Beatty Secondary School

Prelim Examination 2020

Sec 4E/5N

Mathematics Paper 2 (4048/02)

Marking Scheme

1(a)	(i)	$A=\frac{1}{2}h\left(a^2-b^2\right)$ $A=\frac{1}{2}(6)\left(4^2-2.5^2\right)$ A = 29.25 ---------- B1	
	(ii)	$A=\frac{1}{2}h\left(a^2-b^2\right)$ $a^2-b^2=\frac{2A}{h}$ ---------- M1 $b=\pm\sqrt{a^2-\frac{2A}{h}}$ or $a=\pm\sqrt{\frac{a^2h-2A}{h}}$ ---------- A1	
1(b)		$\frac{7}{4x^2-4x+1}-\frac{2}{2x-1}$ $=\frac{7}{(2x-1)^2}-\frac{2}{2x-1}$ ---------- B1 (factorisation) $=\frac{7-2(2x-1)}{(2x-1)^2}$ ---------- M1 $=\frac{7-4x+2}{(2x-1)^2}$ $=\frac{9-4x}{(2x-1)^2}$ ---------- A1	

1(c)	(i)	$\frac{5-4x}{7} \geq \frac{x+4}{2}$ $2(5-4x) \geq 7(x+4)$ ---------- M1 $10-8x \geq 7x+28$ $-15x \geq 18$ $x \leq -1\frac{1}{5}$ or $x \leq -1.2$ ---------- A1	
	(ii)	Greatest integer $x = -2$ ---------- B1	
1(d)		$\frac{x+7}{2} + \frac{2x}{5} = x$ $5(x+7) + 4x = 10x$ ---------- M1 $5x + 35 + 4x = 10x$ $x = 35$ ---------- A1	[11 marks]
2(a)		$\frac{48}{x}$ hours ---------- B1	
2(b)		Time taken = $\frac{18}{60}$ hour ---------- B1 = $\frac{3}{10}$ hour $\frac{48}{x} - \frac{32}{x+6} = \frac{3}{10}$ ---------- M1 $48(x+6) - 32x = \frac{3}{10}x(x+6)$ $10(48x + 288 - 32x) = 3x^2 + 18x$ ---------- M1 $160x + 2880 = 3x^2 + 18x$ $3x^2 - 142x - 2880 = 0$ ---------- A1	

2(c)		$x = \frac{-(-142) \pm \sqrt{(-142)^2 - 4(3)(-2880)}}{2(3)}$ -------- M1 $x = \frac{142 \pm \sqrt{54724}}{6}$ $x = 62.655$ or -15.321 $x = 62.66$ or -15.32 ---------- A2	
2(d)		Time taken = $\frac{48}{62.655} + \frac{32}{62.655 + 6}$ = 1.2321 hours ---------- B1 = 1 hour 14 minutes ---------- B1	[10 marks]
3(a)	(i)	Total mail in 2009 = 1 002 662 Total mail in 2012 = 764 780 Percentage decrease = $\frac{1002662 - 764780}{1002662} \times 100\%$ ----- M1 = 23.7 % ---------- A1	
	(ii)	Mean number of mail per month for 2012 = $\frac{764780}{12}$ ---------- M1 = 63731.66 = 6.37×10^4 (3 sf) ---------- A1	
	(iii)	Year ratio/ fraction 2009 6.9223… 2010 5.6781… 2011 6.0349… 2012 5.9866… ---------- M1 Years 2011 and 2012 ---------- A1	

3(b)		Amount = $8500\left(1+\frac{\frac{1.5}{12}}{100}\right)^{36}$ ---------- M1 Interest = $8500\left(1+\frac{\frac{1.5}{12}}{100}\right)^{36}-8500$ = $ 390.99 ---------- A1	
3(c)		THB 5400 = $ $\frac{5400}{22.66}$ ---------- M1 = $ 238.305 Amount payable = $\frac{101.2}{100}\times 238.305$ --------- M1 = $ 241.16 = $241 (nearest dollars) ---- A1	[11 marks]
4(a)		p = 1.25 ---------- B1	
4(b)		Drawing of graph - Correct plotting of points ---------- G1 - Drawing of curve through all the points ---- G1 - Curve is smooth ---------- G1	
4(c)		$\frac{x^3}{4}-3x-1-2=0$ $\frac{x^3}{4}-3x-1=2$ This equation can be solved by drawing <u>the line y = 2</u> on the same grid. ---------- B1 The line y = 2 <u>meets the curve at 3 intersection points</u>, hence there are 3 solutions for the equation $\frac{x^3}{4}-3x-3=0$. ---------- B1	
4(d)		Drawing of tangent at $(3,-3.25)$ ---------- M1 Gradient = 3.75 (Accept 3 to 4) ---------- A1	

<table>
<tr>
<td>4(e)</td>
<td>(i)</td>
<td>$2y + x = 6$
$y = \frac{6 - x}{2}$

Take any 2 or 3 points to draw the above line.

<table>
<tr><td>x</td><td>-4</td><td>0</td><td>4</td></tr>
<tr><td>y</td><td>5</td><td>3</td><td>1</td></tr>
</table>

Drawing of line on the grid. Ensure that line is drawn for $-4 \leq x \leq 4$. ---------- B1</td>
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<td>(ii)</td>
<td>$\frac{x^3}{4} - 3x - 1 = \frac{6 - x}{2}$ ---------- M1
$x^3 - 12x - 4 = 12 - 2x$
$x^3 - 10x - 16 = 0$ ---------- A1</td>
<td>[11 marks]</td>
</tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr>
<td>5(a)</td>
<td></td>
<td>$\frac{11 + n}{44 + n} = \frac{8}{19}$ ---------- M1

$19(11 + n) = 8(44 + n)$
$209 + 19n = 352 + 8n$
$11n = 143$
$n = 13$ ---------- A1

Number of students in S2 = 31 ---------- B1</td>
<td></td>
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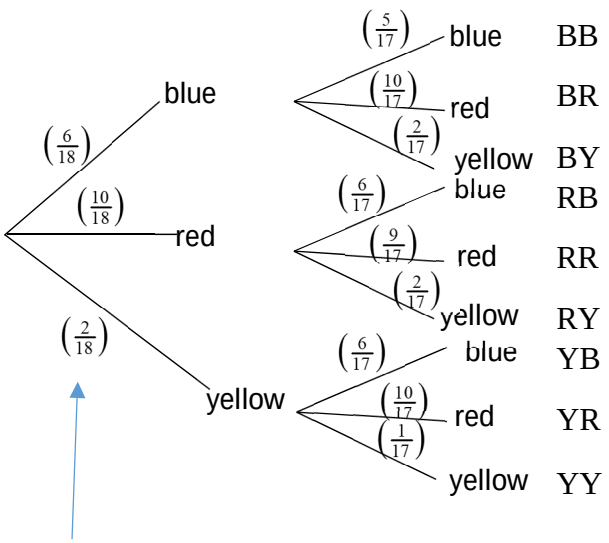
5(b)	(i)	First card / Second card  First branch correct ------- B1 All correct ---------- B1		
	(ii)	(a)	$\frac{6}{18}\times\frac{5}{17}$ $= \frac{5}{51}$ ---------- B1	
		(b)	$\frac{6}{18}\times\frac{10}{17}+\frac{10}{18}\times\frac{6}{17}$ ---------- M1 $= \frac{20}{51}$ ---------- A1	
		(c)	P(no red cards) = $\frac{8}{18}\times\frac{7}{17}$ ---------- M1 $= \frac{28}{153}$ P(at least one red card) = $1-\frac{28}{153}$ $= \frac{125}{153}$ ---------- A1 Alternative method $1-\left(\frac{6}{18}\times\frac{5}{17}+\frac{6}{18}\times\frac{2}{17}\right)-\left(\frac{2}{18}\times\frac{6}{17}+\frac{2}{18}\times\frac{1}{17}\right)$ ----- M1 $= \frac{125}{153}$ ---------- A1	[10 marks]

6	(a)	$\sin 40^\circ = \dfrac{6.8}{AC}$ $AC = \dfrac{6.8}{\sin 40^\circ}$ ---------- M1 $AC = 10.5789$ $AC = 10.58$ m (2 dp) ---------- A1		
	(b)	(i)	$AB = \sqrt{12.7^2 + 10.58^2 - 2(12.7)(10.58)\cos 66^\circ}$ ---- M1 $AB = 12.8032$ $AB = 12.8$ m (3sf) ---------- A1	Can use the given value of AC = 10.58 km
		(ii)	Area $= \frac{1}{2}(12.7)(10.58)\sin 66^\circ + \frac{1}{2}(10.58)(6.8)\sin 50^\circ$ --- M2 $= 88.930$ $=88.9$ m^2 ---------- A1	
	(c)	$\tan 8^\circ = \dfrac{h}{12.8032}$ $h = 12.8032\tan 8^\circ$ ---------- M1 $h = 1.7993$ $\tan\theta = \dfrac{h}{AC}$ $\theta = \tan^{-1}\left(\dfrac{12.8032\tan 8^\circ}{10.58}\right)$ ---------- M1 $\theta = 9.652^\circ$ Angle of depression is 9.7° ---------- A1		[10 marks]
7	(a)	$\mathbf{F} = \begin{pmatrix} 0.47 \\ 0.75 \\ 1.27 \end{pmatrix}$ ---------- B1		
	(b)	$\mathbf{T} = \begin{pmatrix} 16 & 16 & 18 \\ 20 & 21 & 8 \\ 20 & 17 & 0 \end{pmatrix}\begin{pmatrix} 0.47 \\ 0.75 \\ 1.27 \end{pmatrix}$ $= \begin{pmatrix} 7.52+12+22.86 \\ 9.4+15.75+10.16 \\ 9.4+12.75+0 \end{pmatrix}$		

		$= \begin{pmatrix} 42.38 \\ 35.31 \\ 22.15 \end{pmatrix}$ ---------- B1		
	(c)	42.38 represents the total transport cost of all the employees in shift 1 when they come for work from the nearest MRT station. 35.31 represents the total transport cost of all the employees in shift 2 when they come for work from the nearest MRT station. 22.15 represents the total transport cost of all the employees in shift 3 when they come for work from the nearest MRT station. ---------- B1		
	(d)	$\mathbf{N} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$ ---------- B1 $\mathbf{NT} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 42.38 \\ 35.31 \\ 22.15 \end{pmatrix}$ $= \begin{pmatrix} 99.84 \end{pmatrix}$ Total fare = \$99.84 ---------- B1		[5 marks]
8	(a)	(i)	$\angle OAC = \frac{2\pi}{5} \div 2 = \frac{\pi}{5}$ rad Angle at centre = 2 × angle at circumference ----- B1	
		(ii)	$AC^2 = 6^2 + 6^2 - 2(6)(6)\cos\left(\frac{3\pi}{5}\right)$ ---------- M1 AC = 9.7082 = 9.71 cm (3sf) ---------- A1	
		(iii)	M1 M1 Area of $OCE = \frac{1}{2}(9.7082)^2\left(\frac{\pi}{5}\right) - \frac{1}{2}(6)(6)\sin\left(\frac{3\pi}{5}\right)$ = 29.60927 – 17.119 = 12.4903 cm^2 Therefore, Area of shaded region = $\frac{1}{2}(6)^2\left(\frac{2\pi}{5}\right) - 12.4903$ -- M1 = 10.1 cm^2 (3sf) ---------- A1	

	(b)	(i)	$\angle OCB = 48°$ (radius perpendicular to tangent) --- M1 $\angle COB = 84°$ (triangle OBC is isosceles, angle sum of triangle) Therefore, $\angle BAC = 42°$ (angle at centre = 2 × angle at circumference) ---------- A1	– 1 mark for no reason / wrong reason(s)
		(ii)	$\angle ABC = 108°$ (angles in opposite segments) ----- M1 Therefore, $\angle BKC$ = 108 – 42 = 66° (exterior angle of triangle is sum of the 2 opposite interior angles) --- A1	[11 marks]
9	(a)	B1	$5 < h \leq 10$ | 26 $10 < h \leq 15$ | 27	
	(b)	(i)	11.3 m ---------- B1	Accept 11.2 m to 11.3 m
		(ii)	15 – 7.5 ---------- M1 = 7.5 m ---------- A1	
		(iii)	9 m ---------- B1	
	(c)		$\frac{80-75}{80} \times 100 = 6.25\%$ ---------- M1, A1	
	(d)	(i)	6.5 m ---------- B1	
		(ii)	Forest B has taller trees on the average as the median height is larger than the median height of Forest A. ---------- B1 Forest B has lesser spread as the interquartile range is smaller than the interquartile range of Forest A. B1	[10 marks]
10	(a)		Mass of CO produced per hour $= \left(1.83 \times 10^{20} \times 3600\right) \times 4.65 \times 10^{-26}$ ---------- M1 = 0.0306342 kg ---------- M1 $= 3.06342 \times 10^{-2}$ kg ---------- A1	
	(b)		3.06342×10^{-2} kg = $3.06342 \times 10^{-2} \times 10^{6}$ mg = 30634.2 mg ---------- MA1 Mass of CO produced per unit volume of air = $\frac{30634.2\,\text{mg}}{4500\,\text{m}^3}$ = 6.8076 mg/m^3 MA1	

		PPM = $\frac{6.8076\,mg/m^3}{0.811 mg/m^3}$ ---------- P1 = 8.39 ppm < 35 ppm Therefore, I do not agree. ---------- A1	
	(c)	35 ppm = 35 × 0.811 mg/m^3 = 28.385 mg/m^3 ---------- M1 Volume of air when presence of CO is considered dangerous = $\frac{30634.2\,mg}{28.385\,mg/m^3}$ = 1080 m^3 (3sf) ---------- A1 The assumption is that the total number of CO molecules produced in an hour remains unchanged. ---------- B1	[10 marks]