24^2} = 32\,\text{cm}$ $\text{Volume of cone} = \frac{1}{3} \times (24)^2 \times \pi \times 32$ $= 6144\pi\ \text{cm}^3$ (Shown)	M1 M1 A1	
1bi)	$6144\pi = \frac{4}{3} \times \pi \times r^3$ $r = \sqrt[3]{\frac{6144(3)}{4}}$ $=16.641$ $=16.6\ \text{cm}$ (to 3 s.f.)	M1 A1	
1bii)	$\text{Surface area} = 4\pi(16.641)^2$ $= 3479.91555$ $= 3480\,\text{cm}^2$ (to 3 s.f.)	M1 A1	
2a)	$20000\left(1 + \frac{r}{100}\right)^{24} = 21337.05$ $\left(1 + \frac{r}{100}\right)^{24} = \frac{21337.05}{20000}$ $1 + \frac{r}{100} = \sqrt[24]{\frac{21337.05}{20000}}$ $=1.002700002$ $r = 0.270$ (to 3 s.f.)	M1 A1	
2b)	$\text{loan amount} = \$1774.40 \times 84$ $= \$149049.60$ $100\% + 2.78\% \times 7 = 119.46\%$ Loan amount without interest $= \$149049.60 \times \frac{100\%}{119.46\%}$ $= \$124769.4626$ $\text{Price of Car} = \$124769.4626 \times \frac{100\%}{70\%}$ $= \$178242$ (nearest dollar)	 M1 M1 A1	
2c)	April : $\text{SGD}980 = \text{JPY}112210$ June : $\text{JPY}112210 = \text{SGD}949.32318$ $\text{Percentage loss} = \frac{949.32318 - 980}{980} \times 100\%$ $= -3.13\%$ (to 3 s.f.)	M1 M1 A1	Must circle loss.

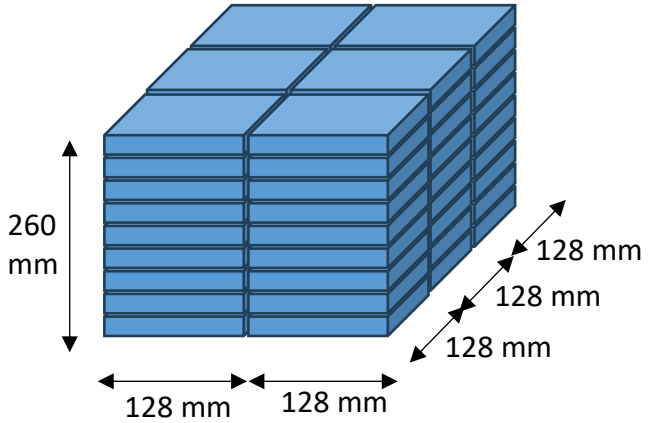
3)	Let the number of couples needed be x. $\frac{16+x}{37+16+x+x} = 0.4$ $16+x = 0.4(53+2x)$ $16+x = 21.2+0.8x$ $x = 26$	M1 M1 A1	
4i)	$a = 0$ $b = 5$	B1 B1	
4ii)	46	B1	
4iii)	mean = 45.1 Standard Deviation = 9.58	B1 B1	
4iv)	The group of 20 students have higher marks since the mean is higher, and they have more consistent marks as their standard deviation is lower.	B1 B1	
5a)	Let y be the number of 50 cent coins and x be the number of 20 cent coins. $y + x = 73$ --------(1) $(y)0.5 + (x)0.2 = 28.10$ ------(2) From (2) $(y) = 56.2 - 0.4(x)$ ----------(3) Sub (3) into (1) $56.2 - 0.4x + x = 73$ $0.6x = 16.8$ $x = 28$ $y = 73 - 28 = 45$ They have 45 50-cent coins and 28 20-cent coins.	M1 M1 M1 A1 A1	Forming 1st equation Forming 2nd equation For performing substitution/elimination A1 for 45 and A1 for 28
5b)	$\frac{4x}{(2x)^2-(5)^2} - \frac{1}{(2x-5)}$ $= \frac{4x}{(2x-5)(2x+5)} - \frac{1}{(2x-5)}$ $= \frac{4x-(2x+5)}{(2x-5)(2x+5)}$ $= \frac{4x-2x-5}{(2x-5)(2x+5)}$ $= \frac{2x-5}{(2x-5)(2x+5)}$ $= \frac{1}{2x+5}$	M1 M1 A1	• Factorise first denominator • Combine into single fraction

6a)	$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA}$ $= \begin{pmatrix} -1 \\ 12 \end{pmatrix} + \begin{pmatrix} -9 \\ 3 \end{pmatrix}$ $= \begin{pmatrix} -10 \\ 15 \end{pmatrix}$	B1	
6b)	$\left\lvert\overrightarrow{BA}\right\rvert = \sqrt{(-10)^2 + (15)^2}$ $= 18.0$ units (to 3 s.f.)	M1 A1	
7i)	Let m be the mid point of AB. $MB = \frac{1}{2}(65.32)$ (property of chord) $= 32.66$ mm $\angle MOB = \sin^{-1}\left(\frac{32.66}{45}\right)$ $= 0.81216$ rad $\angle AOB = 2\angle MOB$ $= 1.624328$ $= 1.6243$ rad (shown)	M1 A1	M1 for using sine
7ii)	$\pi(45+x)^2 - \pi(45)^2 = 1389.29$ $(45+x)^2 - (45)^2 = 445.08953$ $x = 4.699$ $= 4.70$ mm (to 3 s.f.)	M1 A1	
8i)	$\angle ABN_B = 180° - 64° = 116°$ (int $\angle$s, $N_A // N_B$) $\angle ABC = 360° - 127° - 116°$ ($\angle$s at a pt.) $= 117°$ $AC = \sqrt{4.83^2 + 7.24^2 - 2(4.83)(7.24)\cos 117°}$ $= 10.368$ km (shown)	M1 M1 A1	Must give reason Use cosine rule
8ii)	$\frac{\sin 117°}{10.368} = \frac{\sin \angle BCA}{4.83}$ $\angle BCA = 24.52443°$ (5 d.p.) Bearing of A from C Reflex $\angle N_C CB = 360° - 24.52443° - (180° - 127°)$ $= 282.47557°$ Bearing of A from C $= 282.5°$ (1 d.p.)	M1 M1 A1	Use sine rule
8iii)	Shortest distance $= 7.24 \times \sin 24.52443°$ $= 3.00518$ $= 3.01$ km (to 3 s.f.)	M1 A1	
8iv)	Distance of object to $C = 7.24 \times \tan 12°$ $= 1.54$ km (to 3 s.f.)	M1 A1	

9	Let interior angle of polygon be x. $3x+\angle a+\angle b=(5-2)\times 180°$ $3x+72°=540°$ $x=156°$ $n=\dfrac{360°}{180°-156°}$ $=15$	M1 M1 A1	
10ai)	$\dfrac{2}{5}\underset{\sim}{b}$	B1	
10aii)	$\overrightarrow{DE}=\overrightarrow{DC}+\overrightarrow{CO}+\overrightarrow{OE}$ $=-\dfrac{2}{5}\underset{\sim}{b}-\underset{\sim}{a}+\underset{\sim}{b}$ $=-\underset{\sim}{a}+\dfrac{3}{5}\underset{\sim}{b}$ $\overrightarrow{AD}=\dfrac{2}{3}\left(-\underset{\sim}{a}+\dfrac{3}{5}\underset{\sim}{b}\right)$ $=\dfrac{2}{5}\underset{\sim}{b}-\dfrac{2}{3}\underset{\sim}{a}$	M1 M1 A1	
10b)	$\overrightarrow{AC}=\overrightarrow{AD}+\overrightarrow{DC}$ $=-\dfrac{2}{3}\underset{\sim}{a}$ $=-\dfrac{2}{3}\overrightarrow{OC}$ therefor $\overrightarrow{AC}$ is parallel to $\overrightarrow{OC}$.	M1 A1	
10ci)	Let h be the common height of ΔADB and ΔACD from A to CB. $\dfrac{\text{area of }\Delta ADB}{\text{area of }\Delta ACD}=\dfrac{\frac{1}{2}\times DB\times h}{\frac{1}{2}\times CD\times h}$ $=\dfrac{3}{2}$	B1	
10cii)	Since $\dfrac{AC}{AO}=\dfrac{AD}{AE}=\dfrac{CD}{OE}$ ΔADC is similar to ΔAOE. $\dfrac{\text{area of }\Delta ADB}{\text{area of }\Delta ACD}=\left(\dfrac{2}{5}\right)^2$ $=\dfrac{4}{25}$	M1 A1	
10ciii)	Area area of ΔADB : area of ΔACD : area of ΔAOE $=6:4:25$ $\dfrac{\text{area of }\Delta ADB}{\text{area of CDEO}}=\dfrac{6}{25-4}$ $=\dfrac{2}{7}$	M1 A1	M1 for finding the ratio of area of CDEO relative to area of ΔADB

11a)	0	B1	
11b)	Refer to graph • plot all points correctly • smooth curve	 B1 B1	
11ci)	Refer to graph	B1	
11cii)	Gradient $= \frac{0.8-(-1)}{0-3}$ $= -0.6 \quad \pm 0.1$ $y = -0.6x + 0.8$ y-intercept ± 0.1	 M1 A1	
11d)	$3x^2 - 8x + 6 = 0$ $3x - 8 + \frac{6}{x} = 0$ $2x + \frac{6}{x} - 7 = -x + 1$ $y = -x + 1$ Draw graph $y = -x + 1$. $3x^2 - 8x + 6 = 0$ has no solution because $y = 2x + \frac{6}{x} - 7$ and $y = -x + 1$ do not intersect for $0.5 \le x \le 7$	 M1 M1 A1	 M1 for either of these 2 working steps. M1 for drawing graph.
12a)	$\frac{1500}{x}$	B1	
12b)	$\frac{1500}{x} - \frac{17}{\frac{x-22.5}{60}} = 2$ $\frac{1500}{x} - \frac{1020}{x-22.5} = 2$ $1500(x-22.5) - 1020x = 2(x^2 - 22.5x)$ $2x^2 - 525x + 33750 = 0$ (shown)	M1 M1 A1	
12c)	$x = \frac{-(-525) \pm \sqrt{(-525)^2 - 4(2)(33750)}}{2(2)}$ $= \frac{525 \pm \sqrt{5625}}{4}$ $= 150 \quad$ or $\quad = 112.5$	M1 A1 A1	 A1 for each solution
12d)	Total time taken $= x + (x - 22.5) = 2x - 22.5$ minutes When $x = 112.5$ Total time $= 2(112.5) - 22.5$ $= 202.5$ mins (reject since total time >4 hours) When $x = 150$ Total time $= 2(150) - 22.5$ $= 277.5$ mins $= 4$h 37.5min	 M1 A1	 Must reject with reason.

13a)	Percentage difference $= \frac{282-202}{202} \times 100\%$ $= 39.60396$ $= 39.6\%$ (to 3 s.f.)	M1 A1		
13b)	The books can be stacked into columns. The configuration of the columns will fit into boxes. Configuration 1 (1 column) Max height $= 900-128-128 = 644$ Exceed max 400mm Max books $= 400 \div 6 \approx 66$ books Total weight $= 66 \times 75 = 4950$ g Configuration 2 (2 columns) Max height $= 900-128-128-128 = 516$ Exceed max 400mm Max books per column $= 400 \div 6 = 66\frac{2}{3} \approx 66$ books Max books $= 66 \times 2 = 132$ books Total weight $= 132 \times 75 = 9900$ g Configuration 3 (4 columns) Max height $= 900-128-128-128-128 = 388$ Max books per column $= 388 \div 6 = 64\frac{2}{3} \approx 64$ books Max books $= 64 \times 4 = 256$ books Total weight $= 256 \times 75 = 19200$ g	 M1 M1 M1 M1	These 3 M1 can be allocated to any of the 5 configurations, M1 for max height, M1 for max books, M1 for max weight.	Configuration 1  Configuration 2  Configuration 3 

	Configuration 4 (6 columns) Max height $= 900 - 128 - 128 - 128 - 128 - 128 = 260$ Max books per column $= 260 \div 6 = 43\frac{1}{3} = 43$ books Max books $= 43 \times 6 = 258$ books Total weight $= 258 \times 75 = 19350$ g Configuration 5 (9 columns) Max height $= 900 - 128 \times 6 = 132$ Max books per column $= 132 \div 6 = 22$ books Max books $= 22 \times 9 = 198$ books Total weight $= 198 \times 75 = 14850$ g Max books by weight $= 20000 \div 75 \approx 266$ books Pick configuration 4 as most books. Total cost to manufacture and post to Australia $= 202 + 258 \times 1.80$ $= \$666.40$ Price per book $= \frac{666.40 + 3 \times 258}{258}$ $= \$5.58294$ $= \$5.60$ (to nearest ten cents)	M1 A1		Configuration 4  Configuration 5