

BENDEMEER SECONDARY SCHOOL 2024 PRELIMINARY EXAMINATION SECONDARY FOUR NORMAL (ACADEMIC)

| CANDIDATE NAME | MARKING SCHEME | |
|-------------------|--------------------|-----------------|
| CLASS | | INDEX NUMBER |
| MATHEMA | ATICS (SYLLABUS A) | 4045/01 |
| Paper 1 | | 30 July 2024 |
| | | |
| | | 2 Hours |

READ THESE INSTRUCTIONS FIRST

Write your name, register number on all the work you hand in.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

The use of an approved scientific calculator is expected, where appropriate.

Answer **all** the questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question, it must be shown with the answer.

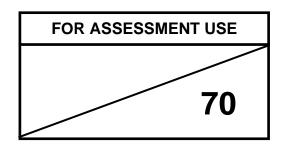
Omission of essential working will result in loss of marks.

The total of the marks for this paper is 70.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.



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Mathematical Formulae

Compound interest

Total amount =
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved Surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$ Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$Mean = \frac{\sum fx}{\sum f}$$

Standard deviation =
$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

| 1 | (a) | $6x^2y$ |
|---|-----|----------------------------------|
| | | Simplify $\frac{6x^2y}{8xy^3}$. |

$$\frac{3x}{4y^2} [B1]$$

Answer[1]

(b) Simplify
$$\left(\frac{x^3}{27}\right)^{-\frac{1}{3}}$$
.

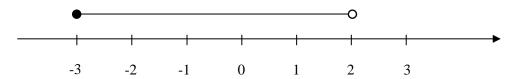
$$\left(\frac{27}{x^3}\right)^{\frac{1}{3}} = \frac{27^{\frac{1}{3}}}{(x^3)^{\frac{1}{3}}} [M1]$$

$$= \frac{3}{x} [A1]$$

Alternate method

$$\left(\frac{x^3}{3^3}\right)^{-\frac{1}{3}} = \frac{x^{-1}}{3^{-1}} [M1]$$
$$= \frac{3}{x} [A1]$$

2 The diagram below shows a solution set for x on a number line.



(a) Write down the range of values of x.

$$-3 \le x < 2 \text{ [B1]}$$

$$Answer \qquad [1]$$

(b) If x is an integer, state the largest value of x.

| In a sale, the price of a handphone is reduced from \$388 to \$302. |
|---|
| Calculate the percentage decrease in price. |
| \$388 - \$302 = \$86 [M1] |
| $\frac{86}{388} \times 100\% = 22.2\%$ (to 3 s.f) [A1] |
| |
| |
| |

| Answer | % | [2] |
|--------|---|-----|
|--------|---|-----|

4 The angles in a triangle are in the ratio 3:4:5. Calculate the size of each angle in degrees.

```
Total number of units = 3+4+5 = 12

12 \text{ units } \rightarrow 180^{\circ} \text{ [M1]}

1 \text{ unit } \rightarrow \frac{180^{\circ}}{12} = 15^{\circ}

3 \text{ units } \rightarrow 15^{\circ} \text{ x } 3 = 45^{\circ}

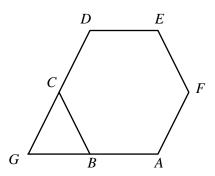
4 \text{ units } \rightarrow 15^{\circ} \text{ x } 4 = 60^{\circ}

5 \text{ units } \rightarrow 15^{\circ} \text{ x } 5 = 75^{\circ}

Ans: 45^{\circ}, 60^{\circ} and 75^{\circ} [A1]
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Answer°,° and° [2]

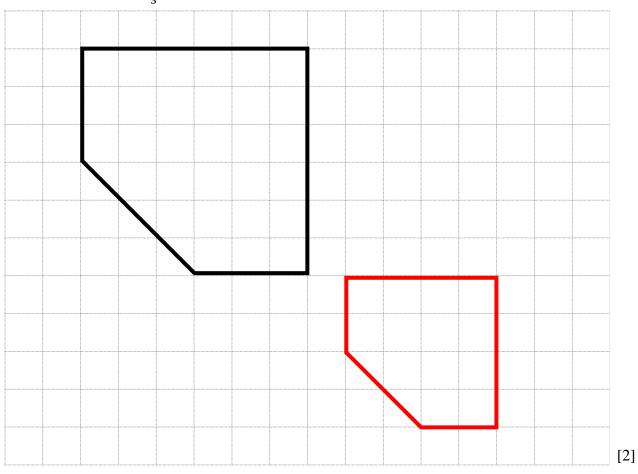
5 The diagram shows a regular hexagon *ABCDEF*. *AB* and *DC* are produced to meet at *G*.



Find angle *BGC*.

Exterior angle of hextagon =
$$\frac{360}{6} = 60^{\circ}$$
 [M1]

6 Use a scale factor of $\frac{2}{3}$ to draw a reduction of the given figure.



7 (a) Solve
$$\frac{x}{x+3} = 24$$
.
 $x = 24(x+3)$
 $x = 24x + 72$
 $23x = -72$

 $x = -3\frac{3}{23} \text{ or } -\frac{72}{23}$. [B1 – No marks if 3 s.f]

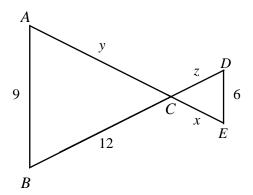
(b) Given that
$$\frac{5^7 \times 5^{-2}}{5^4} = 5^a$$
, find the value of a . [1] $5^{7-2-4} = 5^a$ [M1] $5^1 = 5^a$

a = 1 [A1]

By comparing power

Answer
$$a = \dots$$
 [2]

8 Triangles *ABC* and *DEC* are similar. All the lengths are in centimetres.



(a) Calculate x.

Since Triangles ABC and DEC are similar,

$$\frac{6}{9} = \frac{x}{12}$$
 [M1]

$$x = \frac{12 \times 6}{9} = 8 \text{ [A1]}$$

Answer
$$x = \dots$$
 [2]

(b) Find y in terms of z.

$$\frac{z}{y} = \frac{6}{9} [M1]$$

$$y = \frac{9}{6}z$$

 $y = \frac{3}{2}z$ [A1] – no marks if did not simplify.

| | | 8 | |
|---|---------------|---|-----|
| Л | The fin | rst four terms of a sequence are 5, 9, 13 and 17. | |
| (| (i) | Find an expression, in terms of n , for the n th term of the sequence. | |
| | | Ans: $T_n = 4n + 1$ [B1] [No marks if not simplified i.e. 5+4(n-1)] | |
| (| (ii) | Answer | [1] |
| | | $T_{25} = 4(25) + 1 = 101$ [B1] | |
| | | Answer | [1] |
| (| (iii) | Jane says that 212 is a term in this sequence. Is she correct? Explain your answer. | [1] |
| | | $4n + 1 = 212$ $4n = 211$ $n = \frac{211}{4} = 52.75$ | |
| | | $n = \frac{1}{4} = 52.75$ Since <i>n</i> is not an integer, 212 is not a term in this sequence. Jane is not correct. [B1] | |
| | | | |
| | | | |
| | | | |
| | | Answer | |

[1]

10 A map is drawn to scale of 1: 25 000.

> The distance on the map between the school and Mary's house is 4 cm. Find the actual distance, giving your answer in kilometres.

1cm: 25 000 cm

1km = 1000m = 100,000cm

1 cm : 25000 / 100,000 = 0.25 km [M1]

 $4cm: 0.25 \times 4 = 1 \text{ km [A1]}$

Answerkm [2]

The area of the playground is 75000 m². Calculate the area of the playground **(b)** on the map. Give your answer in cm².

1cm: 250 m

1cm²: 62 500 m² [M1] 1m² : $\frac{1}{62500}$ cm² 75 000 m² : $\frac{75000}{62500}$ = 1.20 cm² [A1]

Answer cm² [2] 11 (a) Write 90 as the product of its prime factors.

| 2 | 90 |
|---|----|
| 3 | 45 |
| 3 | 15 |
| 5 | 5 |
| | 1 |

Ans: 2 x 3² x 5 [B1]

| Answer | | [1] |
|--------|--|-----|
|--------|--|-----|

(b) Find the highest common factor (HCF) of 90 and 216.

$$2^3 \times 3^3 = 216$$

Ans: $2 \times 3^2 = 18$ [B1]

(c) Find the smallest positive integer value of k such that 90k is a perfect square.

Ans: $2 \times 5 = 10$ [B1]

Answer
$$k = \dots$$
 [1]

| 12 | (a) | Ahmad invested \$4500 for 2 years in a savings account of Bank Prosper. He |
|-----------|-----|--|
| | | was paid 4% per annum compound interest. How much did Ahmad have in his |
| | | savings account after 2 years? |

$$A = P \left(1 + \frac{r}{100} \right)^n$$
= 4500 \left(1 + \frac{4}{100} \right)^2 \quad [M1]
= 4867.20 \quad [A1]

[-1m if answer in 1 d.p]

| Answer \$ | [2] |
|-----------|-----|
|-----------|-----|

(b) Ahmad invested another \$4500 for 2 years in an endowment fund of Bank Wealthy. The fund pays an interest rate compounded yearly. At the end of 2 years, he received a total of \$5100. Find the interest rate per annum.

$$5100 = 4500 \left(1 + \frac{r}{100} \right)^{\frac{5100}{4500}} = \left(1 + \frac{r}{100} \right)^{2} [M1]$$

$$\sqrt{\frac{5100}{4500}} - 1 = \frac{r}{100}$$

$$r = 6.46 [A1]$$

| Answer | [2] |
|--------|---------|
| | |

Factorise $x^2 - 49$. 13 (a)

$$(x-7)(x+7)$$
 [B1]

(b) Solve
$$\frac{11}{x+7} - \frac{2}{x^2-49} = 1$$
.

You must show all your working clearly.

From must show all your working clearly
$$\frac{11}{x+7} - \frac{2}{x^2 - 49} = 1$$

$$\frac{11}{x+7} - \frac{2}{(x-7)(x+7)} = 1$$

$$\frac{11(x-7)}{(x-7)(x+7)} - \frac{2}{(x-7)(x+7)} = 1 \quad [M1]$$

$$\frac{11x-77-2}{(x-7)(x+7)} = 1$$

$$11x - 79 = (x - 7)(x + 7)$$

$$x^2 - 49 - 11x + 79 = 0$$

$$x^2 - 11x + 30 = 0 \quad [M1]$$

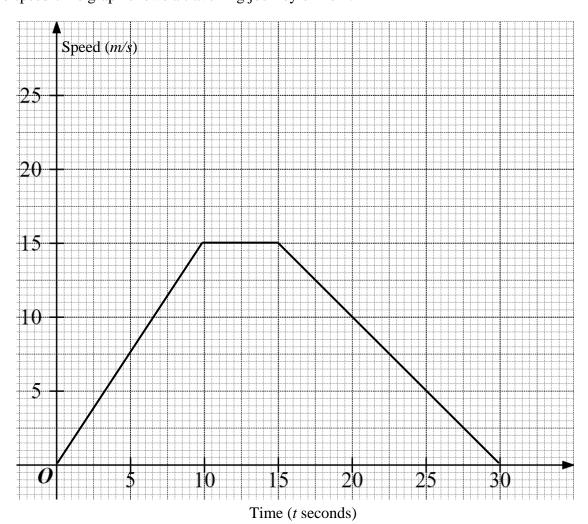
$$(x - 6)(x - 5) = 0 \quad [M1]$$

$$x - 6 = 0 \quad \text{or} \quad x - 5 = 0$$

$$x = 6 \quad \text{or} \quad x = 5 \quad [A1]$$

Answer
$$x = \dots$$
 or $x = \dots$ [4]

14 The speed-time graph shows a travelling journey of Tom.



(a) Describe the motion between t = 10 to t = 15.

Tom is travelling at a constant speed of 15 m/s. [B1]

(b) Calculate the acceleration of the first 10 s.

$$\frac{15}{10} = 1.5$$
 [B1]

Answer m/s^2 [1]

(c) The area under the graph represents the total distance travelled. Calculate the total distance travelled.

| Method 1 | Method 2 |
|--|------------------------------------|
| $\left(\frac{1}{2} \times 10 \times 15\right) + (5 \times 15) + \left(\frac{1}{2} \times 15 \times 15\right) \text{ [M1]}$ | $\frac{1}{2}$ × (5 + 30) × 15 [M1] |
| = 262.5 [A1] | = 262.5 [A1] |

15 The table below shows the distance of some of the asteroids from earth in 2024.

| Name of asteroids | Distance from earth (km) |
|-------------------|--------------------------|
| 2024 GM | 7,160,000 |
| 2024 HS | 7,450,000 |
| 2021 VH2 | 3,560,000 |
| 2024 HD | 2,260,000 |

| (a) | State the distance from earth of 2024 GM in standard form. | | | |
|-----|--|--------|----|-----|
| | Answer: 7.16×10^6 [B1] | | | |
| | | Answer | km | [1] |

(b) What is the distance between 2024 HS and 2021 VH2? Give your answer in million.

| | **** | |
|--------|---------|-----|
| Answer | million | [2] |

(c) The distance between Earth and the moon is about one fifth of the distance between Earth and 2024 HD. Estimate the distance between Earth and the moon, in standard form.

$$\frac{\frac{2,260,000}{5}}{5} [M1]$$
= $452000 = 4.52 \times 10^5 [A1]$

| Answer | km | [2] | ı |
|--------|----|-----|---|
| | | | |

| 16 | The points A and B | have coordinates | (2 1) | and (1 Q |) rocpostivoly |
|----|--------------------|------------------|---------|-----------|----------------|
| 16 | The points A and B | nave coordinates | (-3, 4) | and (1, 8 | respectively. |

| (i |) Ca | lculate | the | length | of AB |
|----|------|---------|-----|--------|---------|
| | | | | | |

length of
$$CD = \sqrt{(8-4)^2 + (1-(-3))^2} = \sqrt{4^2 + 4^2}$$
 [M1]
= $\sqrt{16+16} = \sqrt{32} = 5.66$ [A1]

(ii) Find the equation of the line AB.

Gradient,
$$m = \frac{8-4}{1-(-3)} = \frac{4}{4} = 1$$
 [M1]

Sub. (-3, 4) into
$$y = mx + c$$

$$4 = 1(-3) + c$$

 $c = 4 + 3$
 $c = 7$

Ans:
$$y = x + 7$$
 [A1]

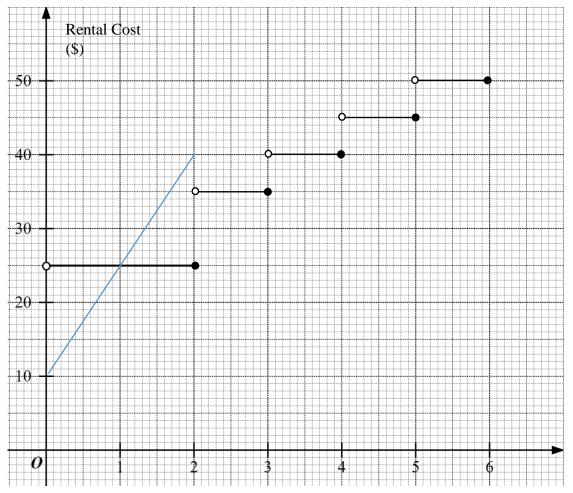
Answer[2]

(iii) Find the coordinates of the point where the line AB cuts the line y = 9

$$9 = x + 7$$
$$x = 2$$

Ans: (2, 9) [A1]

17 Jamal plans to rent a candy floss machine for a class event. The graph shows the rental costs from Company A.



Hours of Rental

(a) How much should she expect to spend if she rent the candy floss machine from 8am to 12.30pm?

Duration of rental 4 hours 30mins From graph, rental cost \$45

| Answer | \$ | [1] |
|-----------|----|-----|
| 111151101 | Ψ | L±J |

- **(b)** Jamal found Company B who rents candy floss machine with a flat rate of \$10 and an additional rate of 25 cents per minute.
 - (i) Convert 25 cents per minute to dollar per hour. $0.25 \times 60 = 15$ [B1]

(ii) Draw the graph of Company B's rental rates for the 1st two hours on the same axes [1] as above.

| (iii) | From the graph, how long should Jamal rent the candy floss machine for her to be |
|-------|--|
| | paying the same amount of rental from both Companies. |

| | | | _ |
|--------|------------|----|---|
| Answer | l hour | 11 | ı |

18 (a) Given that
$$x^2 + 4x - 3 = (x + a)^2 + b$$
, find a and b, such that a and b are integers.

Integers.

$$x^{2} + 4x - 3 = \left(x + \frac{4}{2}\right)^{2} - \left(\frac{4}{2}\right)^{2} - 3 \text{ [M1]}$$

$$(x + 2)^{2} - 4 - 3 = (x + 2)^{2} - 7$$

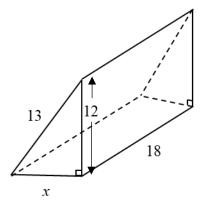
$$a = 2, b = -7 \text{ [A1]}$$

Answer
$$a = \dots$$
 [2]

(b) Hence, solve $x^2 + 4x - 3 = 0$, giving your answers correct to 2 decimal places.

[1 M if got the answer correct by other methods.]
[1 M ecf]

The diagram below shows a prism.
Three of its faces are rectangles.
All lengths are given in centimetres.



(a) Show that *x* is 5 cm.

By Pythagoras Theorem [M1]

$$x^2 = 13^2 - 12^2$$
 [A1]
 $x = \pm \sqrt{25}$
 $x = 5$ (Since length is positive)
[no marks deducted if did not state to reject -5]

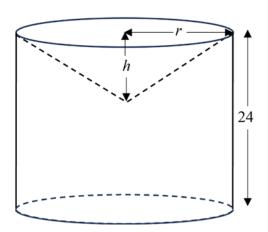
[2]

(b) Calculate the **total** surface area of the prism. Total surface area = $\left(2 \times \frac{1}{2} \times 5 \times 12\right) + (13 + 12 + 5) \times 18$ [M2] = 600 [A1]

[3]

A solid is formed by removing a cone of radius r cm from the top of a cylinder of radius **20** r cm. The height of cylinder is thrice the height of the cone.

The total volume of the solid is $533\frac{1}{3}\pi$ cm³.



Calculate the exact value of r.

Height of cone = $\frac{24}{3}$ = 8 cm Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{8}{3}\pi r^2$ [M1] Volume of cylinder = $\pi r^2 h = 24\pi r^2$ [M1]

Volume of solid: $533\frac{1}{3}\pi = 24\pi r^2 - \frac{8}{3}\pi r^2$ $533\frac{1}{3}\pi = \frac{64}{3}\pi r^2 \text{ [M1]}$ $r^2 = 25$ r = 5 (length is positive) [A1]

Answer
$$r = \dots$$
 [4]

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