Name: _____

Tutorial Class: _____

ANGLO-CHINESE JUNIOR COLLEGE	
MATHEMATICS DEPARTMENT	
MATHEMATICS	8865
Higher 1	24 August 2023
JC1 CA2 (25 Marks)	Time allowed: 45 mins

1 Differentiate
$$\ln(5-6x^2)$$
. [1]

2 The curve *D* has equation $y = x^{x+1} - 1$. Find the numerical value of the gradient of *D* at the point where x = 1. Hence state the equation of the tangent to *D* at this point. [2]

3 (a) Find
$$\int \frac{(3x^2-1)^2}{x} dx.$$
 [3]

(b) Find
$$\int \frac{1}{2\sqrt{1-\pi x}} dx.$$
 [2]

4 The diagram below shows the curve C with equation $y = e^{1-\frac{1}{2}x} + 3x$.



- (i) The finite area, *R*, between the curve *C*, the *x*-axis and the lines x = 0 and x = 2 is given by R = p + 2q, where *p* and *q* are constants. Use integration to find the exact values of *p* and *q*. [4]
- (ii) Hence find the range of values of *m* such that the equation $e^{1-\frac{1}{2}x} = m-3x$ has no real roots. [2]
- 5 [It is given that the curved surface area of a cylinder of base radius r and height h is $2\pi rh$.]



The diagram shows an open top trash bin in the shape of a half cylinder which is made up of three parts.

- The front of the bin is a curved surface of a half cylinder of radius *r* cm and height $\frac{4\pi}{r^2}$ cm.
- The back of the bin is a rectangle of length $\frac{4\pi}{r^2}$ cm and width 2r cm.
- The base of the bin is a semi-circle of radius *r* cm.

The three parts are joined together as shown in the diagram. You may assume that the material use is of negligible thickness.

(i) Show that the total external surface area of the trash bin, A, is $\frac{8\pi}{r} + \frac{4\pi^2}{r} + \frac{1}{2}\pi r^2$ cm².

To reduce the material used to manufacture the trash bin, the total external surface area of the trash bin, *A*, is to be minimised. Using a non-calculator method, (ii) Show that $r^3 = 4\pi + 8$. [3]

Hence find, in terms of π , the ratio of the diameter of the semi-circular base to the height of the trash bin. [2]

6 The National Environment Agency (NEA) has an annual trash bin renewal programme which identifies a total of x trash bins which are either damaged or missing to be replaced with new ones.

In order to ensure a healthy stockpile of new trash bins for use, NEA purchases 500 new trash bins from a manufacturer annually.

The annual budget for NEA's trash bin renewal programme consists of three components.

- the total purchase cost of 500 new trash bins from the manufacturer, whereby each new bins cost \$35.
- the total cost of disposing the damaged trash bins, whereby each damaged bin cost \$5 to dispose.
- an annual government subsidy of $30e^{0.01x}$.

On average, 5% of the x trash bins identified to be replaced are missing and hence do not require any disposal cost.

(i) Show that the annual total budget, C, for the NEA's trash bin renewal programme is given by

$$C = 17500 + 4.75x - 30e^{0.01x}.$$
 [1]

(ii) Using differentiation, find the maximum value of *C*. Justify that this is the maximum value. [4]