

Name:

National Junior College 2016 – 2017 H2 Further Mathematics Topic F7: Further Complex Numbers (Assignment 2)

Suggested Duration: 60 min

1. The complex number *z* satisfies

$$|z-1| \le 2$$
 and $|z+i| \le |z-2-i|$

Sketch the locus of points representing *z* on an Argand diagram. [3]

Hence show that the maximum value of $\arg(z+2+2i)$ is $2\sin^{-1}\left(\frac{2}{\sqrt{13}}\right)$. Show your working clearly.

2. (a) Given that
$$z = \lambda e^{i\theta}$$
, where $\lambda > 0$ and $0 < \theta < \frac{\pi}{2}$, and $w = i\sqrt{3}z$, find

- (i) |z+w|, in terms of λ ,
- (ii) $\arg(z+w)$, in terms of θ . [2]
- (b) The complex number z satisfies the relations

$$\left|z-1+i\sqrt{3}\right| \le \sqrt{3}$$
 and $\arg\left(2z+4i\sqrt{3}\right) = \frac{\pi}{3}$.

- (i) Illustrate, on an Argand diagram, the locus of points representing the complex number *z*. [3]
- (ii) Find the exact value of z that gives the greatest possible value of $|\arg(z)|$. [3]
- 3. On an Argand diagram, the complex number a = 3+4i is represented by A and ia is represented by B. On a single Argand diagram, sketch the following loci:
 - (i) $|z-a| \le 5$, [2]

(ii)
$$|z - ia| \le 5$$
. [2]

(iii) State a single transformation that will map the locus of (i) to the locus of (ii). [1]

The point C represents the complex number a + ia on the Argand diagram.

- (iv) Explain why C lies on the loci of |z-a| = 5 and |z-ia| = 5. [1]
- (v) Find the exact area of the region satisfying both (i) and (ii). [2]
- (vi) If z satisfies both (i) and (ii), find the maximum value of $\arg(z ia + a)$, leaving your answer in exact form. [2]

[3]

[2]

- 4. The complex number z satisfies the relations $\arg(z+3-3i) = -\frac{\pi}{4}$ and $|z-3+3i| \le b$, where b is a constant and $1 \le b \le 3$.
 - (i) Illustrate each of the above relations on a single Argand diagram. [2]
 - (ii) Find the exact least possible value of |z + 5i|. [1]
 - (iii) Given that the least possible value of |z| is $\sqrt{18} 2$,
 - (a) find the value of b, [1]
 - (b) hence find an exact expression for z, in the form x + iy. [2]
 - (c) State the cartesian equation of the locus of the point representing complex variable w such that $|w| = |w z_1|$, where z_1 is the complex number found in part (b). [1]

(ACJC/2014/P2/Q2)