ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS/ 5 NORMAL (ACADEMIC)

Candidate's Name	Class	Register Number
STUDENT SOLUTIONS		

ADDITIONAL MATHEMATICS 4049/02

PAPER 2 9 September 2024

Candidates answer on the Question Paper.
No Additional Materials are required.

2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

For Examiner's Use
90

Setter: Mr Francis Tan
Vetter: Mr Lionel Ang

1	**(a)**	When $t = 0$, $170000 = k + 150000e^0$ $170000 = k + 150000$ $k = 20000$	
	(b)	$140000 = 20000 + 150000e^n$ $120000 = 150000e^n$ $\frac{4}{5} = e^n$ $\ln\frac{4}{5} = n$ $n = -0.223144$ $n = -0.223$	
	(c)	$70000 < 20000 + 150000e^{-0.223t}$ $50000 < 150000e^{-0.223t}$ $\frac{1}{3} < e^{-0.223t}$ $\ln\frac{1}{3} < -0.223t$ $t < 4.92$ $t = 4$ David must sell the car in 2026	

2	**(a)**	General term $\left(px+\frac{2}{x}\right)^7$ $=\binom{7}{r}(px)^{7-r}\left(\frac{2}{x}\right)^r$ Consider the power of x $=7-r-r$ $=7-2r$ If there is an independent term, $7-2r=0$ $r=\frac{7}{2}$ Since r is not a whole number, there is no term independent of x and as such, every term is dependent on x
	(b)	$\left(px+\frac{2}{x}\right)^7(5-2x)$ Since there is no independent term in $\left(px+\frac{2}{x}\right)^7$, the only term to form the independent term is the $\frac{1}{x}$ in the expansion of $\left(px+\frac{2}{x}\right)^7(5-2x)$. Find the $\frac{1}{x}$ term Power of x: $7-2r=-1$ $r=4$ $\frac{1}{x}$ term $=\binom{7}{4}(px)^3\left(\frac{2}{x}\right)^4$ $=\frac{35p^3\times 16}{x}$ $=\frac{560p^3}{x}$

		Thus, the term independent of x $\frac{560p^3}{x}\times -2x=-241920$ $p^3=216$ $p=6$	
3	**(a)**	Consider $\angle ABD=\angle ADC$ (by tangent chord theorem) $\angle ADC=\angle BAD$ (alternate angles) Thus, $\angle ABD=\angle BAD$. Since there are 2 equal angles in triangle ABD, triangle ABD is isosceles.	
	(b)	Let $\angle ABD=x$, Since triangle ABD is isosceles (part(a)), $\angle BDA=180-2x$ (angle sum of triangle) by tangent chord theorem, $\angle ABD=\angle ADC=x$ Or $\angle ADC=180-[x+180-2x]$ $=x$ (interior angles) Since tangents from an external point are equal, triangle CDA is also isosceles. Thus, $\angle ADC=\angle CAD=x$ And $\angle DCA=180-2x=\angle BDA$ (shown)	
4	**(a)**	**(i)** $0^\circ\le\cos^{-1}x\le 180^\circ$ or $0\le\cos^{-1}x\le\pi$	
		(ii) $-90^\circ<\tan^{-1}x<90^\circ$ or $-\frac{\pi}{2}<\tan^{-1}x<\frac{\pi}{2}$	
	(b)	**(i)** $a=-\left(\frac{20-6}{2}\right)=-7$ $b=\frac{2\pi}{12}=\frac{\pi}{6}$ $c=13$	
		(ii) $14<x<16$	

5	**(a)**	$\cos 105$ $= \cos(60+45)$ $= \cos 60 \cos 45 - \sin 60 \sin 45$ $= \frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$ $= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$ $= \frac{\sqrt{2}-\sqrt{6}}{4}$ $\sec 105°$ $= \frac{1}{\cos 105}$ $= \frac{4}{\sqrt{2}-\sqrt{6}}$ $= \frac{4}{\sqrt{2}-\sqrt{6}} \times \frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}+\sqrt{6}}$ $= -\sqrt{2}-\sqrt{6}$		
	(b)	**(i)**	$f(x) = \frac{e^{3x}}{x-1}$ $f'(x) = \frac{3e^{3x}(x-1) - e^{3x}}{(x-1)^2}$ $f'(x) = \frac{e^{3x}(3x-4)}{(x-1)^2}$ $a = 3$ $b = -4$	
		(ii)	At y axis, $x = 0$. When x=0, $y = -1$ $f'(0) = -4$ Thus, gradient of normal $= \frac{1}{4}$ Equation of normal: $y = \frac{1}{4}x + c$ Sub x=0, $y = -1$ $c = -1$ $y = \frac{1}{4}x - 1$ When $y = 0$, $x = 4$ $P(4,0)$	

6 **(a)**

Let $f(x)=10x^3-9x^2-3x+2$

$$f(-\frac{1}{2})=10\left(-\frac{1}{2}\right)^3-9\left(-\frac{1}{2}\right)^2-3\left(-\frac{1}{2}\right)+2$$

$$f(-\frac{1}{2})=0$$

By factor theorem, $2x+1$ is a factor.

$$f(x)=10x^3-9x^2-3x+2$$

$$\begin{array}{rl}
 & 5x^2-7x+2 \\
2x+1 \big) & 10x^3-9x^2-3x+2 \\
- & 10x^3+5x^2 \\
\hline
 & -14x^2-3x+2 \\
- & -14x^2-7x \\
\hline
 & 4x+2 \\
- & 4x+2 \\
\hline
 & 0
\end{array}$$

$$f(x)=(2x+1)\left(5x^2-7x+2\right)$$

	x	-1
$5x$	$5x^2$	$-5x$
-2	$-2x$	2

$$f(x)=(2x+1)(5x-2)(x-1)$$

(b)

Let $3^y=a$.

$$5\left(a^2\right)+\frac{1}{a}=\frac{3}{2}(3a+1)$$

$$10a^2+\frac{2}{a}=9a+3$$

$$10a^3+2=9a^2+3a$$

$$10a^3-9a^2-3a+2=0$$

Comparing with (a)

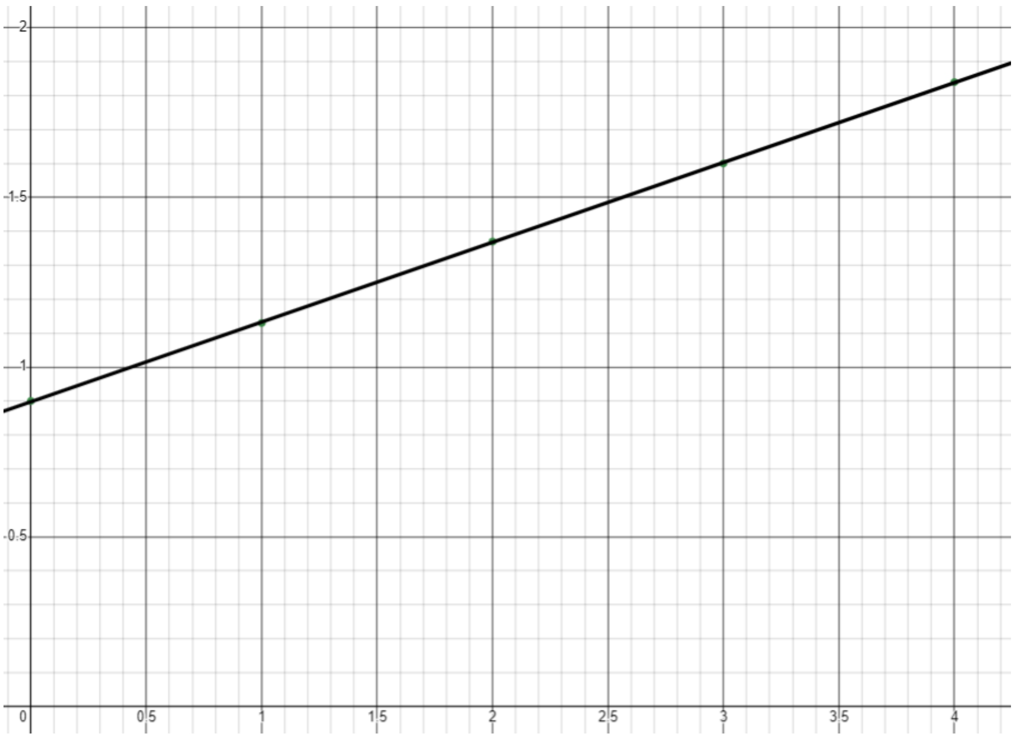
$$(2a+1)(5a-2)(a-1)=0$$

$$a=-\frac{1}{2} \quad \text{or} \quad a=\frac{2}{5} \quad \text{or} \quad a=1$$

		$3^y = -\frac{1}{2}$ or $3^y = \frac{2}{5}$ or $3^y = 1$ (reject as $3^y > 0$) or $y \ln 3 = \ln \frac{2}{5}$ $y = 0$ $y = -0.834$	
7	**(a)**	$v = 10\cos(5-2t) + 50$ Acceleration = $\frac{dv}{dt}$. $\frac{dv}{dt} = 10\left(-\sin(5-2t) \times -2\right)$ $\frac{dv}{dt} = 20\sin(5-2t)$ When $t = 0$, $20\sin(5)$ $= -19.2\ \text{m/s}^2$	
7	**(b)**	$v_c = \int \frac{-24}{(t+2)^2}\ dt$ $v_c = \int -24(t+2)^{-2}\ dt$ $v_c = 24(t+2)^{-1} + c$ When $t = 0$, $v_c = 5$ Thus, $5 = 12 + c$ $c = -7$ $v_c = 24(t+2)^{-1} - 7$ cyclist is instantaneously at rest $\Rightarrow v_c = 0$ $24(t+2)^{-1} - 7 = 0$ $\frac{24}{(t+2)} = 7$ $24 = 7(t+2)$ $24 = 7t + 14$ $t = \frac{10}{7}$	
	(c)	Displacement, $s = \int v_c\ dt$ $s = \int 24(t+2)^{-1} - 7\ dt$ $s = 24\ln(t+2) - 7t + c$	

		When $t=0$, $s=0$ $c=-24\ln 2$ $s=24\ln(t+2)-7t-24\ln 2$ At $t=\frac{10}{7}$, $s=24\ln\left(\frac{10}{7}+2\right)-7\left(\frac{10}{7}\right)-24\ln 2$ $s=2.93591\,\mathrm{m}$ At $t=10$, $s=24\ln(10+2)-7(10)-24\ln 2$ $s=-26.99777$ m  Total distance = 2.93591 + (2.93591+26.99777) =32.9m
8	**(a)**	$PQ=PD+DQ$ Consider triangle APD, $\frac{PD}{3}=\sin\theta$ $PD=3\sin\theta$ Consider triangle DCQ, $\frac{DQ}{2}=\cos\theta$ $DQ=2\cos\theta$ Thus, $PQ=3\sin\theta+2\cos\theta$
	(c)	$PQ=\sqrt{13}\sin(\theta+33.7)$ Maximum value = $\sqrt{13}$ $\sqrt{13}\sin(\theta+33.7)=\sqrt{13}$ $\sin(\theta+33.7)=1$ $\theta+33.7=90$ $\theta=56.3^\circ$
9	**(a)**	At time t, Distance from NEX to P = $10t$ Distance from NEX to Q = $1000-5t$ $PQ^2=(10t)^2+(1000-5t)^2$

		$PQ^2 = 100t^2 + 1000000 - 10000t + 25t^2$ $PQ = \sqrt{125t^2 - 10000t + 1000000}$ $s = \sqrt{1000000 - 10000t + 125t^2}$	
	(b)	Least distance occurs at minimum point i.e. $\frac{\mathrm{d}s}{\mathrm{d}t} = 0$ $\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{1}{2}\left(1000000 - 10000t + 125t^2\right)^{-\frac{1}{2}}\left(250t - 10000\right)$ $\frac{1}{2}\left(1000000 - 10000t + 125t^2\right)^{-\frac{1}{2}}\left(250t - 10000\right) = 0$ $250t - 10000 = 0$ $t = 40$ When $t = 40$, $s = \sqrt{1000000 - 10000(40) + 125(40)^2}$ $s = 894.4$ $s = 894\,\mathrm{m}$	

10	**(a)**		

Year	1980	1990	2000	2010	2020
x	0	1	2	3	4
P	2.45	3.09	3.94	4.95	6.30
$\ln P$	0.90	1.13	1.37	1.60	1.84

	(b)	From the graph,	

		m = 0.235 (± 0.2) c = 0.90 (± 0.2) $\ln P = 0.235x + 0.9$ $P = e^{0.235x+0.9}$ $P = 2.45e^{0.235x}$	
	(c)	When P = 13 $13 = 2.45e^{0.235x}$ $t = 7.10$ First year in the interval would be 2050.	

End of paper