H

ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

CANDIDATE NAME

TUTORIAL/ FORM CLASS

INDEX NUMBER

MATHEMATICS 9758/01

Paper 1 **29 August 2019**

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of __ printed pages.

Anglo-Chinese Junior College **[Turn Over**

1 The points $A(2,-3)$ and $B(-3,1)$ are on a curve with equation $y = \mathrm{f}(x)$. The corresponding points on the curve $y = \mathrm{f}\big(a(x-b)\big)$ are $A'(7,-3)$ and $B'(-1,1)$. Find the values of a and b. [3]

2 Use differentiation to find the area of the largest rectangle with sides parallel to the coordinate axes, lying above the x-axis and below the curve with equation $y = 44 + 4x - x^2$. [5]

3 Solve the equation $\left|\dfrac{x^2+3x}{x-1}\right| = 2x+3$ exactly. [4]

Hence, by sketching appropriate graphs, solve the inequality $\left|\dfrac{x^2+3x}{x-1}\right| < 2x+3$ exactly. [2]

4 A kite 50 m above ground is being blown away from the person holding its string in a direction parallel to the ground at a rate 5 m per second. Assuming that the string is taut, at the instant when the length of the string already let out is 100 m, find, leaving your answers in exact form,

(i) the rate of change of the angle between the string and the ground, [3]

(ii) the rate at which the string of the kite should be let out, [4]

5 Given that $y = \tan\left(1-\mathrm{e}^{3x}\right)$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = k\mathrm{e}^{3x}\left(1+y^2\right)$, where k is a constant to be determined. By further differentiation of this result, or otherwise, find the first three non-zero terms in the Maclaurin series for $\tan\left(1-\mathrm{e}^{3x}\right)$. [5]

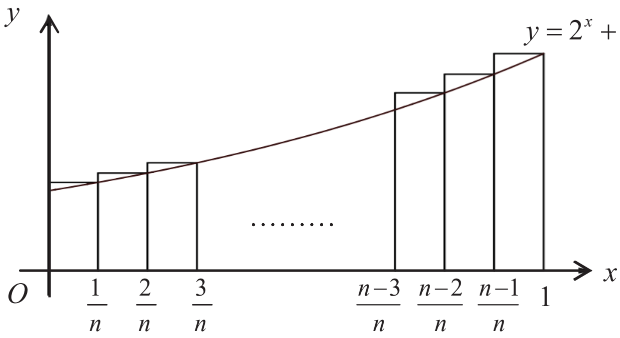
The first two terms in the Maclaurin series for $\tan\left(1-\mathrm{e}^{3x}\right)$ are equal to the first two non-zero terms in the series expansion of $\dfrac{x}{a+bx}$. Find the constants a and b. [3]

6 The diagram below shows the graph of $y = 2^x + 1$ for $0 \le x \le 1$. Rectangles, each of width $\frac{1}{n}$, are also drawn on the graph as shown.

Show that the total area of all n rectangles, S_n, is given by

$$S_n = \frac{2^{\frac{1}{n}}}{n(2^{\frac{1}{n}} - 1)} + 1.$$ [3]

Find the exact value of $\lim_{n\to\infty} S_n$. [2]



7 **(a)** Find $\int \sin px \cos qx \, dx$ where p and q are positive integers such that $p \ne q$. [2]

(b) Show that $\int x \sin nx \, dx = -\frac{x \cos nx}{n} + \frac{\sin nx}{n^2} + c$ where n is a positive integer and c is an arbitrary constant. [1]

Hence find

(i) $\int_0^{\pi} x \sin nx \, dx$, giving your answers in the form $\frac{k\pi}{n}$ where the possible values of k are to be determined, [2]

(ii) $\int_0^{\frac{\pi}{2}} |x \sin 3x| \, dx$ in terms of π. [3]

8 **Do not use a calculator in answering this question.**

(a) The complex numbers z and w satisfy the following equations

$$w-2z=9,$$
$$3w-wz^{*}=17-30\mathrm{i}.$$

Find w and z in the form $a+b\mathrm{i}$, where a and b are real and $\mathrm{Re}(z)<0$. [4]

(b) **(i)** Given that $-\mathrm{i}$ is a root of the equation

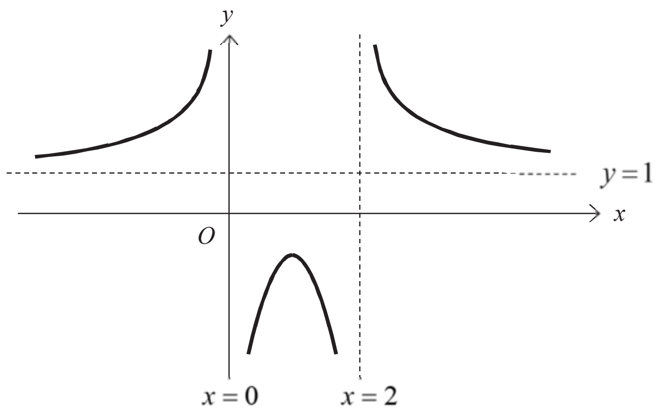
$$z^3+kz^2+\left(8+2\sqrt{2}\,\mathrm{i}\right)z+8\,\mathrm{i}=0,$$

where k is a constant to be determined, find the other roots, leaving your answers in exact cartesian form $x+y\mathrm{i}$, showing your working. [3]

(ii) Hence solve the equation $\mathrm{i}z^3+kz^2+\left(2\sqrt{2}-8\,\mathrm{i}\right)z-8\,\mathrm{i}=0$, leaving your answers in exact cartesian form. [2]

(iii) Let z_0 be the root in **(i)** such that $\arg(z_0)>0$. Find the smallest positive integer value of n such that $(\mathrm{i}z_0)^n$ is a purely imaginary number. [2]

9 **(a)** The diagram below shows the graph of $y=\dfrac{1}{\mathrm{f}(x)}$ with asymptotes $x=0,\ x=2,$ and $y=1$, and turning point $(1,-2)$.



(i) Given that $\mathrm{f}(0)=\mathrm{f}(2)=0$, sketch the graph of $y=\mathrm{f}(x)$, stating clearly the coordinates of any turning points and points of intersection with the axes, and the equations of any asymptotes. [3]

(ii) The function f is now defined for $x>k$ such that f^{-1} exists.
State the smallest value of k. On the same diagram, sketch the graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$, showing clearly the geometrical relationship between the two graphs. [3]

(b) The function g is defined for $x > 0$ as

$$g : x \mapsto 2^n x - 1, \quad \frac{1}{2^n} \le x < \frac{1}{2^{n-1}}, \text{ where } n \in \square .$$

(i) Fill in the blanks.

$$g(x) = \begin{cases} \boxed{}, & \frac{1}{4} \le x < \frac{1}{2} \\ \boxed{}, & \frac{1}{2} \le x < 1 \end{cases}$$

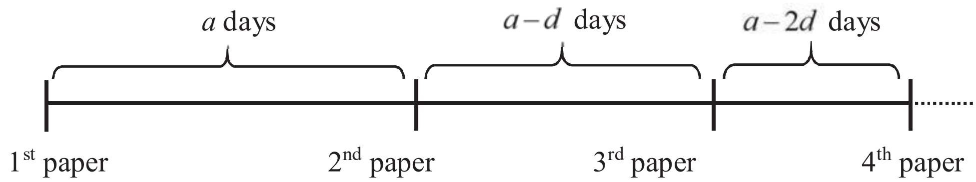
Hence sketch the graph $y = g(x)$ for $\frac{1}{4} \le x < 1$. [3]

(ii) Show that $g(x) = g\left(\frac{x}{2}\right)$. [2]

(iii) Find the number of solutions of $g(x) = x$ for $0.001 < x < 1$. [2]

10 David is preparing for an upcoming examination with 9 practice papers to complete in 90 days. The examination is on the 91st day. He is planning to spread out the practice papers according to the following criteria, and illustrated in the diagram below.

- He only completes 1 practice paper a day.
- He attempts the first practice paper on the first day.
- The duration between the first and the second practice paper is *a* days.
- The duration between each subsequent paper decreases by *d* days.
- He completes the last practice paper as close to the examination date as possible.



(i) By first writing down two inequalities in terms of *a* and *d*, determine the values of *a* and *d*. [4]

The mark for his *n*-th practice paper, u_n, can be modelled by the formula

$$u_n = 92 - 65(b)^n \text{ where } 0 < b < 1.$$

(ii) What is the significance of the number 92 in the formula? [1]

(iii) Find *m*, his average mark, for the nine practice papers he completed, leaving your answer in terms of *b*. [3]

(iv) Given that he scored higher than *m* from his fourth practice paper onwards, find the range of values of *b*. [2]

11 A toy paratrooper is dropped from a building and the attached parachute opens the moment it is released. The toy drops vertically and the distance it drops after t seconds is x metres. The motion of the toy can be modelled by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+k\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2=10,$$

where k is a constant.

By substituting velocity, $v=\frac{\mathrm{d}x}{\mathrm{d}t}$, write down a differential equation in v and t. [1]

Given that $\frac{\mathrm{d}v}{\mathrm{d}t}=6$ when $v=\sqrt{10}$, and that the initial velocity of the toy is zero, show that

$$v=\frac{5(1-\mathrm{e}^{-4t})}{1+\mathrm{e}^{-4t}},$$

and deduce the velocity of the toy in the long run. [6]

The toy is released from a height of 10 metres. Find the time it takes for the toy to reach the ground. [5]

12 In air traffic control, coordinates (x, y, z) are used to pinpoint the location of an aircraft in the sky within certain air space boundaries. In a particular airfield, the base of the control tower is at $(0,0,0)$ on the ground, which is the x-y plane. Assuming that the aircrafts fly in straight lines, two aircrafts, F_1 and F_2, fly along paths with equations

$$\mathbf{r}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}+\lambda(2\mathbf{i}-4\mathbf{j}+\mathbf{k}) \text{ and } x+2=\frac{y-1}{m}=\frac{3-z}{7}$$

respectively.

(i) What can be said about the value of m if the paths of the two aircrafts do not intersect? [3]

(ii) The signal detecting the aircrafts is the strongest when an aircraft is closest to the controller, who is in the control tower 3 units above the base. Find the distance of F_1 to the controller when the signal detecting it is the strongest. [3]

In a choreographed flying formation, the aircraft F_3 takes off from the point $(1,1,0)$ and flies in the direction parallel to $\mathbf{i}-\mathbf{k}$. The path taken by another aircraft, F_4, is the reflection of the path taken by F_2 along the path taken by F_3.

For the case when $m=5$, find

(iii) the cartesian equation of the plane containing all three flight paths. [2]

(iv) the vector equation of the line that describes the path taken by F_4. [4]

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H

Higher 2

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MATHEMATICS 9758/01

Paper 2 **3 September 2019**

3 hours

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This document consists of **5** printed pages.

Anglo-Chinese Junior College

[Turn Over

Section A: Pure Mathematics [40 marks]

1 Referred to the origin O, the points A and B have position vectors **a** and **b**, where **a** and **b** are not parallel, **b** is a unit vector, and $\angle AOB = 45^\circ$. The point R has position vector given by $\mathbf{r} = 3\mathbf{a} + 5\mathbf{b}$.
Find

(i) the position vector of the point where OR meets AB, [3]

(ii) the length of projection of $\overrightarrow{OR}$ on $\overrightarrow{OB}$, leaving your answer in terms of $|\mathbf{a}|$. [3]

2 By considering $\dfrac{1}{r(r-2)} - \dfrac{1}{r(r+2)}$, show that

$$\sum_{r=3}^{n} \frac{1}{(r-2)r(r+2)} = a + \frac{b}{(n-1)(n+1)} + \frac{c}{n(n+2)},$$

where a, b and c are constants to be determined. [4]

(i) State the value of $\displaystyle\sum_{r=3}^{\infty} \frac{1}{(r-2)r(r+2)}$. [1]

(ii) Find $\displaystyle\sum_{r=5}^{n} \frac{1}{r(r+2)(r+4)}$ in terms of n. [3]

3 **(a)** An ellipse has equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ where $b > a > 0$.

(i) Show that the area A of the region enclosed by the ellipse is given by

$$A = \frac{4b}{a}\int_0^a \sqrt{a^2 - x^2}\ \mathrm{d}x.$$ [1]

By using the substitution $x = a\sin\theta$, find A in terms of a, b and π. [4]

(ii) The region enclosed by the ellipse is rotated about the x-axis through π radians to form an ellipsoid. Find the volume of the ellipsoid formed, in terms of a, b and π. [3]

(b) When a continuous function, $y = \mathrm{f}(x)$, $a \le x \le b$, is rotated completely about the x-axis, the resulting curved surface area is given by

$$S = \int_a^b 2\pi y\sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\ \mathrm{d}x.$$

By considering the circle with equation $x^2 + y^2 = r^2$ where $-r \le x \le r$, show that the surface area of a sphere with radius r is $4\pi r^2$. [4]

4 The folium of Descartes is an algebraic curve defined by the equation

$$x^3 + y^3 = 3axy,$$

where a is a real constant.
Consider the curve when $a = 1$.
Show that by setting $y = xt$, where t is a parameter such that $t \in \mathbb{R}$,

$$x = \frac{3t}{1+t^3}, \quad t \neq k,$$

where k is a real number to be determined.
Hence write down the expression for y in terms of t. [3]

(i) Show that

$$x + y = \frac{3t}{t^2 - t + 1},$$

and, by considering limits or otherwise, find the equation of the oblique asymptote of the curve. [3]

(ii) Sketch the graph of the folium of Descartes when $a = 1$, showing clearly the coordinates of any intersection with the axes, and the equation of any asymptote(s). [2]

(iii) Find the equation of the tangent to the curve at $t = 2$, and determine if the tangent cuts the curve again, giving the coordinates of the intersection if it does. [6]

Section B: Probability and Statistics [60 marks]

5 A school tennis team comprises of 6 boys and 6 girls.

(i) Given that the members of the tennis team all have different heights, find the number of ways a group of 4 students of increasing heights can be chosen from the team. [1]

(ii) A delegation of at least one student from the team is to be chosen to attend a convention. Find the number of ways this delegation can be chosen. [2]

(iii) Andy, a boy, and Beth, a girl, are members of the tennis team. A group of 8 students from the team is to be chosen to attend a dinner, seated at a round table.
Find the probability that the boys and the girls in the group alternate such that the group includes Andy and Beth who are seated next to each other. [2]

6 A class proposes the following game for their school's fundraising event.
Four discs numbered '1', '3', '5' and '7' respectively are placed in Bag A. Another four discs numbered '2', '4', '6', and '8' respectively are placed in Bag B. A player chooses two discs at random and without replacement from each of the two bags. It is assumed that selections from the two bags are independent of each other.
Let X represent the difference between the numbers on the two discs drawn from Bag A, and Y represent the difference between the numbers on the two discs drawn from Bag B. The player wins \$$X$ if $X = Y$. Otherwise, the player wins nothing.

(i) Show that $\mathrm{P}(X = 2) = \frac{1}{2}$ and find the probability distribution of X. [3]

(ii) Find the probability that the player wins a prize. [2]

The game is to be played using a \$$k$ coupon, where k is a positive integer. Find the minimum value of k in order for the class to earn a profit for each game, justifying your answer. [2]

7 A concert promoter claims that the mean price of a ticket to a pop concert is \$200. A media company collected information on ticket price, \$$x$, for 50 randomly chosen people who bought pop concert tickets. The results are summarised as follows.

$$\sum(x-200) = 450 \qquad \sum(x-200)^2 = 55\ 000$$

(i) Test, at the 4% level of significance, the concert promoter's claim that the mean price of a ticket to a pop concert is \$200. You should state your hypotheses and define any symbols you use. [5]

(ii) The media company took another random sample of n tickets, and found that the average ticket price for this sample is \$206. If the standard deviation of ticket price is now known to be \$32.25, find the maximum value of n such that there is insufficient evidence at the 4% level of significance to reject the concert promoter's claim. [2]

8 A company that organizes live concerts believes that the popularity of an artiste affects his/her concert ticket sales. The popularity of an artiste can be measured by an index, x, such that $1 \le x \le 10$, where 10 indicates most popular and 1 indicates least popular. A study was conducted over six months to investigate the relationship between the popularity index of eight artistes and their concert ticket sales. The results are summarised in the following table.

Artiste	A	B	C	D	E	F	G	H
Popularity Index, x	1.2	2.0	2.7	3.8	4.8	5.6	6.9	8.0
Concert Ticket Sales (hundreds of thousands), \$$y$	2.2	4.5	5.8	7.3	7.4	9.0	9.9	10.8

(i) Draw a scatter diagram for these values, labelling the axes. [1]

(ii) It is thought that concert ticket sales y can be modelled by one of the formulae

$$y = ax^2 + b \qquad \text{or} \qquad y = c\ln x + d,$$

where a, b, c and d are positive constants.

Use your diagram in **(i)** to explain which of the two is a more appropriate model, and calculate its product moment correlation coefficient, correct to 4 decimal places. [2]

(iii) The data for a particular artiste appears to be recorded wrongly. Indicate the corresponding point on your diagram by labeling it P. Find the equation of the least squares regression line for the remaining points using the model that you have chosen in **(ii)**. [2]

(iv) The ticket sales for a new artiste is found to be \$800, 000. Estimate the popularity index of this artiste and comment on the reliability of your estimate. Explain why neither the regression line of $\ln x$ on y nor x^2 on y should be used. [3]

9 The teacher in-charge of the Harmonica Ensemble observed that a pair of twins often turned up late for practice. Attendance records show that the older twin, Albert, is late for practice 65% of the time. When the younger twin, Benny, is late for practice, Albert is also late 97.5% of the time. When Benny is not late for practice, Albert is late 56.875% of the time.

(i) Show that the probability that Benny is late for a practice is 0.2. [3]

(ii) Find the probability that only one of the twins will be late for the next practice. [2]

The teacher observed that another student, Carl, is late for practice 50% of the time. The probability that all three will be late for practice is 0.098. Given that the event of either twin being late for practice is independent of the event that Carl is late for practice, find the probability that neither the twins nor Carl is late for a practice. [3]

10 In this question you should state the parameters of any distributions you use.
Mary runs a noodle stall by herself. The noodles are prepared to order, so she prepares the noodles only after each order, and will only take the next customer's order after the previous customer is served his noodles.
The time taken for a customer to place an order at the stall follows a normal distribution with mean 60 s and standard deviation σ s. The time taken for Mary to prepare and serve a customer's order also follows a normal distribution, with mean 300 s and standard deviation 50 s.
The probability that a customer takes not more than 40 s to place an order is 0.16. Show that $\sigma = 20.11$, correct to 2 decimal places. [2]

(i) Let A be the probability that a randomly chosen customer takes between 57 s and 63 s to place an order with Mary, and B be the probability that a randomly chosen customer takes between 49 s and 55 s to place an order with Mary. Without calculating A and B, explain, with the aid of a diagram, how A and B compare with each other. [2]

(ii) There is a 0.1% chance that a randomly chosen customer has to wait for more than k seconds for his noodles to be served after placing his order. Find k. [1]

(iii) A man visits the noodle stall on 10 separate occasions. Find the probability that his average waiting time per visit after placing his order is less than 4.5 minutes. [2]

(iv) In an effort to shorten wait time, Mary improves the ordering and cooking processes such that the time taken for a customer to place an order is reduced by 5%, while the time taken to prepare and serve a customer his noodles is reduced by 10%. Find the largest number of customers she can serve in 1 hour for at least 80% of the time. State one assumption you made in your calculations. [5]

11 A factory manufactures a large number of wine glasses. It is found that on average, a proportion p of the wine glasses are chipped. A distributor purchases batches of wine glasses from the factory, each batch consisting of n wine glasses. A batch of wine glasses is rejected if it has more than 1 chipped wine glass. Let X be the number of wine glasses that are chipped in one batch. State two assumptions for X to be well-modelled by a binomial distribution. [2]
Show that A, the probability that one batch of wine glasses will not be rejected, is given by the formula

$$A = (1-p)^{n-1}\left[1+(n-1)p\right].$$ [2]

(a) Given that $p = 0.02$, and the probability that a batch of wine glasses is not rejected is at least 0.9, find the largest possible value of n. [2]

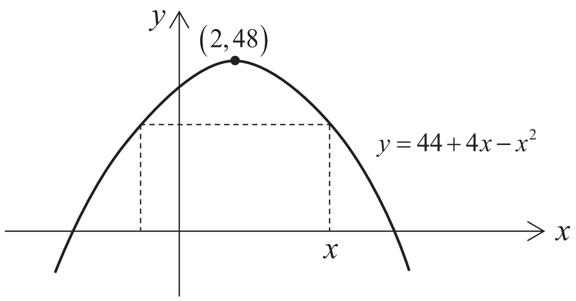
(b) The distributor also purchases wine glasses from another factory in batches of 40 and it is found that $A = 0.73131$.

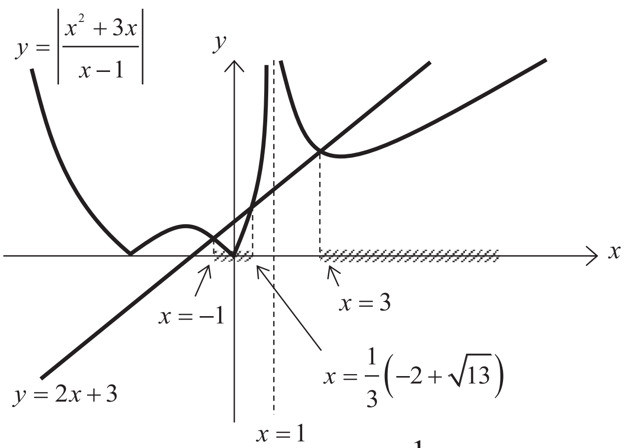
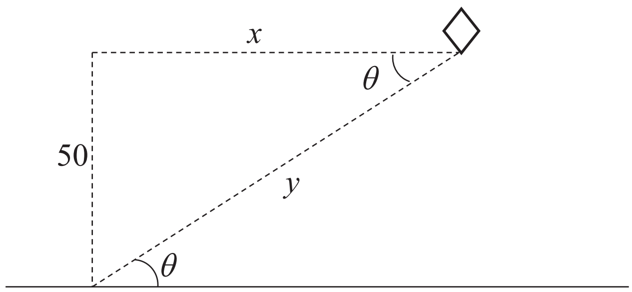
(i) Find p correct to 5 decimal places. [1]

(ii) The mean and standard deviation of X are denoted by μ and σ respectively. Find $\mathrm{P}\left(\mu-\sigma < X < \mu+\sigma\right)$. [2]

(iii) The distributor has a one-year contract with this factory such that every week, the factory produces 20 batches of wine glasses. According to the contract, the distributor will receive a compensation of \$100 for each batch of wine glasses it rejects every week. Assuming that there are 52 weeks in a year, find the probability that the total compensation in the one-year contract period is more than \$30 000. [4]

2019 ACJC H2 Math Prelim P1 Marker's Report

Qn	Solutions	Comments
1	$y=\mathrm{f}(x)\xrightarrow[\text{by factor } \frac{1}{a}]{\text{scaling // } x\text{-axis}} y=\mathrm{f}(ax)$ $\xrightarrow[\text{the positive } x \text{ direction}]{\text{translate } b \text{ units in}} y=\mathrm{f}(a(x-b))$ $A:(2,-3)\to\left(\frac{2}{a},-3\right)\to\left(\frac{2}{a}+b,-3\right)=(7,-3)$ $B:(-3,1)\to\left(-\frac{3}{a},1\right)\to\left(-\frac{3}{a}+b,1\right)=(-1,1)$ $\left.\begin{array}{l}\frac{2}{a}+b=7\\ -\frac{3}{a}+b=-1\end{array}\right\}$ solving gives $a=\frac{5}{8}, b=\frac{19}{5}$	**Badly done:** Common errors: (1) f(a(x – b)) is taken as translate b units in the positive x-direction then scale // x-axis by factor 1/a. Thus (2+b)/a = 7 and (-3 + b)/a = -1 were commonly seen. (2) Equations a(2-b) = 7 and a(-3-b) = -1 commonly seen. The correct equations should be a(7-b) = 2 and a(-1-b) = -3
2	 Area of rectangle, $A=2(x-2)y$ $=2(x-2)(44+4x-x^2)$ $\frac{\mathrm{d}A}{\mathrm{d}x}=0 \Rightarrow 2(x-2)(4-2x)+(44+4x-x^2)(2)=0$ i.e. $-6x^2+24x+72=0$ i.e. $x^2-4x-12=0$ Hence $x=-2$ or 6. Check that $\left.\frac{\mathrm{d}^2A}{\mathrm{d}x^2}\right\vert_{x=6}=-48<0$, therefore A is maximum when $x=6$. Maximum $A=2(6-2)(44+24-36)=256$ sq. units.	**Badly done** (1) Many assume that the area of the rectangle is xy or 2xy. (2) Some even differentiate y = 44 + 4x – x^2 to find the maximum value of y = 48 Thus area = 48 x 2 = 96 (3) Quite a number of candidates did not check that the area is maximum by checking the 2$^{\text{nd}}$ derivative
3	$\left\vert\frac{x^2+3x}{x-1}\right\vert=2x+3$ Note that for the equation to have any solution, $2x+3\geq 0\Rightarrow x\geq -\frac{3}{2}$.	Squaring both sides would lead to tedious working unless students were able to apply a^2-b^2. Another tedious method was to consider 4 different regions according to x = −3, 0, 1.

	$\frac{x^2+3x}{x-1}=2x+3$ $x^2+3x=(2x+3)(x-1)$ $x^2-2x-3=0$ $(x-3)(x+1)=0$ $\therefore x=3,-1$ $\frac{x^2+3x}{x-1}=-(2x+3)$ $x^2+3x=-(2x+3)(x-1)$ $3x^2+4x-3=0$ $x=\frac{-4\pm\sqrt{16-4(3)(-3)}}{2(3)}$ $=\frac{1}{3}\left(-2\pm\sqrt{13}\right)$ Reject $\frac{1}{3}\left(-2-\sqrt{13}\right)$(since $\approx -1.87<-1.5$) $\therefore x=-1, \frac{1}{3}\left(-2+\sqrt{13}\right)$ or 3.	Final answers need to be simplified: $\frac{\sqrt{52}}{6}=\frac{\sqrt{13}}{3}$ & $\frac{4}{6}=\frac{2}{3}$. Many students were not aware of the need to reject $\frac{1}{3}\left(-2-\sqrt{13}\right)$. Of those who rejected this negative root, few were able to provide reason that $x\geq -\frac{3}{2}$.
	 From graph, solution is $-1<x<\frac{1}{3}\left(-2+\sqrt{13}\right)$ or $x>3$	"Hence, **by sketching appropriate graphS**, solve…" was not followed: • $y=\left\|\frac{x^2+3x}{x-1}\right\|-2x-3$ was drawn instead. • Sign test was seen instead. • No graphs were seen in some scripts. Missing asymptote of $x = 1$ resulted in a pointed graph as shown in the GC.
4	 Given that $\frac{dx}{dt}=5$.	**Very badly done:** - wrong understanding of question. The rate of change given in the question is not the length of the string but the horizontal distance of the kite and the person. - many assume that the length of the string is constant at 100 and resulted in $\cos\theta=\frac{x}{100}$ and similar expressions.
(i)	From the diagram, $\tan\theta=\frac{50}{x}$. Hence, $x=\frac{50}{\tan\theta}=50\cot\theta$. Differentiating with respect to θ,	- There were many who did average rate of change, rather than instantaneous rate of change - when doing differentiation or integration, the angle is always in radians, a significant number left the answer in °/sec.

<table>
<tr>
<td></td>
<td>$\dfrac{dx}{d\theta} = -50\,\text{cosec}^2\theta$.

When $y = 100$, $\sin\theta = \dfrac{1}{2}$ and $\dfrac{dx}{dt} = 5$, hence
$\dfrac{dx}{dt} = \dfrac{dx}{d\theta}\cdot\dfrac{d\theta}{dt} \Rightarrow 5 = \left(-50\,\text{cosec}^2\theta\right)\cdot\dfrac{d\theta}{dt}$
$\Rightarrow 5 = \left(-50\cdot(2)^2\right)\cdot\dfrac{d\theta}{dt}$
$\Rightarrow \dfrac{d\theta}{dt} = \dfrac{5}{-50(4)} = -\dfrac{1}{40}$ rad s^{-1}.

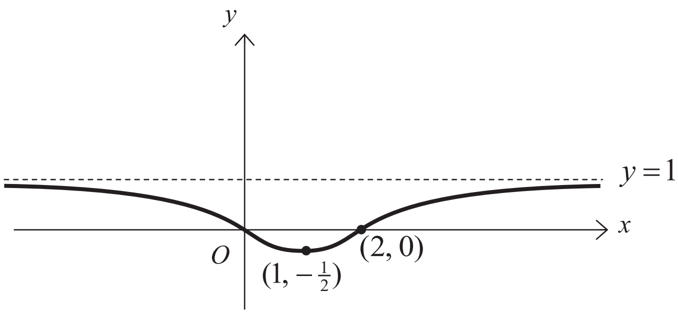
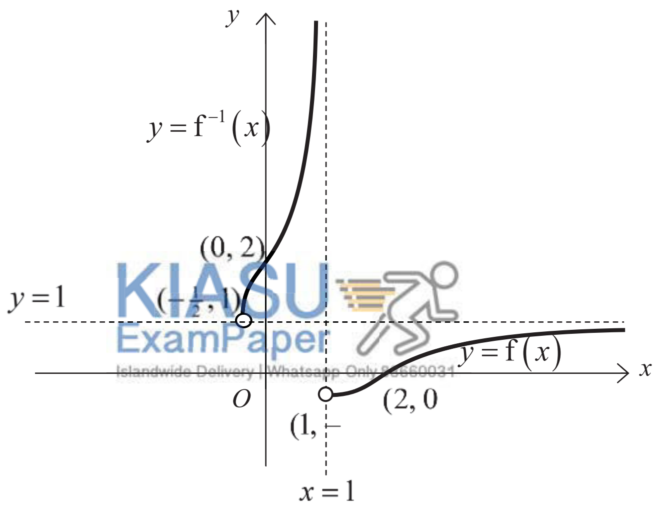
ALTERNATIVELY,
$\tan\theta = \dfrac{50}{x}$
At the instant when $y = 100$,
$x = \sqrt{100^2 - 50^2} = \sqrt{7500} = 50\sqrt{3}$, and $\theta = \sin^{-1}\dfrac{1}{2} = \dfrac{\pi}{6}$.
Differentiating (*) with respect to t,
$\sec^2\theta\dfrac{d\theta}{dt} = -\dfrac{50}{x^2}\dfrac{dx}{dt}$.
When $x = 50\sqrt{3}$, $\theta = \dfrac{\pi}{6}$ and $\dfrac{dx}{dt} = 5$,
$\sec^2\left(\dfrac{\pi}{6}\right)\cdot\dfrac{d\theta}{dt} = -\dfrac{50}{\left(50\sqrt{3}\right)^2}\cdot(5)$
$\Rightarrow \dfrac{d\theta}{dt} = -\dfrac{250}{(7500)\left(\dfrac{2}{\sqrt{3}}\right)^2} = -\dfrac{1}{40}$ rad s^{-1}.</td>
<td>- there are some who used
$\theta = \cos^{-1}\dfrac{x}{\sqrt{x^2+50^2}}$ or
$\theta = \sin^{-1}\dfrac{50}{\sqrt{x^2+50^2}}$ which will result in very tedious differentiation.
- many did not use chain rule when differentiating
$\theta = \tan^{-1}\dfrac{50}{x}$ and similar expressions, resulting in
$\dfrac{d\theta}{dx} = \dfrac{1}{1+\left(\frac{50}{x}\right)^2}$ which is wrong
- many students still do not know how to differentiate
sec, cot, cosec directly, resulting in unnecessary working</td>
</tr>
<tr>
<td>(ii)</td>
<td>Want to find $\dfrac{dy}{dt}$ when $y = 100$.
Now $x^2 + 50^2 = y^2 \Rightarrow y = \sqrt{x^2+50^2}$

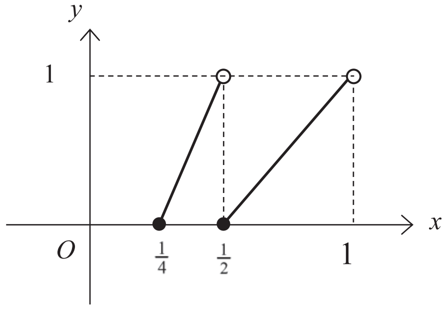
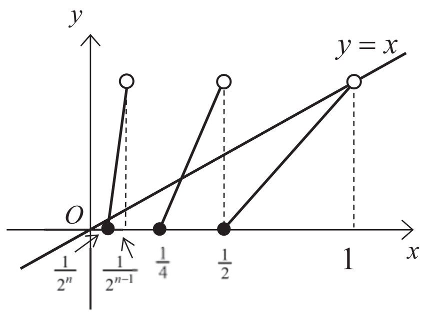
$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2}\left(x^2+50^2\right)^{-\frac{1}{2}}(2x) = \dfrac{x}{\sqrt{x^2+50^2}}$
By chain rule, $\dfrac{dy}{dt} = \dfrac{dy}{dx}\cdot\dfrac{dx}{dt}$.
Hence when $y = 100$,
$\dfrac{dy}{dt} = \dfrac{dy}{dx}\cdot\dfrac{dx}{dt}$
$\dfrac{dy}{dt} = \left(\dfrac{x}{\sqrt{x^2+50^2}}\right)\cdot 5 = \left(\dfrac{50\sqrt{3}}{\sqrt{\left(50\sqrt{3}\right)^2+50^2}}\right)\cdot 5 = \dfrac{5\sqrt{3}}{2}$ cm s^{-1}.</td>
<td>This part was left empty in most scripts.
For those who did this, many did not simplify the final answer.</td>
</tr>
</table>

5	$y=\tan\left(1-e^{3x}\right) \quad \Rightarrow \quad \tan^{-1} y = 1-e^{3x}$. Differentiating with respect to x, $\frac{1}{1+y^2}\frac{dy}{dx} = -3e^{3x}$ $\frac{dy}{dx} = -3e^{3x}\left(1+y^2\right)$. Hence $k=-3$. Differentiating again with respect to x, $\frac{d^2y}{dx^2} = -3\left(2y\frac{dy}{dx}\right)e^{3x} - 3\left(1+y^2\right)\left(3e^{3x}\right)$ $= -6y\frac{dy}{dx}e^{3x} - 9e^{3x}\left(1+y^2\right)$ $= -3e^{3x}\left(2y\frac{dy}{dx}+3+3y^2\right)$ $\frac{d^3y}{dx^3} = -3e^{3x}\left(2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2+6y\frac{dy}{dx}\right)-9e^{3x}\left(2y\frac{dy}{dx}+3+3y^2\right)$ When $x=0$, $y=0$, $\frac{dy}{dx}=-3$, $\frac{d^2y}{dx^2}=-9$ and $\frac{d^3y}{dx^3}=-81$. Hence, $y=\tan\left(1-e^{3x}\right)$ $=0-3x+\frac{(-9)}{2!}x^2+\frac{(-81)}{3!}x^3+\dots$ $\approx -3x-\frac{9}{2}x^2-\frac{27}{2}x^3$.	Most approach it this way: differentiate $\tan\left(1-e^{3x}\right)$ to get $-3e^{3x}\sec^2\left(1-e^{3x}\right)$ and use trigo identity to show k. A lot of complete and accurate work, as many of them make careless mistakes/slips: • $\frac{d}{dx}\left(1+y^2\right)=1+2y\frac{dy}{dx}$ • $\frac{d}{dx}\left(y\frac{dy}{dx}\right)=y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)$ And some did not use product rule: • $\frac{d}{dx}\left(y\frac{dy}{dx}\right)=\frac{dy}{dx}\frac{d^2y}{dx^2}$ Clearly taught to avoid quotient rule for implicit differentiation but some students still proceed in that direction.
	$\frac{x}{a+bx}=x(a+bx)^{-1}$ $=\frac{x}{a}\left(1+\frac{bx}{a}\right)^{-1}$ $=\frac{x}{a}\left(1-\frac{bx}{a}+\left(\frac{bx}{a}\right)^2+\dots\right)$ $\approx \frac{x}{a}-\frac{bx^2}{a^2}$ Hence $-3x-\frac{9}{2}x^2=\frac{x}{a}-\frac{bx^2}{a^2}$. Comparing coefficients, x: $\quad -3=\frac{1}{a} \quad \Rightarrow \quad a=-\frac{1}{3}$ x^2: $\quad -\frac{9}{2}=-\frac{b}{a^2} \quad \Rightarrow \quad b=\frac{1}{2}$	Instead of using binomial series to expand, more students differentiated twice, use Maclaurins again, compared coefficient of x with $f'(0)$, but many made mistake in comparing the coefficient of x^2 where they equated $f''(0)=-\frac{9}{2}$.

6	Total area of n rectangles $=\frac{1}{n}\left[(2^{\frac{1}{n}}+1)+(2^{\frac{2}{n}}+1)+...(2^{\frac{n-1}{n}}+1)+(2^1+1)\right]$ $=\frac{1}{n}\left[2^{\frac{1}{n}}+2^{\frac{2}{n}}+...2^{\frac{n-1}{n}}+2^1\right]+\frac{1}{n}\left[1+1+...1+1\right]$ $=\frac{1}{n}\sum_{r=1}^{n}2^{\frac{r}{n}}+1$ $=\frac{1}{n}\left(\frac{2^{\frac{1}{n}}(1-(2^{\frac{1}{n}})^n)}{1-2^{\frac{1}{n}}}\right)+1=\frac{2^{\frac{1}{n}}(-1)}{n(1-2^{\frac{1}{n}})}+1=\frac{2^{\frac{1}{n}}}{n(2^{\frac{1}{n}}-1)}+1$	Most students could identify the area of the r[th] rectangle having width $\frac{1}{n}$ and length $(2^{\frac{r}{n}}+1)$. Many fail to recognize that they should **split** the following sum: $\frac{1}{n}\sum_{r=1}^{n}(2^{\frac{r}{n}}+1)$ and proceed with S_{GP} and sum of constant. As this is an answer given question, students have to show the **full details** when applying the GP formula.
	$\lim_{n\to\infty}S_n=\int_0^1 2^x+1\,dx=\left[\frac{2^x}{\ln 2}+x\right]_0^1=\left(\frac{2^1}{\ln 2}+1\right)-\frac{1}{\ln 2}=\frac{1}{\ln 2}+1$	Most students aren't aware that the limit of the sum of the area of n rectangles is the area under curve. Many students concluded that $n(2^{\frac{1}{n}}-1)$ tends to zero when the table in the GC indicates that $n(2^{\frac{1}{n}}-1)$ tends to 0.69339. But since the question asked for exact answers, students can't use this method.
7(a)	$\int\sin px\cos qx\,dx=\frac{1}{2}\int\sin(p+q)x+\sin(p-q)x\,dx$ $=-\frac{\cos(p+q)x}{2(p+q)}-\frac{\cos(p-q)x}{2(p-q)}+c$	Half of the cohort not aware the need to use factor formula. They attempted to solve by parts.
(b)	$\int x\sin nx\,dx=x(-\frac{\cos nx}{n})-\int-\frac{\cos nx}{n}\,dx$ $=-\frac{x\cos nx}{n}+\int\frac{\cos nx}{n}\,dx=-\frac{x\cos nx}{n}+\frac{\sin nx}{n^2}+c$	Most able to use by parts correctly.
(b)(i)	$\int_0^{\pi}x\sin nx\,dx=\left[-\frac{x\cos nx}{n}+\frac{\sin nx}{n^2}\right]_0^{\pi}=-\frac{\pi\cos n\pi}{n}=\pm\frac{\pi}{n}$ $\therefore k=\pm 1$	Many have no idea what to do with $\cos n\pi$.

(b)(ii)	$\int_0^{\frac{\pi}{2}} \lvert x\sin 3x\rvert \,\mathrm{d}x = \int_0^{\frac{\pi}{3}} x\sin 3x\,\mathrm{d}x - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} x\sin 3x\,\mathrm{d}x$ $= \left[-\frac{x\cos 3x}{3}+\frac{\sin 3x}{9}\right]_0^{\frac{\pi}{3}} - \left[-\frac{x\cos 3x}{3}+\frac{\sin 3x}{9}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ $= -\frac{\pi\cos\pi}{9} - \left(-\frac{\pi\cos\frac{3\pi}{2}}{6}+\frac{\sin\frac{3\pi}{2}}{9}\right)+\left(-\frac{\pi\cos\pi}{9}\right)$ $=\frac{2\pi+1}{9}$	Some students used 1 instead of $\frac{\pi}{3}$. They gotten the number from GC.
8(a)	From $w-2z=9$, $w=9+2z$. Substitute into $3w-wz^*=17-30\mathrm{i}$: $3(9+2z)-(9+2z)z^*=17-30\mathrm{i}$ $\Rightarrow\ 27+6z-9z^*-2zz^*=17-30\mathrm{i}$ Let $z=a+b\mathrm{i}$, then $27+6(a+b\mathrm{i})-9(a-b\mathrm{i})-2(a+b\mathrm{i})(a-b\mathrm{i})=17-30\mathrm{i}$ i.e. $27+6a-9a-2a^2-2b^2+6b\mathrm{i}+9b\mathrm{i}=17-30\mathrm{i}$ i.e. $27-3a-2a^2-2b^2+15b\mathrm{i}=17-30\mathrm{i}$. Comparing coefficients, Imaginary: $15b=-30 \ \Rightarrow\ b=-2$ Real: $27-3a-2a^2-2b^2=17 \ \Rightarrow\ 2-3a-2a^2=0$ solving for a, $a=\frac{1}{2}$ or -2. Since $\mathrm{Re}(z)<0$, $a=-2$. Therefore, $z=-2-2\mathrm{i}$, and $w=9+2(-2-2\mathrm{i})=5-4\mathrm{i}$.	It is good practice to work to eliminate one variable when solving simultaneous equations before substituting $z=a+b\mathrm{i}$. Many students who started by using $z=a+b\mathrm{i}$ and $w=c+d\mathrm{i}$ made careless mistakes in their computation and were not successful in arriving at the correct answer.
8(b)(i)	$-\mathrm{i}$ is a root of the equation $z^3+kz^2+\left(8+2\sqrt{2}\,\mathrm{i}\right)z+8\,\mathrm{i}=0$ hence $(-\mathrm{i})^3+k(-\mathrm{i})^2+\left(8+2\sqrt{2}\,\mathrm{i}\right)(-\mathrm{i})+8\,\mathrm{i}=0$ $\mathrm{i}-k-8\mathrm{i}+2\sqrt{2}+8\,\mathrm{i}=0$ $\mathrm{i}-k+2\sqrt{2}=0$ $\therefore\ k=2\sqrt{2}+\mathrm{i}$. $z^3+\left(2\sqrt{2}+\mathrm{i}\right)z^2+\left(8+2\sqrt{2}\,\mathrm{i}\right)z+8\,\mathrm{i}=0$ $\Rightarrow\ (z+\mathrm{i})\left(z^2+bz+8\right)=0$ Comparing coefficients of z, $8+2\sqrt{2}\,\mathrm{i}=8+b\mathrm{i}$. Hence $2\sqrt{2}=b$. $(z+\mathrm{i})\left(z^2+2\sqrt{2}\,z+8\right)=0$ $\therefore\ z=-\mathrm{i}$ or $z=\frac{-2\sqrt{2}\pm\sqrt{8-32}}{2}=-\sqrt{2}\pm\sqrt{6}\,\mathrm{i}$. The other roots are $-\sqrt{2}+\sqrt{6}\,\mathrm{i}$ and $-\sqrt{2}-\sqrt{6}\,\mathrm{i}$.	The given equation is NOT a real polynomial as the coefficients are not all real numbers. Hence conjugate of $-\mathrm{i}$ is NOT a root. k is a constant does not mean it is a real number. In this case it is a complex constant. So when $\mathrm{i}-k+2\sqrt{2}=0$ $\Rightarrow\ k=2\sqrt{2}+\mathrm{i}$ Instead, some students **wrongly** proceeded to equate real and imaginary parts.

(b)(ii)	$\mathrm{i}z^3 + kz^2 + \left(2\sqrt{2} - 8\,\mathrm{i}\right)z - 8\,\mathrm{i} = 0$ $\Rightarrow \quad -(\mathrm{i}z)^3 - k(\mathrm{i}z)^2 - \left(8 + 2\sqrt{2}\,\mathrm{i}\right)(\mathrm{i}z) - 8\,\mathrm{i} = 0$ i.e. $\quad (\mathrm{i}z)^3 + k(\mathrm{i}z)^2 + \left(8 + 2\sqrt{2}\,\mathrm{i}\right)(\mathrm{i}z) + 8\,\mathrm{i} = 0$. Hence from **(i),** $\mathrm{i}z = -\mathrm{i}, \quad -\sqrt{2} + \sqrt{6}\,\mathrm{i}, \quad$ or $\quad -\sqrt{2} - \sqrt{6}\,\mathrm{i}$ $\therefore \quad z = -1, \quad -\sqrt{6} + \sqrt{2}\,\mathrm{i}, \quad$ or $\quad \sqrt{6} + \sqrt{2}\,\mathrm{i}$.	Not well done. A substitution is needed here.
(b)(iii)	$z_0 = -\sqrt{2} + \sqrt{6}\,\mathrm{i}$ $\arg z_0 = \dfrac{2\pi}{3}$ For $(\mathrm{i}z_0)^n$ to be purely imaginary, $\arg(\mathrm{i}z_0)^n = \pm\dfrac{\pi}{2} \quad \Rightarrow \quad n\arg(\mathrm{i}z_0) = \pm\dfrac{k\pi}{2}$ where k is odd i.e. $\quad n\left[\arg \mathrm{i} + \arg z_0\right] = \pm\dfrac{k\pi}{2}$ i.e. $\quad n\left[\dfrac{\pi}{2} + \dfrac{2\pi}{3}\right] = \pm\dfrac{k\pi}{2}$ i.e. $\quad n\left[-\dfrac{5\pi}{6}\right] = \pm\dfrac{k\pi}{2}$ Hence smallest positive integer value of n is 3.	Not well done. Some errors in the method to find argument of a complex number.
9(ai)		Generally well done, except some students who totally do not know how to sketch reciprocal graph. Common errors are - both tails tend to infinity - left tail tends to infinity - wrong y-value for minimum point
(a)(ii)	$k = 1$ 	Some did not get the mark $k = 1$ even though their (i) is correct. Weird! Common mistakes for $y = \mathrm{f}^{-1}(x)$ graph - draw $y = \mathrm{f}'(x)$ instead - the left end points are on the respective asymptotes - missing labelling of points, especially the y-intercept - the point $(1, -\frac{1}{2})$ becomes $(-1, \frac{1}{2})$ - missing vertical asymptote

(b)(i)	$\mathrm{g}(x)=\begin{cases}\boxed{4x-1}, & \frac{1}{4}\le x<\frac{1}{2}\\ \boxed{2x-1}, & \frac{1}{2}\le x<\frac{1}{4}\end{cases}$ 	Most students get the 2 marks if they attempt it Common mistakes in graph - the y-value of both end points are at 1, so they should reach the same height - scale on the x-axis, there were a significant number who drew the same width for both pieces - swap the rule for the domains
(b)(ii)	$\mathrm{g}(x)=2^n x-1,\ \frac{1}{2^n}\le x<\frac{1}{2^{n-1}}$ $\mathrm{g}\left(\frac{x}{2}\right)=2^n\left(\frac{x}{2}\right)-1,\ \frac{1}{2^n}\le\frac{x}{2}<\frac{1}{2^{n-1}}$ $=2^{n-1}x-1,\ \frac{1}{2^{n-1}}\le x<\frac{1}{2^{n-2}}$ $=2^k x-1,\ \frac{1}{2^k}\le x<\frac{1}{2^{k-1}}$ (optional) $=\mathrm{g}(x)$	About half did not attempt this. Most who attempted it got partial marks. Partial marks were given if - Obtain the rule for the general case in simplified form - obtain the rule for the case in **(i)** with the correct domain
(b)(iii)	Consider $\frac{1}{2^n}<0.001<\frac{1}{2^{n-1}}$ $n>\frac{\ln 0.001}{\ln\left(\frac{1}{2}\right)}=9.97$ $\therefore \frac{1}{2^{10}}<0.001<\frac{1}{2^9}$  When $n=10$, $\mathrm{g}(x)=2^{10}x-1,\ \frac{1}{2^{10}}\le x<\frac{1}{2^9}$. Solving $\mathrm{g}(x)=x$ when $n=10$, $2^{10}x-1=x \Rightarrow x=0.000978<0.001$. Hence there is no solution when $n=10$. There are therefore **8** solutions (since n =1 also has has no solution).	Most students assume that the solution is in the region of the graph drawn in **b(i)**. Few who attempted this got the correct answer, which is fine.
10(i)	As there are 9 papers, there are 8 durations in between the papers.	Very few students manage to write down **both** inequalities correctly. Many students wrote $S_8\le 90$ as they may have miss out on the fact that the 1st practice paper is

	$S_8 = \frac{8}{2}\left(2a+(8-1)(-d)\right) < 90$ $\Rightarrow 8a - 28d < 90$ $\Rightarrow a < 11.25 + 3.5d$ $T_8 = a + (8-1)(-d) > 0$ $\Rightarrow a > 7d$ For the last paper to be as close to the exam date as possible, a and S_8 must be as large as possible, By trial and error, $d = 1, a < 14.75 \Rightarrow a = 14 (> 7(1))$ and $S_8 = 84$ $d = 2, a < 18.25 \Rightarrow a = 18 (> 7(2))$ and $S_8 = 88$ $d = 3, a < 21.75 \Rightarrow a = 21 (\not> 7(3))$ Therefore $d = 2, a = 18$.	already attempted on the 1st day. By writing $S_8 \le 90$, this implies that the 9th practice paper could be on the 91st day (examination day) which is not what the question wants. Quite a number of student also wrote $T_8 \ge 0$. This is incorrect as $T_8 \ge 0$ means that it is possible for the 8th and 9th practice paper to be done on the same day which is also not what the question wants. Even fewer students realised of the need to determine the values of a and d by trial and error. Quite a number of students attempt to "solve" the 2 inequalities by treating them as "equations" and attempting to "solve" them "simultaneously" which is incorrect.
10(ii)	92 is the highest (theoretical) mark that he will get even if he practise many many times. OR 92 is the highest (theoretical) mark that he will get based on his aptitude and ability.	This part was quite well attempted as students generally know the significance of the number 92. However, their answer can be improved on by being clearer in stating the reason why 92 is the highest mark David will get.
10(iii)	$m = \frac{1}{9}\sum_{n=1}^{9} u_n = \frac{1}{9}\sum_{n=1}^{9}\left(92 - 65(b^n)\right)$ $= \frac{1}{9}\left(92(9) - 65\sum_{n=1}^{9} b^n\right)$ $= 92 - \frac{65}{9}\left(\frac{b(1-b^9)}{1-b}\right)$	Many students assume wrongly that $\sum_{n=1}^{9} u_n$ is an AP and took the first term as $\left(92 - 65(b)\right)$ and the last term as $\left(92 - 65(b^9)\right)$ which are incorrect. Quite a number of student could not recall the S_n of GP correctly.
10(iv)	As he scored higher than m from his 4th paper onwards,	Many students could write the inequality but could not solve as they did not realise that GC can be used to help them solve.

	$92-\frac{65}{9}\left(\frac{b(1-b^9)}{1-b}\right)<92-65(b^4)$ $\Rightarrow 1-b^9>9b^3(1-b)$ $\Rightarrow b^9-9b^4+9b^3-1<0$ From GC, $0<b<0.726$	
11	$\frac{d^2x}{dt^2}+k\left(\frac{dx}{dt}\right)^2=10$ Substitute $v=\frac{dx}{dt}$ and $\frac{dv}{dt}=\frac{d^2x}{dt^2}$ into DE, $\therefore \frac{dv}{dt}+kv^2=10$	Majority are able to get the differential equation. A handful of students left blank. Many students misinterpreted the question as $v=\frac{x}{t}$.
	when $v=\sqrt{10}$, $\frac{dv}{dt}=6$ $6+k(\sqrt{10})^2=10 \therefore k=0.4$ $\frac{dv}{dt}=10-0.4v^2$ $\int \frac{1}{10-0.4v^2}\,dv=\int dt$ $\frac{1}{0.4}\int \frac{1}{25-v^2}\,dv=t+c$ $\frac{1}{0.4}\left(\frac{1}{2(5)}\right)\ln\left\|\frac{5+v}{5-v}\right\|=t+c$ $\frac{1}{4}\ln\left\|\frac{5+v}{5-v}\right\|=t+c$ $\ln\left\|\frac{5+v}{5-v}\right\|=4t+d, \quad d=4c$ $\frac{5+v}{5-v}=Ae^{4t}, \quad A=\pm e^d$ $\frac{5-v}{5+v}=Be^{-4t}, \quad B=\frac{1}{A}$ when $t=0,\ v=0 \ \therefore A=1$ $\frac{5-v}{5+v}=e^{-4t}$ $5-v=(5+v)e^{-4t}$ $\therefore v=\frac{5(1-e^{-4t})}{1+e^{-4t}}$ As $t\to\infty$, $v\to 5$ ms^{-1}	Those who managed to obtain $\frac{dv}{dt}+kv^2=10$, are able to get $k=0.4$ easily. With $\frac{dv}{dt}+kv^2=10$, majority knew how to separate the variables. However, only a handful include modulus. A common mistake made is $\int \frac{5}{50-2v^2}\,dv$ $=\frac{5}{2(\sqrt{50})}\ln\left\|\frac{\sqrt{50}+\sqrt{2}v}{\sqrt{50}-\sqrt{2}v}\right\|$. Students did not realise that coefficient of v^2 must be 1 if they are applying the same formula from MF26. Students have no idea why $\frac{5(e^{4t}-1)}{e^{4t}+1}=\frac{5(1-e^{-4t})}{1+e^{-4t}}$. Working must be shown explicitly. Students didn't read question. At least a quarter of the cohort missed this part. Those who attempted this part managed to answer correctly.

	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{5(1-\mathrm{e}^{-4t})}{1+\mathrm{e}^{-4t}}$ $x=\int\frac{5(1-\mathrm{e}^{-4t})}{1+\mathrm{e}^{-4t}}\,\mathrm{d}t$ $=5\int\frac{1}{1+\mathrm{e}^{-4t}}-\frac{\mathrm{e}^{-4t}}{1+\mathrm{e}^{-4t}}\,\mathrm{d}t=5\int\frac{\mathrm{e}^{4t}}{1+\mathrm{e}^{4t}}-\frac{\mathrm{e}^{-4t}}{1+\mathrm{e}^{-4t}}\,\mathrm{d}t$ $=\frac{5}{4}\left[\ln(1+\mathrm{e}^{4t})+\ln(1+\mathrm{e}^{-4t})\right]+c$ when $t=0$, $x=0$ $\therefore c=-2.5\ln 2$ Hence, $x=\frac{5}{4}\left[\ln(1+\mathrm{e}^{4t})+\ln(1+\mathrm{e}^{-4t})\right]-\frac{5}{2}\ln 2$. From G.C., when $x=10$, $t=2.3465\approx 2.35$ s Alternatively, $\int_0^t\frac{5(1-\mathrm{e}^{-4t})}{1+\mathrm{e}^{-4t}}\,\mathrm{d}t=10$ From G.C., $t=2.3465\approx 2.35$ s	Badly done. Many students did not attempt this part. Those who attempted, majority have no idea that the question is solving for the particular solution of x. Many students differentiated $\frac{5(1-\mathrm{e}^{-4t})}{1+\mathrm{e}^{-4t}}$. There is still a handful of students who got $x=\int\frac{5(1-\mathrm{e}^{-4t})}{1+\mathrm{e}^{-4t}}\,\mathrm{d}t$. However, most of them stuck at $\int\frac{1}{1+\mathrm{e}^{-4t}}\,\mathrm{d}t$, which many mistook as $\ln\left(1+\mathrm{e}^{-4t}\right)$ or tangent inverse. Some students interpreted the question wrongly as "when $t=0$, $x=10$". There are some impressive solutions, but a handful of them didn't realise that they need to make use of G.C to solve.
12(i)	F_1: $\mathbf{r}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}+\lambda(2\mathbf{i}-4\mathbf{j}+\mathbf{k})$ F_2: $\mathbf{r}=-2\mathbf{i}+\mathbf{j}+3\mathbf{k}+\mu(\mathbf{i}+m\mathbf{j}-7\mathbf{k})$ Since $\begin{pmatrix}2\\-4\\1\end{pmatrix}\neq k\begin{pmatrix}1\\m\\-7\end{pmatrix}$, the paths cannot be parallel. If the paths intersect, $\begin{pmatrix}1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}2\\-4\\1\end{pmatrix}=\begin{pmatrix}-2\\1\\3\end{pmatrix}+\mu\begin{pmatrix}1\\m\\-7\end{pmatrix}$ $2\lambda-\mu=-3$ .... (1) $\lambda+7\mu=0$ .... (2) $4\lambda+m\mu=1$ .... (3) Solving (1) and (2), $\lambda=-\frac{7}{5}$, $\mu=\frac{1}{5}$ From (3), $m=33$ Since the paths do not intersect, they are skew lines $\therefore m\neq 33$	Shocking that a significant number of students did not know how or made mistakes/slips when converting from Cartesian to vector form. For non-intersecting lines, many considered parallel lines and concluded that m is not multiples of 4. A lot of students did not even consider case of skew lines.
(ii)	Signal is located at $S(0, 0, 3)$. <u>Method 1:</u> Let $A(1,2,3)$ be a point on F_1. Then $\overrightarrow{AS}=\begin{pmatrix}0\\0\\3\end{pmatrix}-\begin{pmatrix}1\\2\\3\end{pmatrix}=\begin{pmatrix}-1\\-2\\0\end{pmatrix}$	Many students did not write the position of signal correctly, <u>base</u> of control towel at (0, 0, 0) (info given earlier and found on a different page).

	Perpendicular dist $=\left\|\overrightarrow{AS}\times\hat{\mathbf{b}}\right\|$ $=\left\|\begin{pmatrix}-1\\-2\\0\end{pmatrix}\times\frac{1}{\sqrt{21}}\begin{pmatrix}2\\-4\\1\end{pmatrix}\right\|=\frac{1}{\sqrt{21}}\left\|\begin{pmatrix}-2\\1\\8\end{pmatrix}\right\|$ $=\frac{\sqrt{(-2)^2+1+8^2}}{\sqrt{21}}=\sqrt{\frac{23}{7}}$ Method 2: $\left\|\overrightarrow{AF}\right\|=\left\|\overrightarrow{AS}.\hat{\mathbf{b}}\right\|=\left\|\begin{pmatrix}-1\\-2\\0\end{pmatrix}.\frac{1}{\sqrt{21}}\begin{pmatrix}2\\-4\\1\end{pmatrix}\right\|=\frac{6}{\sqrt{21}}$ $SF=\sqrt{AS^2-AF^2}=\sqrt{(\sqrt{5})^2-(\frac{6}{\sqrt{21}})^2}=\sqrt{\frac{23}{7}}$ Method 3: Let $\overrightarrow{OF}=\begin{pmatrix}1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}2\\-4\\1\end{pmatrix}$ for some λ. $\overrightarrow{SF}=\begin{pmatrix}1+2\lambda\\2-4\lambda\\3+\lambda\end{pmatrix}-\begin{pmatrix}0\\0\\3\end{pmatrix}=\begin{pmatrix}1+2\lambda\\2-4\lambda\\\lambda\end{pmatrix}$ $\overrightarrow{SF}.\mathbf{b}=0$ $\begin{pmatrix}1+2\lambda\\2-4\lambda\\\lambda\end{pmatrix}.\begin{pmatrix}2\\-4\\1\end{pmatrix}=0\Rightarrow 2(1+2\lambda)-4(2-4\lambda)+\lambda=0\therefore\lambda=\frac{2}{7}$ $\overrightarrow{SF}=\begin{pmatrix}1+2(2/7)\\2-4(2/7)\\2/7\end{pmatrix}=\frac{1}{7}\begin{pmatrix}11\\6\\2\end{pmatrix}$ $\left\|\overrightarrow{SF}\right\|=\frac{\sqrt{11^2+6^2+2^2}}{7}=\frac{\sqrt{161}}{7}$ Method 4: $\left\|\overrightarrow{SF}\right\|=\left\|\begin{pmatrix}1+2\lambda\\2-4\lambda\\3+\lambda\end{pmatrix}-\begin{pmatrix}0\\0\\3\end{pmatrix}\right\|=\left\|\begin{pmatrix}1+2\lambda\\2-4\lambda\\\lambda\end{pmatrix}\right\|$ $=\sqrt{(1+2\lambda)^2+(2-4\lambda)^2+\lambda^2}$ $=\sqrt{5-12\lambda+21\lambda^2}$ $=\sqrt{\frac{23}{7}+21(\lambda-\frac{6}{21})^2}$ $\therefore$ shortest dist $=\sqrt{\frac{23}{7}}$	

(iii)	$F_2 : \mathbf{r} = \begin{pmatrix}-2\\1\\3\end{pmatrix} + \mu\begin{pmatrix}1\\5\\-7\end{pmatrix}$ $F_3 : \mathbf{r} = \begin{pmatrix}1\\1\\0\end{pmatrix} + \alpha\begin{pmatrix}1\\0\\-1\end{pmatrix}$ $\mathbf{n} = \begin{pmatrix}1\\0\\-1\end{pmatrix} \times \begin{pmatrix}1\\5\\-7\end{pmatrix} = \begin{pmatrix}5\\6\\5\end{pmatrix}$ $\mathbf{a.n} = \begin{pmatrix}1\\1\\0\end{pmatrix} . \begin{pmatrix}5\\6\\5\end{pmatrix} = 11$ $\therefore \mathbf{r.}\begin{pmatrix}5\\6\\5\end{pmatrix} = 11 \Rightarrow 5x + 6y + 5z = 11$	Many students did not know what **cartesian** form "$ax+by+cz=p$" of equation of plane is and left answer in parametric form: $\mathbf{r} = \begin{pmatrix}1\\1\\0\end{pmatrix} + \alpha\begin{pmatrix}1\\0\\-1\end{pmatrix} + \mu\begin{pmatrix}1\\5\\-7\end{pmatrix}$.
(iv)	When F_2 and F_3 intersect, $\begin{pmatrix}-2+\mu\\1+5\mu\\3-7\mu\end{pmatrix} = \begin{pmatrix}1+\alpha\\1\\-\alpha\end{pmatrix} \quad \therefore \mu = 0, \alpha = -3$ ie. $\begin{pmatrix}-2\\1\\3\end{pmatrix}$ Let B be another point on F_2 ($\mu = 1$). $\overrightarrow{OB} = \begin{pmatrix}-2+1\\1+5(1)\\3-7(1)\end{pmatrix} = \begin{pmatrix}-1\\6\\-4\end{pmatrix}$ Find p.v. of foot of perpendicular to F_3: <u>Method 1:</u> Let $\overrightarrow{ON} = \begin{pmatrix}1\\1\\0\end{pmatrix} + \alpha\begin{pmatrix}1\\0\\-1\end{pmatrix}$ for some α. $\overrightarrow{BN} = \begin{pmatrix}1+\alpha\\1\\-\alpha\end{pmatrix} - \begin{pmatrix}-1\\6\\-4\end{pmatrix} = \begin{pmatrix}2+\alpha\\-5\\4-\alpha\end{pmatrix}$ $\overrightarrow{BN}\mathbf{.b} = 0$ $\begin{pmatrix}2+\alpha\\-5\\4-\alpha\end{pmatrix} . \begin{pmatrix}1\\0\\-1\end{pmatrix} = 0 \Rightarrow (2+\alpha) - (4-\alpha) = 0 \therefore \alpha = 1$ $\therefore \overrightarrow{ON} = \begin{pmatrix}2\\1\\-1\end{pmatrix}$ $\overrightarrow{ON} = \dfrac{\overrightarrow{OB} + \overrightarrow{OB'}}{2} \Rightarrow \overrightarrow{OB'} = 2\overrightarrow{ON} - \overrightarrow{OB}$ $\overrightarrow{OB'} = 2\begin{pmatrix}2\\1\\-1\end{pmatrix} - \begin{pmatrix}-1\\6\\-4\end{pmatrix} = \begin{pmatrix}5\\-4\\2\end{pmatrix}$ $\overrightarrow{PB'} = \begin{pmatrix}5\\-4\\2\end{pmatrix} - \begin{pmatrix}-2\\1\\3\end{pmatrix} = \begin{pmatrix}7\\-5\\-1\end{pmatrix} \quad \therefore F_4 : \mathbf{r} = \begin{pmatrix}-2\\1\\3\end{pmatrix} + \mu\begin{pmatrix}7\\-5\\-1\end{pmatrix}$ <u>Method 2:</u>	Most students just gave up. Out of those who did not, many used the intersection of F_2 and F_3 (−2, 1, 3) to find foot of perpendicular, which failed.

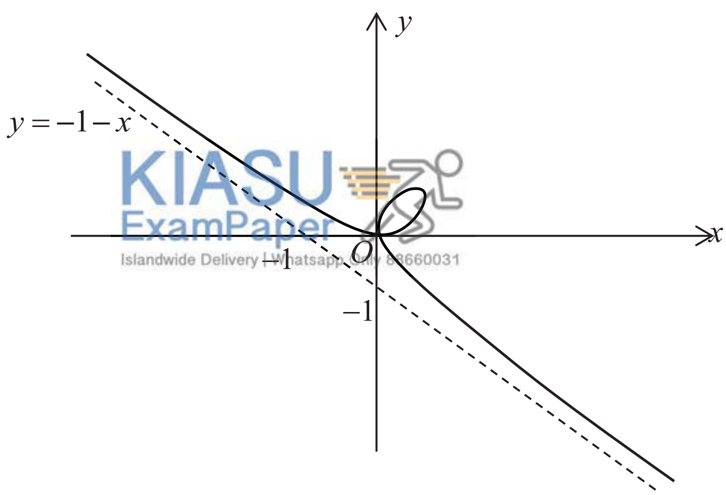
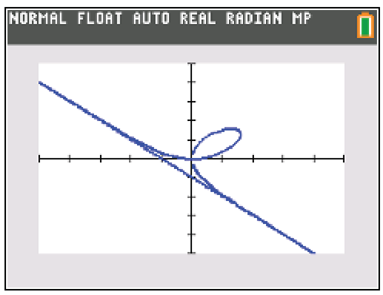
	$\begin{pmatrix}-1\\6\\-4\end{pmatrix}-\begin{pmatrix}-2\\1\\3\end{pmatrix}=\begin{pmatrix}1\\5\\-7\end{pmatrix}$ $\overrightarrow{PN}=\left(\overrightarrow{PB}.\hat{\mathbf{b}}\right).\hat{\mathbf{b}}=\left(\begin{pmatrix}1\\5\\-7\end{pmatrix}.\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix}\right).\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix}=4\begin{pmatrix}1\\0\\-1\end{pmatrix}$ $\overrightarrow{PN}=\frac{\overrightarrow{PB}+\overrightarrow{PB'}}{2}\Rightarrow\overrightarrow{PB'}=2\overrightarrow{PN}-\overrightarrow{PB}$ $\overrightarrow{PB'}=8\begin{pmatrix}1\\0\\-1\end{pmatrix}-\begin{pmatrix}1\\5\\-7\end{pmatrix}=\begin{pmatrix}7\\-5\\-1\end{pmatrix}$ $\therefore F_4:\mathbf{r}=\begin{pmatrix}-2\\1\\3\end{pmatrix}+\mu\begin{pmatrix}7\\-5\\-1\end{pmatrix}$	

2019 JC2 H2 Prelim Exam P2 Markers Report

1(i)	$k(3\mathbf{a}+5\mathbf{b})=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})$ Comparing coefficient of **a** and **b**, $\left.\begin{array}{l}3k=1-\lambda\\ 5k=\lambda\end{array}\right\} \therefore \lambda=\frac{5}{8},\ k=\frac{1}{8}$ $\therefore$p.v of the point of intersection is $\frac{3}{8}\mathbf{a}+\frac{5}{8}\mathbf{b}$.	Some made the mistake of using vector AB as the line equation Line AB is $r=a+\lambda(b-a)$ not $r=b-a$ Some assumed that the intersection point is R, making the mistake of equating $(3\mathbf{a}+5\mathbf{b})=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})$ when it should have been $k(3\mathbf{a}+5\mathbf{b})=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})$
(ii)	$\mathbf{a}\cdot\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos 45^o=\frac{\lvert\mathbf{a}\rvert}{\sqrt{2}}$ $\lvert\overrightarrow{OR}\cdot\hat{\mathbf{b}}\rvert=\lvert(3\mathbf{a}+5\mathbf{b})\cdot\mathbf{b}\rvert=\lvert 3\mathbf{a}\cdot\mathbf{b}+5\mathbf{b}\cdot\mathbf{b}\rvert$ $=(3\mathbf{a}\cdot\mathbf{b}+5\lvert\mathbf{b}\rvert^2)=\frac{3\lvert\mathbf{a}\rvert}{\sqrt{2}}+5$	Some wrote the dot product correctly in (i) but didn't put it to use in any way.
2	$\frac{1}{r(r-2)}-\frac{1}{r(r+2)}=\frac{(r+2)-(r-2)}{(r-2)r(r+2)}=\frac{4}{(r-2)r(r+2)}$ $\sum_{r=3}^{n}\frac{1}{(r-2)r(r+2)}=\frac{1}{4}\sum_{r=3}^{n}\left(\frac{1}{r(r-2)}-\frac{1}{r(r+2)}\right)$ $=\frac{1}{4}\left(\frac{1}{3(1)}-\frac{1}{3(5)}\right.$ $+\frac{1}{4(2)}-\frac{1}{4(6)}$ $+\frac{1}{5(3)}-\frac{1}{5(7)}$ $+\cdots$ $+\frac{1}{(n-2)(n-4)}-\frac{1}{(n-2)n}$ $+\frac{1}{(n-1)(n-3)}-\frac{1}{(n-1)(n+1)}$ $\left.+\frac{1}{n(n-2)}-\frac{1}{n(n+2)}\right)$ $=\frac{1}{4}\left(\frac{1}{3}+\frac{1}{8}-\frac{1}{(n-1)(n+1)}-\frac{1}{n(n+2)}\right)$ $=\frac{11}{96}+\frac{-\frac{1}{4}}{(n-1)(n+1)}+\frac{-\frac{1}{4}}{n(n+2)}$ $a=\frac{11}{96},b=c=-\frac{1}{4}$	A minority of students did not consider the given expression. Out of those who did use the result from considering the given expression, ~10-20% of them used the result wrongly (multiplying by 4 instead of by ¼). MOD was done well generally, with minor slips.

(i)	From previous result, $\sum_{r=3}^{n}\frac{1}{(r-2)r(r+2)}=\frac{11}{96}-\frac{1}{4(n-1)(n+1)}-\frac{1}{4n(n+2)}$ As $n\to\infty$, $\frac{1}{(n-1)(n+1)}, \frac{1}{n(n+2)}\to 0$, therefore $\sum_{r=3}^{\infty}\frac{1}{(r-2)r(r+2)}=\frac{11}{96}$.	Done well, but the presentation was off for 20-30%. Some even wrote As $r\to\infty$, $\frac{1}{(r-2)r(r+2)}\to\frac{11}{96}$. Students are reminded to <u>answer the question</u>.
(ii)	$\sum_{r=5}^{n}\frac{1}{r(r+2)(r+4)}$ Replace r by $r-2$, $\sum_{r-2=5}^{r-2=n}\frac{1}{(r-2)r(r+2)}=\sum_{r=7}^{n+2}\frac{1}{(r-2)r(r+2)}$ $=\sum_{r=3}^{n+2}\frac{1}{(r-2)r(r+2)}-\sum_{r=3}^{6}\frac{1}{(r-2)r(r+2)}$ $=\left(\frac{11}{96}-\frac{1}{4(n+1)(n+3)}-\frac{1}{4(n+2)(n+4)}\right)$ $-\left(\frac{11}{96}-\frac{1}{4(6-1)(6+1)}-\frac{1}{4(6)(6+2)}\right)$ $=\frac{83}{6720}-\frac{1}{4(n+1)(n+3)}-\frac{1}{4(n+2)(n+4)}$	Poorly done. A variety of similar approaches can be applied, but there were many conceptual errors with regard to how to the general term interacts with the dummy variable in the summation. There was a small number of scripts where the working was filled with errors arriving at the right answer (or close to). Marks may not have been awarded if the terms used in the working were not equal at any juncture.
3(a)(i)	$y^2=b^2\left(1-\frac{x^2}{a^2}\right)=\frac{b^2(a^2-x^2)}{a^2}$ $A=4\int_0^a y\ \mathrm{d}x=4\int_0^a\sqrt{b^2\left(1-\frac{x^2}{a^2}\right)}\ \mathrm{d}x$ $=4\int_0^a\sqrt{\frac{b^2(a^2-x^2)}{a^2}}\ \mathrm{d}x=\frac{4b}{a}\int_0^a\sqrt{a^2-x^2}\ \mathrm{d}x.$ (shown) $A=\frac{4b}{a}\int_0^a\sqrt{a^2-x^2}\ \mathrm{d}x$ $=\frac{4b}{a}\int_0^{\frac{\pi}{2}}\sqrt{a^2-(a\sin\theta)^2}(a\cos\theta)\ \mathrm{d}\theta$ $=\frac{4b}{a}\int_0^{\frac{\pi}{2}}\sqrt{a^2(1-\sin^2\theta)}(a\cos\theta)\ \mathrm{d}\theta$ $=\frac{4b}{a}\int_0^{\frac{\pi}{2}}a^2\cos^2\theta\ \mathrm{d}\theta$ $=2ab\int_0^{\frac{\pi}{2}}2\cos^2\theta\ \mathrm{d}x=2ab\int_0^{\frac{\pi}{2}}1+\cos2\theta\ \mathrm{d}\theta$ $=2ab\left[\theta+\frac{1}{2}\sin2\theta\right]_0^{\frac{\pi}{2}}=2ab(\frac{\pi}{2})=\pi ab$	For the 'show' part, it is a simple result that aims to test the students' conceptual understanding of the application of integration to find area under a curve. Students' presentation has to demonstrate that understanding. Generally very well done, though the usual mistakes were still present (not changing the limits, making errors substituting in $\frac{\mathrm{d}x}{\mathrm{d}\theta}$). Some students did not know how to proceed with the integrand thereafter (but only minority). Of those who applied the cosine double angle formula, there was a significant minority who integrated $\cos2\theta$ wrongly.

(a)(ii)	$V=2\pi\int_0^a y^2\ \mathrm{d}x$ or $\pi\int_{-a}^a y^2\ \mathrm{d}x$ $=2\pi\int_0^a \frac{b^2(a^2-x^2)}{a^2}\ \mathrm{d}x=\frac{2\pi b^2}{a^2}\int_0^a a^2-x^2\ \mathrm{d}x$ $=\frac{2\pi b^2}{a^2}\left[a^2x-\frac{x^3}{3}\right]_0^a=\frac{2\pi b^2}{a^2}\left(\frac{2a^3}{3}\right)$ $=\frac{4\pi ab^2}{3}$	Poorly done by quite a number of students. These students may have been confused by the integral they showed in (i), not helped by their poor understanding of finding volume of revolution by applying integration. Simplify answers too!
(b)	$x^2+y^2=a^2$ $2x+2y\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{x}{y}$ $S=\int_a^b 2\pi y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\ \mathrm{d}x$ $=\int_{-r}^r 2\pi y\sqrt{1+\left(-\frac{x}{y}\right)^2}\ \mathrm{d}x$ $=\int_{-r}^r 2\pi\sqrt{x^2+y^2}\ \mathrm{d}x$ $=\int_{-r}^r 2\pi\sqrt{x^2+(r^2-x^2)}\ \mathrm{d}x$ $=\int_{-r}^r 2\pi r\ \mathrm{d}x=2\pi r\ [x]_{-r}^r=4\pi r^2$ Alternative: $y=\sqrt{a^2-x^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-x}{\sqrt{a^2-x^2}}$ $S=\int_a^b 2\pi y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\ \mathrm{d}x$ $=\int_{-r}^r 2\pi\sqrt{r^2-x^2}\sqrt{1+\left(\frac{-x}{\sqrt{r^2-x^2}}\right)^2}\ \mathrm{d}x$ $=\int_{-r}^r 2\pi\sqrt{r^2-x^2}\sqrt{1+\frac{x^2}{r^2-x^2}}\ \mathrm{d}x$ $=\int_{-r}^r 2\pi\sqrt{(r^2-x^2)+x^2}\ \mathrm{d}x$ $=\int_{-r}^r 2\pi r\ \mathrm{d}x=2\pi r\ [x]_{-r}^r=4\pi r^2$	Students generally made good progress with this part, applying the formula with ease. A common conceptual error was integrating r with respect to x to obtain $\frac{r^2}{2}$.

4	When $a=1$, $x^3+y^3=3xy$. Let $y=xt$, then $x^3+(xt)^3=3x(xt)$ $\Rightarrow \quad x^3+x^3t^3=3x^2t$ $\Rightarrow \quad x^3(1+t^3)=3x^2t$ $\Rightarrow \quad x=\frac{3t}{1+t^3}$. (shown) For x to be defined, $1+t^3\neq 0$ Hence $t\neq -1$. Therefore $k=-1$ Since $y=xt$, hence $y=\left(\frac{3t}{1+t^3}\right)t=\frac{3t^2}{1+t^3}$.	The proof for x was generally well done. Working for find y in terms of t varied from a one-liner to a full page working when the question says "write down the expression…" which students should suspect that the answer is obvious. Some students neglected to write down the value of k.
(i)	$x+y=\frac{3t}{1+t^3}+\frac{3t^2}{1+t^3}$ $=\frac{3t(1+t)}{1+t^3}$ $=\frac{3t(1+t)}{(1+t)(t^2-t+1)}$ $=\frac{3t}{t^2-t+1}$. (shown) Oblique asymptote is when $x\to\pm\infty$. Since $x=\frac{3t}{1+t^3}$, $x\to\pm\infty$ when $t\to -1$. When $t\to -1$, $x+y=\frac{3t}{t^2-t+1}\to\frac{-3}{(-1)^2-(-1)+1}=\frac{-3}{3}=-1$. i.e. $x+y\to -1$ $\Rightarrow \quad y\to -1-x$. Therefore, the oblique asymptote of the curve is $y=-1-x$.	In this show question, students either did not know how to simplify $\frac{3t(1+t)}{1+t^3}$, or some just did $\frac{3t(1+t)}{1+t^3}=\frac{3t}{t^2-t+1}$ without showing the factorization. Students need to realise that nothing in a "show" or "prove" question should be assumed as obvious. Only a few students knew that oblique asymptote occurs when $x\to\pm\infty$, not when $t\to\pm\infty$.
(ii)		Graph was badly drawn. Students need to know how to interpret their G.C. sketch: 

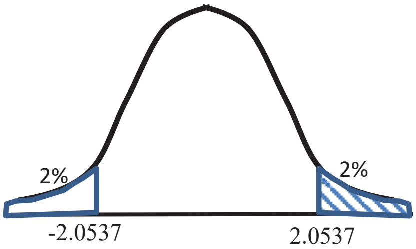
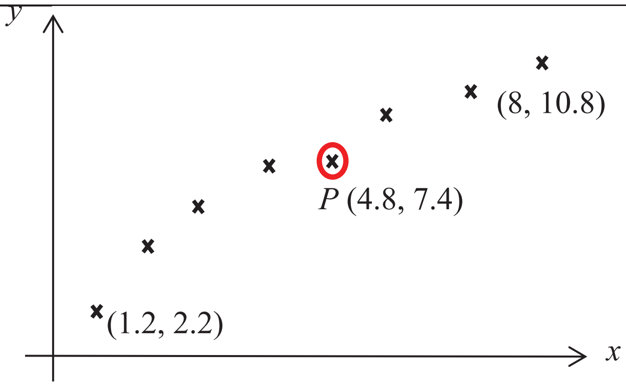
		1. Why is there a "hole" in the graph near the origin? 2. Is that line that looks like a straight line part of the graph? Sketches that included a sharp turn to draw the straight line as part of the graph did not get any credit. Nor did sketches with a gap near the origin.
(iii)	Differentiating the cartesian equation implicitly, $3x^2+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}=3x\frac{\mathrm{d}y}{\mathrm{d}x}+3y$ $\Rightarrow\ (y^2-x)\frac{\mathrm{d}y}{\mathrm{d}x}=y-x^2$ i.e. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y-x^2}{y^2-x}$. When $t=2$, $x=\frac{2}{3}$, $y=\frac{4}{3}$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{4}{3}-\left(\frac{2}{3}\right)^2}{\left(\frac{4}{3}\right)^2-\frac{2}{3}}=\frac{4}{5}$. Hence equation of tangent when $t=2$: $\frac{y-\frac{4}{3}}{x-\frac{2}{3}}=\frac{4}{5}\ \Rightarrow\ y=\frac{4}{5}x+\frac{4}{5}$. If the tangent cuts the curve again, using the parametric equations of the curve to substitute into the equation of the tangent, $\frac{3t^2}{1+t^3}=\frac{4}{5}\left(\frac{3t}{1+t^3}\right)+\frac{4}{5}$ $\Rightarrow\ 3t^2=\frac{4}{5}(3t)+\frac{4}{5}(1+t^3)$ $\Rightarrow\ 15t^2=12t+4+4t^3$ $\Rightarrow\ 4t^3-15t^2+12t+4=0$ $\Rightarrow\ t=-\frac{1}{4}$, or $t=2$ (tangent here) Therefore the tangent cuts the curve again at $t=-\frac{1}{4}$, with coordinates $\left(-\frac{16}{21},\frac{4}{21}\right)$.	This part of the question required students to choose between the given Cartesian equation or the equivalent parametric equations to use to find $\frac{\mathrm{d}y}{\mathrm{d}x}$. Most students chose to differentiate the parametric equations, with many many remembering quotient rule wrongly. Some used product rule instead but also fumbled with the algebra. The most efficient method is to differentiate the Cartesian equation implicitly, then substituting $t=2$ to find x and y at the point to find gradient of the tangent. The intersection between tangent and original curve should be a familiar question. The parametric equations should be substituted into the equation of the tangent to solve for t. Technically, even without having done the preceding parts, this question is a standard one that could have easily been done with just the Cartesian equation and the G.C.
5(i)	Number of ways $={}^{12}C_4$ $=495$	Most common error was to use ${}^{12}P_4$ instead. But order in this case is already

<table>
<tr><td></td><td></td><td>predetermined by heights, so we just need to find how many ways a group of 4 can be chosen and there is only 1 way to arrange them by height.</td></tr>
<tr><td>5(ii)</td><td>Method 1
Number of ways
$= 2^{12} - 1$
$= 4095$

Method 2
Number of ways
$= {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 ... + {}^{12}C_{12}$
$= 4095$</td><td>This question, unintentionally, has two interpretations:
1. Total number of ways to choose delegations with at least one student. This was the intended interpretation.
2. If n is assumed constant, then number of ways to choose n students from the team is just ${}^{12}C_n = \frac{12!}{n!(12-n)!}$. For this approach we need to see the formula in order to gain the second mark.</td></tr>
<tr><td>5(iii)</td><td>Probability
$= \frac{{}^5C_3 3! \times {}^5C_3 3! \ \times 2}{{}^{12}C_8 7!}$
$= \frac{2}{693}$ (or 0.00289)</td><td>This question proved a challenge for many with a majority neglecting to choose the number of boys and girls to be seated with Andy and Beth. The team has 12 members but only 8 are chosen to attend the dinner. Another problem is not knowing that the total number of ways is to count how many ways to choose any 8 to seat at a round table.</td></tr>
<tr><td>6(i)</td><td>$P(X=2) = 2P(\{1,3\}) + 2P(\{3,5\}) + 2P(\{5,7\})$
$= 2\left(\frac{1}{4}\right)\left(\frac{1}{3}\right) + 2\left(\frac{1}{4}\right)\left(\frac{1}{3}\right) + 2\left(\frac{1}{4}\right)\left(\frac{1}{3}\right)$
$= \frac{1}{2}$ (shown)
$P(X=4) = 2P(\{1,5\}) + 2P(\{3,7\})$
$= 2\left(\frac{1}{4}\right)\left(\frac{1}{3}\right) + 2\left(\frac{1}{4}\right)\left(\frac{1}{3}\right) = \frac{1}{3}$
<table><tr><td>x</td><td>2</td><td>4</td><td>6</td></tr><tr><td>$P(X=x)$</td><td>$\frac{1}{2}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{6}$</td></tr></table></td><td>(i) Unclear presentation for P(X=2) = 0.5 for many students.

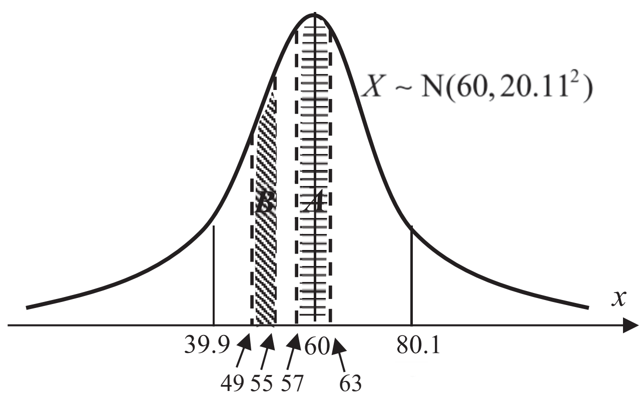
Some wrote $6\left(\frac{1}{4}\right)\left(\frac{1}{3}\right) = \frac{1}{2}$ or $2\left(\frac{1}{4}\right)\left(\frac{1}{3}\right) + 4\left(\frac{1}{4}\right)\left(\frac{1}{3}\right)$ without explaining what these numbers meant.</td></tr>
<tr><td>6(ii)</td><td>Similarly
<table><tr><td>y</td><td>2</td><td>4</td><td>6</td></tr><tr><td>$P(Y=y)$</td><td>$\frac{1}{2}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{6}$</td></tr></table></td><td>(ii) some made the mistake
$E(W) = 2\left(\frac{1}{2}\right) + 4\left(\frac{1}{3}\right) + 6\left(\frac{1}{6}\right)$</td></tr>
</table>

	P(player wins a prize) $= P(X=2)P(Y=2)+P(X=4)P(Y=4)+P(X=6)P(Y=6)$ $=\left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{6}\right)^2$ $=\frac{7}{18}$	when it should have been $E(W)=2\left(\frac{1}{2}\right)^2+4\left(\frac{1}{3}\right)^2+6\left(\frac{1}{6}\right)^2$
6(ii)	Let W be the winnings per game. $\begin{array}{c\|c\|c\|c\|c} w & 0 & 2 & 4 & 6 \\ \hline P(W=w) & \frac{11}{18} & \left(\frac{1}{2}\right)^2 & \left(\frac{1}{3}\right)^2 & \left(\frac{1}{6}\right)^2 \end{array}$ $E(W)=2\left(\frac{1}{2}\right)^2+4\left(\frac{1}{3}\right)^2+6\left(\frac{1}{6}\right)^2=\frac{10}{9}$ Since the expected winnings of player $=\$\frac{10}{9} > \$1$, the game is to be played using $2 coupon in order to have profit per game. $k=2$	Some didn't realise that to win a prize you need P(X=Y), thus making the mistake of calculating E(X) = 10/3 Some students did not realise that amount won is associated with their different probabilities, they made the mistake of using prob $\frac{7}{18}$ with W (winning per game). Some found E(W) $=\frac{10}{9}$ but made wrong conclusion for k = $1
7(i)	$\bar{x}=\frac{\sum(x-200)}{50}+200=209$ $s^2=\frac{1}{49}\left[55000-\frac{(450)^2}{50}\right]=\frac{50950}{49}$ or 1039.8 (5 s.f.) Let μ be the population mean price of all concert tickets. To test $H_0:\mu=200$ against $H_1:\mu\neq200$ at the 4% level of significance. Under H_0, $Z=\frac{\bar{X}-200}{s/\sqrt{50}}\sim N(0,1)$ or $\bar{X}\sim N\left(200,\frac{50950}{49(50)}\right)$ approximately by central limit theorem since sample size, 50, is large. Value of test statistic $z=1.97$ p-value = 0.0484 > 0.04 (do not reject H_0) There is **insufficient** evidence at the **4% level of significance** that the mean concert ticket price is not $200.	Most students are able to present first step, i.e. $H_0:\mu=200$ $H_1:\mu\neq200$ at 4% level of sig. Many DID NOT define μ. Some students made the mistake of putting the wrong population mean 209 in place of the value 200 in the distribution, $\bar{X}\sim N\left(200,\frac{50950}{49(50)}\right)$ Some forgot to quote central limit theorem. Phrasing of the conclusion was contradicting for some students, even though they knew it was 'do not reject H_0'.
(ii)	Now given the standard deviation is 32.25 Under H_0, $\bar{X}\sim N\left(200,\frac{32.25^2}{n}\right)$ by CLT since n is large	Some students misread the question.

	 For H_0 to not be rejected, the test statistic, z, must not be in critical region. Hence $-2.0537 < z < 2.0537$ i.e. $-2.0537 < \dfrac{206-200}{\frac{32.25}{\sqrt{n}}} < 2.0537$ i.e. $-2.0537 < \dfrac{6\sqrt{n}}{32.25} < 2.0537$ i.e. $-11.039 < \sqrt{n} < 11.039$ Hence largest value of n is 121.	(ii) Some made the mistake to assuming it is upper tail, writing $\dfrac{206-200}{\frac{32.25}{\sqrt{n}}} < 2.0537$ instead of $-2.0537 < \dfrac{206-200}{\frac{32.25}{\sqrt{n}}} < 2.0537$ Some also made the mistake of confusing it with (i) information, writing it as $-2.0537 < \dfrac{209-206}{\frac{32.25}{\sqrt{n}}} < 2.0537$
8(i)	 	Students should be using a cross ‘x’ to make the points instead of a small square box (as seem on the calculator screen).
(ii)	The scatter diagram suggests that as x increases, y increases at a decreasing rate, which is consistent with the model $y = c\ln x + d$ where c and d are positive. The model $y = ax^2 + b$ where a and b are positive suggests instead that as x increases, y increases at an increasing rate, which is not suitable for the data given. $r = 0.9927$ (4dp) 	Quite a number of students said that the points in the scatter diagram were distributed along a line, hence $y = c\ln x + d$ was the appropriate model (incorrectly assuming $\ln x$ is a **linear** function?) Some students mentioned the turning point in $y = ax^2 + b$. This is not relevant to the current question as we are only looking at a limited range of values of x. Some students also calculated r for both models before deciding $y = c\ln x + d$ was the better model as its value of r was closer to 1. This was

		not accepted as it did not make use of the scatter diagram. Finally, many students did not give r to 4 decimal places as stated in the question.
(iii)	Least squares regression line is $y = 1.3755 + 4.4612 \ln x$ (5 s.f.) $= 1.38 + 4.46 \ln x$ (3 s.f.)	A large number of students gave $y = 4.46 + 1.38 \ln x$, or otherwise chose the wrong model in part (ii) and hence lost marks here. Most students chose P correctly; some students thought it was the bottom left point (1.2, 2.2).
(iv)	When $y = 8$, $8 = 1.3755 + 4.4612 \ln x$ Hence $\ln x = \dfrac{8 - 1.3755}{4.4612}$ $\Rightarrow x = 4.42$ (3 s.f.) This estimate is reliable as $r_{y,\ln x}$ is very close to +1 and $y = 8$ is within the data range of y ($2.2 \le y \le 10.8$) used to obtain the regression line of y on ln$x$. The popularity index (variable x) is the independent variable in this context, therefore it is not appropriate to use the ln x on y or the x^2 on y regression lines to estimate popularity index x.	Some students did not use the regression line in (iii), but recalculated a new regression line based on a linear model. Some students left their answer as $x = \dfrac{8 - 1.3755}{4.4612} = 1.48$. Many students omitted to mention that r is close to 1. Some students wrote that it was unreliable as the popularity index of a new artiste might not be accurate. This was not accepted. Finally, a large number of students stated the two models were inappropriate for various reasons – such as the fact that y could not be negative, or that the graph did not have a turning point. This reflects a misconception that the regression model can be extrapolated to values of x beyond those of the data.
9(i)	Let A be the event that the older twin Albert is late for practice, and B be the event that the younger twin Benny is late for practice. Given: $\text{P}(A) = 0.65$, $\text{P}(A \mid B) = 0.975$, $\text{P}(A \mid B') = 0.56875$.	Majority of students were able to do this. Those who could not wrongly interpreted the

	$P(A \mid B) = 0.975 \Rightarrow \dfrac{P(A \cap B)}{P(B)} = 0.975$ $\Rightarrow P(A \cap B) = 0.975\, P(B)$ $P(A \mid B') = 0.56875 \Rightarrow \dfrac{P(A \cap B')}{P(B')} = 0.56875$ $\Rightarrow P(A \cap B') = 0.56875\, P(B')$ Now, $P(A) = P(A \cap B) + P(A \cap B')$ $0.975P(B) + 0.56875[1 - P(B)] = 0.65$ $P(B) = 0.2$ (shown)	probability of 0.975 as $P(A \cap B)$ instead of $P(A \mid B)$. Some students had difficulty combining $P(A \cap B)$ and $P(A \cap B')$, not realizing that drawing a simple venn diagram would clearly lead to $P(A) = P(A \cap B) + P(A \cap B')$.
9(ii)	$\dfrac{P(A \cap B)}{0.2} = 0.975 \Rightarrow P(A \cap B) = 0.195$ Probability $= P(A \cap B') + P(A' \cap B)$ $= [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)]$ $= 0.65 - 0.195 + 0.2 - 0.195$ $= 0.46$ OR Probability $= P(B')P(A \mid B') + P(B)P(A' \mid B)$ $= (0.8)(0.56785) + (0.2)(1 - 0.975)$ $= 0.46$ OR $P(A' \cap B') = 1 - P(A \cup B)$ $= 1 - [P(A) + P(B) - P(A \cap B)]$ $= 1 - (0.65 + 0.2 - 0.195)$ $= 0.345$ Probability $= 1 - P(A \cap B) - P(A' \cap B')$ $= 1 - 0.195 - 0.345$ $= 0.46$	Very common mistake: $P(A \cap B') + P(A' \cap B)$ $= P(A)P(B') + P(A')P(B)$ $= (0.65)(0.8) + (0.35)(0.2)$. Student need to realise that the question did not state that *A* and *B* are independent, and they must not assume this. In this question, indeed, *A* and *B* are NOT independent.
9(iii)	$P(A' \cap B' \cap C')$ $= 1 - P(A \cup B \cup C)$ $= 1 - \begin{bmatrix} P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) \\ -P(A \cap C) + P(A \cap B \cap C) \end{bmatrix}$ $= 1 - [0.65 + 0.2 + 0.5 - 0.195 - (0.2)(0.5) - (0.65)(0.5) + 0.098]$ $= 0.172$	A common mistake: $P(A' \cap B' \cap C')$ $= P(A' \cap B')P(C')$, where students assumed that $A' \cap B'$ is independent of *C*'. Some students wrongly interpreted the phrase 'event of either twin being late for practice is independent of the event that C is late for practice' as $A \cup B$ independent of *C*, and wrote $P((A \cup B) \cap C)$ $= P(A \cup B) \times P(C)$.

10	Let X be the random variable for the time taken to place an order and Y be the random variable for the time taken to prepare and serve an order. $X \sim \mathrm{N}(60,\sigma^2), Y \sim \mathrm{N}(300,50^2)$ Given: $\mathrm{P}(X<40)=0.16$ $\Rightarrow \mathrm{P}\left(Z<\dfrac{40-60}{\sigma}\right)=0.16$ $\Rightarrow \dfrac{-20}{\sigma}=-0.9944578907$ $\Rightarrow \sigma=20.11146$ $\Rightarrow \sigma=20.11$(to 2 d.p.) (shown)	This is generally well-done. However, no credit is given if student used the value of 20.11 and compared the probabilities of $\mathrm{P}(X<40)$ for $\sigma=20.10,\ 20.11$ and 20.12. For a question that requires the student to show the result, student must not use the value to be shown in the working. There are also some students who wrote the standardized value wrongly as $\dfrac{60-40}{\sigma}$, instead of $\dfrac{40-60}{\sigma}$.
10(i)	 Since normal curve is symmetrical about mean 60, $\mathrm{P}(49<X<55)<\mathrm{P}(57<X<63)$. So $A>B$.	Students need to read the question carefully. Many students wrongly interpreted A and B as events rather than probabilities. Hence the following mistakes: 1. Drawing a venn diagram showing A and B as 2 sets 2. Writing $\mathrm{P}(A)>\mathrm{P}(B)$. Some drew 2 different normal curves, without realizing that it's the same random variable X.
10 (ii)	$Y \sim \mathrm{N}(300,50^2)$ $\mathrm{P}(Y>k)=0.001$ From GC, $k=454.5116154=455$ s (to 3 s.f.) 	Not well-done despite it being an easy question. Common mistakes: 1. Very careless in interpreting the probability of 0.1%, often writing it as 0.1, or 0.01. 2. Some considered distribution of X, or $X+Y$ instead. Many did additional step of standardizing Y. This is not necessary since the mean and variance of Y are given.

<table>
<tr>
<td>10 (iii)</td>
<td>$\bar{Y} = \dfrac{Y_1 + Y_2 + ... + Y_{10}}{10} \sim \text{N}(300, \dfrac{50^2}{10})$
$\text{P}(\bar{Y} < 270) = 0.028897188$
$= 0.0289$ (to 3 s.f.)</td>
<td>This is generally well-done. Some, however, wrongly quoted the use of CLT.
Common mistakes:
1. Considering distribution of $\dfrac{X_1 + ... + X_{10} + Y_1 + ... + Y_{10}}{10}$
2. Writing variance of $\bar{Y}$ as $10(50^2)$.
3. Not converting 4.5 mins to 270s</td>
</tr>
<tr>
<td>10 (iv)</td>
<td>Let T be random variable for time taken to take the order and prepare the order of 1 customer.
$T = 0.95X + 0.9Y$
$T \sim \text{N}(0.95(60) + 0.9(300), 0.95^2(20.11146^2) + 0.9^2(50^2))$
$T \sim \text{N}(327, 2390.01822)$

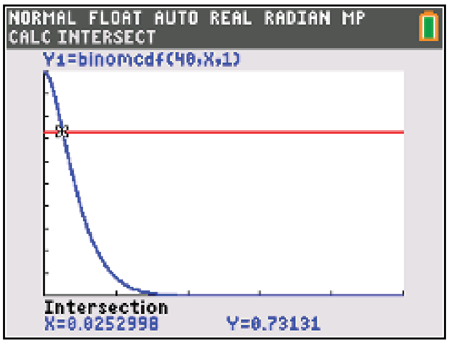
Time taken to serve n customers is $T_1 + T_2 + ... + T_n$
$T_1 + T_2 + ... + T_n \sim \text{N}(327n, 2390.01822n)$

Given: $\text{P}(T_1 + ... + T_n < 3600) \geq 0.8$
$\text{P}(Z < \dfrac{3600 - 327n}{\sqrt{2390.01822n}}) \geq 0.8$
$\dfrac{3600 - 327n}{\sqrt{2390.01822n}} \geq 0.84162$
From GC, let $Y_1 = \dfrac{3600 - 327n}{\sqrt{2390.01822n}}$
<table>
<tr><th>n</th><th>Y_1</th></tr>
<tr><td>9</td><td>4.4796 > 0.84162</td></tr>
<tr><td>10</td><td>2.1346 > 0.84162</td></tr>
<tr><td>11</td><td>0.0185 < 0.84162</td></tr>
</table>

Largest n = 10

Assume that the time taken to place an order is independent of the time taken to prepare and serve an order for a customer
OR
Assume that the time taken to place an order for a customer is independent of the time taken to place an order for another customer.</td>
<td>Common mistakes:
1. Writing Var(T) as $20.11146^2 + 50^2$ or $(0.95)(20.11146^2) + (0.9)(50^2)$
2. Interpreting the time taken for n customers as nT instead of $T_1 + T_2 + ... + T_n$, and hence writing Var(nT) as $2390.01822n^2$

Students should realise that for normal distributions, independence of random variables is a necessary assumption.
There are students who simply assume that the new improved time for serving customers is normally distributed, and even quote use of CLT.</td>
</tr>
<tr>
<td>11</td>
<td>The probability of a randomly chosen wine glass being chipped is constant.
Selections of chipped wine glasses are independent of each other.</td>
<td>Keywords for <u>assumptions</u>:
• probability … constant
• Selections/events … independent
probability … independent is WRONG!
Note the difference between conditions & assumptions.</td>
</tr>
</table>

<table>
<tr>
<td></td>
<td>Let X be the number of chipped wine glasses in a batch of n glasses.
$X \sim B(n, p)$
$A = P(X \le 1)$
$= P(X = 0) + P(X = 1)$
$= \binom{n}{0} p^0 (1-p)^n + \binom{n}{1} p^1 (1-p)^{n-1}$
$= (1-p)^n + np(1-p)^{n-1}$
$= (1-p)^{n-1}(1-p+np)$
$= (1-p)^{n-1}[1+(n-1)p]$ (shown)</td>
<td>Generally well done except –
• Some students mistook A to be $P(X > 1)$.
• Some only had one of $P(X = 0)$ or $P(X = 1)$.
• Some had missing 2nd last step of factoring out $(1 - p)^{n-1}(1 - p + np)$</td>
</tr>
<tr>
<td>11
(a)</td>
<td>Given $p = 0.02$, $X \sim B(n, 0.02)$
$A = P(\text{batch of glasses will not be rejected}) = P(X \le 1) \ge 0.9$
From GC,
<table>
<tr><th>n</th><th>$A = P(X \le 1)$</th></tr>
<tr><td>25</td><td>0.9114 > 0.9</td></tr>
<tr><td>26</td><td>0.9052 > 0.9</td></tr>
<tr><td>27</td><td>0.8989 < 0.9</td></tr>
</table>
Thus largest $n = 26$</td>
<td>Generally well done except that some students wasted time in writing out a few steps of working before using GC, instead of comparing with 0.9 directly. Another common mistake is $n = 27$ because of wrong inequality or using equation instead.</td>
</tr>
<tr>
<td>11
(b)</td>
<td>Given that $n = 40$, $A = P(X \le 1) = 0.73131$ where $X \sim B(40, p)$


From GC, $p = 0.02530$ (5 d.p.)</td>
<td>Common mistake – ignoring '5 decimal places' in the question. Answer such as 0.025300 was common.</td>
</tr>
<tr>
<td>11
(b)
(i)</td>
<td>$X \sim B(40, 0.02530)$
$\mu = E(X) = 40 \times 0.02530 = 1.012$
$\sigma^2 = Var(X) = 40 \times 0.02530 \times (1 - 0.02530) = 0.9863964$
$P(\mu - \sigma < X < \mu + \sigma)$
$= P(0.018805 < X < 2.005175)$
$= P(X = 1) + P(X = 2)$
$= 0.56107$
$= 0.561$ (to 3 s.f.)</td>
<td>Confusion in the distribution of X is common:
$X \sim B(40, 0.02530)$ at the start becomes
$X \sim N(1.1012, 0.9863964)$ a few steps later!

A few students took the values of $n = 26$ and $p = 0.02$ from part (a) instead of using the current values stated in part (b) with $n = 40$ and p found in (b) (i)</td>
</tr>
<tr>
<td>11
(b)
(ii)</td>
<td>Let R be number of rejected batches out of 20×52 batches in a year.
$R \sim B(20 \times 52, 1 - 0.73131)$</td>
<td>There were a few scripts with no mention of any distributions!</td>
</tr>
</table>

	$R \sim \mathrm{B}(1040, 0.26869)$ $\mathrm{P}(\text{total compensation} > \$30000) = \mathrm{P}(R > 300)$ $= 1 - \mathrm{P}(R \le 300) = 0.0711$ <u>OR</u> Let C be number of rejected batches out of 20 batches in a week. $C \sim \mathrm{B}(20, 1 - 0.73131)$ $C \sim \mathrm{B}(20, 0.26869)$ $\mathrm{E}(C) = 20 \times 0.26869 = 5.3738$ $\mathrm{Var}(C) = 20 \times 0.26869 \times 0.73131 = 3.929913678$ Since n = 52 is large, by Central Limit Theorem, $C_1 + C_2 + ... + C_{52} \sim \mathrm{N}(279.4376, 204.3555113)$ approximately Let total compensation be W. $W = 100(C_1 + ... + C_{52})$ $\mathrm{E}(W) = 100(279.4376) = 27943.76$ $\mathrm{Var}(W) = 100^2(204.3555113) = 2043555.113$ $W \sim \mathrm{N}(27943.76, 2043555.113)$ $\mathrm{P}(W > 30000) = 0.075160$ $= 0.0752$ (to 3 s.f.)	Central Limit Theorem was wrongly used for 20 batches instead of 52 weeks. Students should note the random variables involved with their respective n and p: • $X \sim \mathrm{B}(40, 0.02530)$ for the number of chipped wine glass • $C \sim \mathrm{B}(20, 1 - 0.73131)$ for the number of rejected batches