RAFFLES INSTITUTION
2019 YEAR 6 PRELIMINARY EXAMINATION

CANDIDATE NAME

CLASS 19

MATHEMATICS **9758/01**

PAPER 1

3 hours

Candidates answer on the Question Paper
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

FOR EXAMINER'S USE					
Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**
Q7	**Q8**	**Q9**	**Q10**	**Q11**	**Total**

This document consists of **6** printed pages.

RAFFLES INSTITUTION
Mathematics Department

[Turn over

1 A curve C has equation

$$y = \frac{a}{x^3} + bx + c\,,$$

where a, b and c are constants. It is given that C has a stationary point $(-1.2, 6.6)$ and it also passes through the point $(2.1, -4.5)$.

(i) Find the values of a, b and c, giving your answers correct to 1 decimal place. [4]

(ii) One asymptote of C is the line with equation $x = 0$. Write down the equation of the other asymptote of C. [1]

2 Two variables u and v are connected by the equation $\frac{1}{u} + \frac{1}{v} = \frac{1}{20}$. Given that u and v both vary with time t, find an equation connecting $\frac{du}{dt}, \frac{dv}{dt}, u$ and v. Given also that u is decreasing at a rate of 2 units per second, calculate the rate of increase of v when $u = 60$ units.

[4]

3

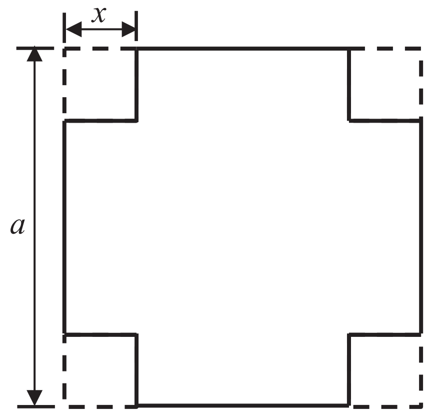


Fig. 1

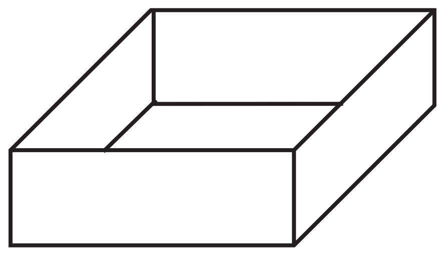


Fig. 2

Fig. 1 shows a square sheet of metal with side a cm. A square x cm by x cm is cut from each corner. The sides are then bent upwards to form an open box as shown in Fig. 2. Use differentiation to find, in terms of a, the maximum volume of the box, proving that it is a maximum. [6]

4 A curve C has parametric equations

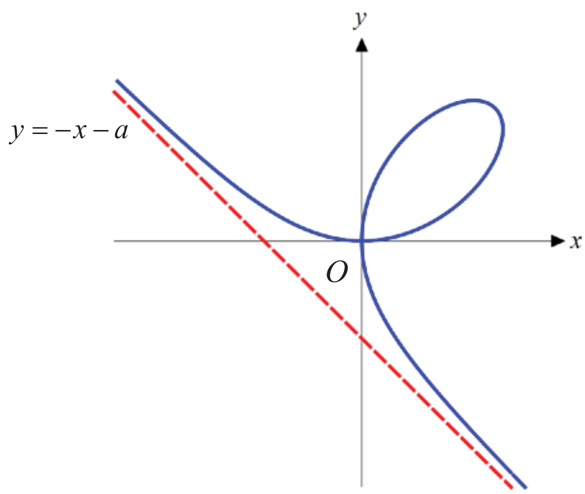
$$x=(1+t)^2, \qquad y=2(1-t)^2.$$

(i) Find the coordinates of the point A where the tangent to C is parallel to the x-axis. [4]

(ii) The line $y=-x+d$ intersects C at the point A and another point B. Find the exact coordinates of B. [4]

(iii) Find the area of the triangle formed by A, B and the origin. [1]

5



The diagram shows the graph of Folium of Descartes with cartesian equation

$$x^3+y^3=3axy,$$

where a is a positive constant. The curve passes through the origin, and has an oblique asymptote with equation $y=-x-a$.

(i) Given that (0, 0) is a stationary point on the curve, find, in terms of a, the coordinates of the other stationary point. [5]

(ii) Sketch the graph of

$$|x|^3+y^3=3a|x|y,$$

including the equations of any asymptotes, coordinates of the stationary points and the point where the graph crosses the x-axis. [3]

6 **(a)** Given that $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$, find an expression for $\sum_{r=n+1}^{2n} \left(r(r-1)\right)$, simplifying your answer. [3]

(b) **(i)** Use the method of differences to find $\sum_{r=1}^{n} \left(\frac{(r+1)-\mathrm{e}r}{\mathrm{e}^r} \right)$. [3]

(ii) Hence find $\sum_{r=2}^{n} \left(\frac{\mathrm{e}+r-\mathrm{e}r}{\mathrm{e}^{r-1}} \right)$. [2]

7 **(a)** The complex numbers $3+2\mathrm{i}$, z, $4-6\mathrm{i}$ are the first three terms in a geometric progression. Without using a calculator, find the two possible values of z. [4]

(b) **(i)** The complex number w is such that $w = a + \mathrm{i}b$, where a and b are non-zero real numbers. The complex conjugate of w is denoted by w^*. Given that $\frac{w^2}{w^*}$ is a real number, find the possible values of w in terms of a only. [4]

(ii) Hence, find the exact possible arguments of w if a is positive. [2]

8 **(a)** Find the exact value of m such that

$$\int_0^3 \frac{1}{9+x^2}\,\mathrm{d}x = \int_0^{\frac{1}{2m}} \frac{1}{\sqrt{1-m^2x^2}}\,\mathrm{d}x. \qquad [5]$$

(b) **(i)** Use the substitution $u = \sin 2x$ to show that

$$\int_0^{\frac{\pi}{4}} \sin^3 2x \cos^3 2x \;\mathrm{d}x = \frac{1}{2}\int_0^1 \left(u^3 - u^5\right)\;\mathrm{d}u. \qquad [4]$$

(ii) Hence find the exact value of $\int_0^{\frac{\pi}{4}} \sin^3 2x \cos^3 2x \;\mathrm{d}x$. [2]

9 **(i)** Write down $\int \frac{1}{a^2 - v^2} \, dv$, where a is a positive constant. [1]

(ii) In the motion of an object through a certain medium, the medium furnishes a resisting force proportional to the square of the velocity of the moving object. Suppose that a body falls vertically through the medium, the model used to describe the velocity, v ms^{-1} of the body at time t seconds after release from rest is given by the differential equation

$$\frac{a^2}{10}\frac{dv}{dt} = a^2 - v^2,$$

where a is a positive constant.

(a) Show that $v = a\left(\frac{e^{\frac{20t}{a}} - 1}{e^{\frac{20t}{a}} + 1}\right)$. [8]

(b) The rate of change of the displacement, x metres, of the body from the point of release is the velocity of the body. Given that $a = 2$, find the value of x when $t = 1$, giving your answer correct to 3 decimal places. [3]

10 The curve C_1 has equation $x^2 + y^2 = 25, \quad y \le 0$.

The curve C_2 has equation $y = x + 3 + \frac{24}{x-3}$.

(i) Verify that $(-3, -4)$ lies on both C_1 and C_2. [1]

(ii) Sketch C_1 and C_2 on the same diagram, stating the coordinates of any stationary points, points of intersection with the axes and the equations of any asymptotes. [4]

The region bounded by C_1 and C_2 is R.

(iii) Find the exact volume of solid obtained when R is rotated through 2π radians about the x-axis. [6]

The region bounded by C_1, the x-axis and the vertical asymptote of C_2, where $x > 3$, is S.

(iv) Write down the equation of the curve obtained when C_1 is translated by 3 units in the negative x-direction.

Hence, or otherwise, find the volume of solid obtained when S is rotated through 2π radians about the vertical asymptote of C_2. [4]

11 Path integration is a predominant mode of navigation strategy used by many animals to return home by the *shortest* possible route during a food foraging journey. In path integration, animals continuously compute a homebound global vector relative to their starting position by integrating the angles steered and distances travelled during the entire foraging run. Once a food item has been found, the animal commences its homing run by using the homebound global vector, which was acquired during the outbound run.

(a) A Honeybee's hive is located at the origin O. The Honeybee travels 6 units in the direction $-\mathbf{i}+2\mathbf{j}-2\mathbf{k}$ before moving 15 units in the direction $3\mathbf{i}-4\mathbf{k}$. The Honeybee is now at point A.

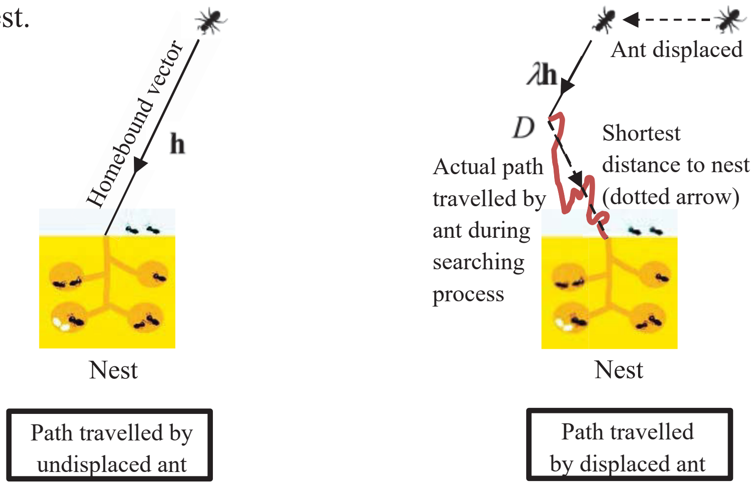
(i) Show that the homebound global vector $\overrightarrow{AO}$ is $-7\mathbf{i}-4\mathbf{j}+16\mathbf{k}$. Hence find the exact distance the Honeybee is from its hive. [3]

(ii) Explain why path integration may fail. [1]

A row of flowers is planted along the line $\dfrac{x-3}{5}=y+2,\ z=2$.

(iii) The Honeybee will take the shortest distance from point A to the row of flowers. Find the position vector of the point along the row of flowers which the Honeybee will fly to. [4]

(b) To further improve their chances of returning home, apart from relying on the path integration technique, animals depend on visual landmarks to provide directional information. When an ant is displaced to distant locations where familiar visual landmarks are absent, its initial path is guided solely by the homebound global vector, $\mathbf{h}$, until it reaches a point D and begins a search for their nest (see diagram). During the searching process, the distance travelled by the ant is 2.4 times the shortest distance back to the nest.



Let an ant's nest be located at the origin O. The ant has completed its foraging journey and is at a point with position vector $4\mathbf{i}+3\mathbf{j}$. A boy picks up the ant and displaces it 4 units in the direction $-\mathbf{i}$. Given $\lambda(-4\mathbf{i}-3\mathbf{j})$ as the initial path taken by the ant before it begins a search for its nest, find the value of λ which gives the minimum total distance travelled by the ant back to the nest. [4]

[It is not necessary to verify the nature of the minimum point in this part.]

RAFFLES INSTITUTION
2019 YEAR 6 PRELIMINARY EXAMINATION

CANDIDATE NAME

CLASS 19

MATHEMATICS 9758/02

PAPER 2

3 hours

Candidates answer on the Question Paper
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FOR EXAMINER'S USE							
SECTION A: PURE MATHEMATICS							
Q1	Q2	Q3	Q4	Total			TOTAL
SECTION B: PROBABILITY AND STATISTICS							
Q5	Q6	Q7	Q8	Q9	Q10	Total	
							100

This document consists of **7** printed pages.

RAFFLES INSTITUTION
Mathematics Department

[Turn over

Section A: Pure Mathematics [40 Marks]

1 The function f is defined as follows.

$$\mathrm{f}: x \mapsto \frac{1}{x^2}, \text{ for } x \in \mathbb{R},\ x \neq 0.$$

(i) Sketch the graph of $y = \mathrm{f}(x)$. [1]

(ii) If the domain of f is further restricted to $x > k$, state with a reason the least value of k for which the function f^{-1} exists. [2]

In the rest of this question, the domain of f is $x \in \mathbb{R},\ x \neq 0$, as originally defined.

A function h is said to be an odd function if $\mathrm{h}(-x) = -\mathrm{h}(x)$ for all x in the domain of h. The function g is defined as follows.

$$\mathrm{g}: x \mapsto \frac{2}{3^x - 1} + m, \text{ for } x \in \mathbb{R},\ x \neq 0.$$

(iii) Given that g is an odd function, find the value of m.

(iv) Using the value of m found in part **(iii)**, find the range of fg.

2 **(a)** The curve $y = \mathrm{f}(x)$ passes through the point (0, 81) and has gradient given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{1}{3}y - 15x\right)^{\frac{1}{3}}.$$

Find the first three non-zero terms in the Maclaurin series for y. [4

(b) Let $\mathrm{g}(x) = \dfrac{4 - 3x + x^2}{(1+x)(1-x)^2}$.

(i) Express $\mathrm{g}(x)$ in the form $\dfrac{A}{1+x} + \dfrac{1}{1-x} + \dfrac{B}{(1-x)^2}$, where A and B are constants to be determined.

The expansion of $\mathrm{g}(x)$, in ascending powers of x, is

$$c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots + c_r x^r + \ldots.$$

(ii) Find the values of c_0, c_1, and c_2 and show that $c_3 = 3$.

(iii) Express c_r in terms of r.

3 Referred to an origin O, the position vectors of three non-collinear points A, B and C are **a**, **b** and **c** respectively. The coordinates of A, B and C are $(-2,4,2)$, $(1,3,1)$ and $(0,1,2)$ respectively.

(i) Find $(\mathbf{a}-\mathbf{b})\times(\mathbf{c}-\mathbf{b})$. [1]

(ii) Hence

(a) find the exact area of triangle ABC,

(b) show that the cartesian equation of the plane ABC is $3x+2y+7z=16$.

(iii) The line l_1 has equation $\mathbf{r}=\begin{pmatrix}1\\1\\6\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\-1\end{pmatrix}$ for $\lambda\in\mathbb{R}$.

(a) Show that l_1 is parallel to the plane ABC but does not lie on the plane ABC.

(b) Find the distance between l_1 and plane ABC.

(iv) The line l_2 passes through B and is perpendicular to the xy-plane. Find the acute angle between l_2 and its reflection in the plane ABC, showing your working clearly.

4 **(a)** The non-zero numbers a, b and c are the first, third and fifth terms of an arithmetic series respectively.

(i) Write down an expression for b in terms of a and c. [1

(ii) Write down an expression for the common difference, d, of this arithmetic series in terms of a and b. [1

(iii) Hence show that the sum of the first ten terms can be expressed as

$$\frac{5}{4}(9c-a).$$ [2]

(iv) If a, b and c are also the fourth, third and first terms, respectively of a geometric series, find the common ratio of this series in terms of a and b and hence show that

$$(2b-a)a^2=b^3.$$

(b) The nth term of a geometric series is $\left(2\sin^2\alpha\right)^{n-1}$.

(i) Find all the values of α, where $0\le\alpha\le 2\pi$, such that the series is convergent.

(ii) For the values of α found in part **(i)**, find the sum to infinity, simplifying your ans

Section B: Probability and Statistics [60 Marks]

5 **(a)** The probability that a hospital patient has a particular disease is p. A test for the disease has a probability of 0.99 of giving a positive result when the patient has the disease and a probability of 0.95 of giving a negative result when the patient does not have the disease. A patient is given the test.

For a general value of p, the probability that a randomly chosen patient has the disease given that the result of the test is positive is denoted by $\mathrm{f}(p)$.

Find an expression for $\mathrm{f}(p)$ and show that f is an increasing function for $0 < p < 1$. Explain what this statement means in the context of this question. [5

(b) For events A and B, it is given that $\mathrm{P}(A) = 0.7$ and $\mathrm{P}(B) = k$.

(i) Given that A and B are independent events, find $\mathrm{P}(A \cap B')$ in terms of k.

(ii) Given instead that A and B are mutually exclusive events, state the range of values of k.

Find $\mathrm{P}(B \mid A')$ in terms of k.

6 Five objects a, b, c, d and e are to be placed in five containers A, B, C, D and E, with one in each container. An object is said to be correctly placed if it is placed in the container of the same letter (e.g. a in A) but incorrectly placed if it is placed in any of the other four containers. Find

(i) the number of ways the objects can be placed in the containers so that a is correctly placed and b is incorrectly placed, [2]

(ii) the number of ways the objects can be placed in the containers so that both a and b are incorrectly placed, [3]

(iii) the number of ways the objects can be placed in the containers so that there are at least 2 correct placings. [3]

7 Based on past records, at government polyclinics, on average each medical consultation lasts 15 minutes with a standard deviation of 10 minutes.

(i) Give a reason why a normal distribution, with this mean and standard deviation, would not give a good approximation to the distribution of consultation duration. [2]

From recent patient and doctor feedback, a polyclinic administrator claims that the average consultation duration at government polyclinic is taking longer. State suitable null and alternative hypotheses to test this claim.

(ii) Given that the null hypothesis will be rejected if the sample mean from a random sample of size 30 is at least 18, find the smallest level of significance of the test. State clearly any assumptions required to determine this value.

The administrator suspects the average consultation duration at private clinics is actually less than 15 minutes. A survey is carried out by recording the consultation duration, y, in minutes, from 80 patients as they enter and leave a consultation room at a private clinic. The results are summarized by

$$\sum(y-15) = -50\,,\ \sum(y-15)^2 = 555\,.$$

(iii) Calculate unbiased estimates of the population mean and variance of the consultation duration at private clinics. Determine whether there is sufficient evidence at the 5% level of significance to support the administrator's claim.

8 A biased cubical die has its faces marked with the numbers 1, 3, 5, 7, 11 and 13. The random variable X is defined as the score obtained when the die is thrown, with probabilities given by

$$\mathrm{P}(X = r) = kr\,,\ r = 1,3,5,7,11,13,$$

where k is a constant.

(i) Show that $\mathrm{P}(X = 3) = \dfrac{3}{40}$. [3]

(ii) Find the exact value of Var(X).

The die is thrown 15 times and the random variable R denotes the number of times that a score less than 10 is observed.

(iii) Find $\mathrm{P}(R \geq 5)$.

(iv) Find the probability that the last throw is the 8^{th} time that a score less than 10 is observed.

9 Rroar Tyre Company develops Brand R tyres. The working lifespan in kilometres of a Brand R tyre is a random variable with the distribution $N(64000, 8000^2)$.

(i) Find the probability that a randomly selected Brand R tyre has a working lifespan of at least 70000 km. [1

(ii) Rroar Tyre Company wishes to advertise that 98% of their Brand R tyres have working lifespans of more than t kilometres. Determine the value of t, correct to the nearest kilometre. [2

Ssoar Tyre Company, a rival company, develops Brand S tyres. The working lifespan in kilometres of a Brand S tyre is a random variable with the distribution $N(68000, \alpha^2)$.

(iii) A man selects 50 Brand S tyres at random. Given that $\alpha = 7500$, find the probability that the average of their working lifespans exceeds 70000 km.

(iv) Using $\alpha = 8000$, find the probability that the sum of the working lifespans of 3 randomly chosen Brand R tyres is less than 3 times the working lifespan of a randomly chosen Brand S tyre.

(v) Find the range of α, correct to the nearest kilometre, if there is a higher percentage of Brand S tyres than Brand R tyres lasting more than 50000 km.

(vi) State clearly an assumption needed for your calculations in parts **(iii)**, **(iv)** and **(v)**.

10 **(a)** With the aid of suitable diagrams, describe the differences between the least square linear regression line of y on x and that of x on y. [2]

(b) The government of the Dragon Island Country is doing a study on its population growth in order to implement suitable policies to support the aging population of the country. The population sizes, y millions in x years after Year 2000, are as follows.

x	9	10	11	12	14	15	16	17	18
Population size, y	5.05	5.21	5.32	5.41	5.51	5.56	5.60	5.65	5.67

(i) Draw a scatter diagram of these values, labelling the axes clearly. Use your diagram to explain whether the relationship between x and y is likely to be well modelled by an equation of the form $y = ax + b$, where a and b are constants.

(ii) Find, correct to 6 decimal places, the value of the product moment correlation coefficient between

(a) $\ln x$ and y,

(b) x^2 and y.

(iii) Use your answers to part **(ii)** to explain which of $y = a + b\ln x$ or $y = a + bx^2$ is the better model.

(iv) It is required to estimate the year in which the population size first exceed 6.5 millions. Use the model that you identified in part **(iii)** to find the equation of a suitable regression line, and use your equation to find the required estimate.

Comment on the reliability of this estimate.

(v) As the population size in Year 2013 is not available, a government statistician uses the regression line in part **(iv)** to estimate the population size in 2013. Find this estimate.

(vi) It was later found that in Year 2013, the population size was in fact 5.31 millions. Comment on this figure with reference to the estimate found in part **(v)**, providing a possible reason in context.

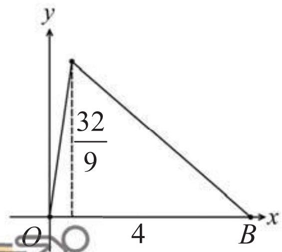
RAFFLES INSTITUTION
2019 YEAR 6 PRELIMINARY EXAMINATION

MATHEMATICS 9758/01 Suggested Solutions

SOLUTION		**COMMENTS**
1(i) **[4]**	$y=\frac{a}{x^3}+bx+c$ $\frac{dy}{dx}=-\frac{3a}{x^4}+b$ At $x=-1.2$, $\frac{dy}{dx}=0$, we have $-\frac{3a}{(-1.2)^4}+b=0$ ...(1) At $(-1.2,6.6)$, we have $\frac{a}{(-1.2)^3}-1.2b+c=6.6$ ...(2) At $(2.1,-4.5)$, we have $\frac{a}{(2.1)^3}+2.1b+c=-4.5$ ...(3) Using GC to solve (1), (2) and (3): $a=-2.03260\approx-2.0$, $b=-2.94068\approx-2.9$, $c=1.89491\approx1.9$	Generally well done for most students, except for a small number. These are the points to note: 1. Some students could not get the 3 equations as they didn't realise that the point $(-1.2, 6.6)$ could result in 2 equations instead of only one. 2. There were a number of students who could get the 3 equations but they end up with the wrong solutions. Do be careful when keying the equations into the GC! 3. A number of students differentiated wrongly: Eg. $\frac{dy}{dx}=-\frac{3a}{x^2}+b$.
(ii) **[1]**	$y=-2.9x+1.9$ (1 d.p.)	From equation of C, the other asymptote is $y=bx+c$. Most students were able to obtain a mark for this.

SOLUTION		COMMENTS
2 **[4]**	$\frac{1}{u}+\frac{1}{v}=\frac{1}{20}$ Differentiating w.r.t t, $-\frac{1}{u^2}\frac{du}{dt}-\frac{1}{v^2}\frac{dv}{dt}=0$ --------------- (1) When $u=60$, $\frac{1}{60}+\frac{1}{v}=\frac{1}{20}$ $\frac{1}{v}=\frac{1}{20}-\frac{1}{60}$ $\frac{1}{v}=\frac{1}{30}$ $v=30$ Substituting $v=30$ and $\frac{du}{dt}=-2$ into (1), $-\frac{1}{(60)^2}(-2)-\frac{1}{(30)^2}\frac{dv}{dt}=0$ $\frac{1}{1800}-\frac{1}{900}\frac{dv}{dt}=0$ $\frac{dv}{dt}=\frac{900}{1800}$ $=\frac{1}{2}$ $\therefore$Rate of increase of $v=\frac{1}{2}$ units/s	Probably 70% of students managed to do this. Some points to note: 1. Students are strongly advised to use implicit differentiation instead of making u or v the subject and applying quotient/product rule which often ends up with long and complicated expressions. 2. Some students mixed up differentiation and integration Eg. $\frac{d}{du}\left(\frac{1}{u}\right)=\ln\lvert u\rvert$. 3. Students are again reminded of the need to write clearly and properly as they mixed up u and v and hence obtained the wrong answer eventually.

SOLUTION		COMMENTS
3 **[6]**	$V=(a-2x)^2x=a^2x-4ax^2+4x^3$ $\frac{\mathrm{d}V}{\mathrm{d}x}=a^2-8ax+12x^2=0$ $(6x-a)(2x-a)=0$ $x=\frac{a}{6}$ or $x=\frac{a}{2}$ (rejected $\because V=0$) $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}=-8a+24x=-4a<0$ when $x=\frac{a}{6}$ Maximum $V=\left(a-2\left(\frac{a}{6}\right)\right)^2\left(\frac{a}{6}\right)=\frac{2a^3}{27}$ cm^3.	This question is generally well done. Some points to note are • Some students did not justify why the value of $x=\frac{a}{2}$ was rejected. Many students checked that this gives a minimum value using the second derivative test, a good strategy. • Many students solve for x by completing the square (of which a number made careless mistakes) when a direct factorization is much faster. Good for some of them to relearn this method. • Students who used the first derivative test to check for the nature of the root need to be reminded that the expression for dV/dx need to be in factorized form. • A small number of students were not aware that the a in the question is a constant and proceed to differentiate wrt a. Such students also tend to bring in expressions involving surface area showing that they are not sure of what the question is asking.

SOLUTION		COMMENTS
4 **(i)** **[4]**	$x=(1+t)^2, \quad y=2(1-t)^2.$ $\frac{\mathrm{d}x}{\mathrm{d}t}=2(1+t), \quad \frac{\mathrm{d}y}{\mathrm{d}t}=-4(1-t)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-4(1-t)}{2(1+t)}=\frac{-2(1-t)}{1+t}$ Tangent parallel to x-axis, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2(1-t)}{1+t}=0 \Rightarrow t=1$ Thus, the coordinates of point A is $(4,0)$.	This question is well done. The main mistakes for **(i)** are • giving $\frac{\mathrm{d}y}{\mathrm{d}t}=4(1-t)$ • the gradient of the tangent parallel to the x-axis is undefined.
(ii) **[4]**	Let the coordinates of B be $\left((1+b)^2, 2(1-b)^2\right)$, where $b \in \mathbb{R}$. $y=-x+d$ passes through $A(4,0) \Rightarrow d=4$ Sub $B: 2(1-b)^2=-(1+b)^2+4$ $2-4b+2b^2=-1-2b-b^2+4$ $3b^2-2b-1=0$ $(3b+1)(b-1)=0$ $b=-\frac{1}{3}$ or $b=1$ (point A) Thus, the coordinates of B is $\left(\frac{4}{9},\frac{32}{9}\right)$.	This part of the question asked for the **exact** coordinates, which means that the use of the GC is **not** allowed.
(iii) **[1]**	Area of triangle OAB $=\frac{1}{2}(4)\left(\frac{32}{9}\right)$ $=\frac{64}{9}$  	This part is well done.

<table>
<tr><th colspan="2">SOLUTION</th><th>COMMENTS</th></tr>
<tr><td>5
(i)
[5]</td><td>$x^3+y^3=3axy$
Differentiate with respect to x,
$3x^2+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}=3a\left(x\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)$
When $\frac{\mathrm{d}y}{\mathrm{d}x}=0$,
$3x^2=3ay\Rightarrow y=\frac{x^2}{a}$
Substitute $y=\frac{x^2}{a}$ into the given curve $3x^2=3ay\Rightarrow y=\frac{x^2}{a}$,
$x^3+y^3=3axy,$
$x^3+\frac{x^6}{a^3}=3x^3$
$x^6-2a^3x^3=0$
$x^3\left(x^3-2a^3\right)=0$
$x=0$ or $x=2^{\frac{1}{3}}a$
and, $y=0$ or $y=2^{\frac{2}{3}}a$
The required coordinates are $(0,0)$ (shown) and $\left(2^{\frac{1}{3}}a,2^{\frac{2}{3}}a\right)$.

For the following working, note that we need $y^2\neq ax$ to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.
$3x^2+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}=3a\left(x\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)$
$\left(y^2-ax\right)\frac{\mathrm{d}y}{\mathrm{d}x}=ay-x^2$
$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{ay-x^2}{y^2-ax}$ if $y^2\neq ax$ ...(*)
Letting (*) = 0 and solving will also give (0,0) as an answer, but would need to be rejected as $y^2\neq ax$.</td><td>Differentiation was well done. A few students made the mistakes:
- did not apply chain rule
- did not apply product rule

Many did not substitute $y=\frac{x^2}{a}$ back into the given curve.

Note a stationary point lies on the curve and has 0 gradient. It does NOT lie on the asymptote $y=-x-a$ or any other lines such as $y=x$.

Many made mistakes when manipulating the indices.

Some students did not simply their answers. In addition, please leave the answers in exact form when the question explicitly states that the coordinates are to be expressed "in terms of a".</td></tr>
<tr><td>(ii)
[3]</td><td>$x^3+y^3=3axy\xrightarrow{\text{Replace } x \text{ by } \lvert x\rvert}\lvert x\rvert^3+y^3=3a\lvert x\rvert y$</td><td>Some common mistakes:
- incomplete graph
- graph not symmetrical about y-axis
- (0, 0) was not drawn as stationary point with zero gradient</td></tr>
</table>

		- stationary points are the points with "maximum" y values, **NOT** the points furthest away from the origin. - label the asymptotes wrongly - curve not approaching the asymptotes as $x \to \infty$ - not labelling/labelling the required coordinates wrongly

SOLUTION		**COMMENTS**
6 (a) [3]	$\sum_{r=n+1}^{2n}\left(r(r-1)\right)$ $=\sum_{r=n+1}^{2n}\left(r^2-r\right)$ $=\sum_{r=n+1}^{2n} r^2-\sum_{r=n+1}^{2n} r$ $=\sum_{r=1}^{2n} r^2-\sum_{r=1}^{n} r^2-\frac{n}{2}(n+1+2n)$ $=\frac{1}{6}(2n)(2n+1)(4n+1)-\frac{1}{6}(n)(n+1)(2n+1)-\frac{1}{2}n(3n+1)$ $=\frac{1}{6}n\left[2(2n+1)(4n+1)-(n+1)(2n+1)-3(3n+1)\right]$ $=\frac{1}{6}n\left(14n^2-2\right)$ $=\frac{1}{3}n\left(7n^2-1\right)$ Alternatively, $\sum_{r=n+1}^{2n}\left(r^2-r\right)$ $=\sum_{r=n+1}^{2n} r^2-\sum_{r=n+1}^{2n} r$	This part was well done. However, quite a number of students lost 1 mark for not simplifying their answer. Another common mistake seen was miscalculation of the number of terms in the sum of AP , $\sum_{r=n+1}^{2n} r$.

	$=\sum_{r=1}^{2n} r^2-\sum_{r=1}^{n} r^2-\left(\sum_{r=1}^{2n} r-\sum_{r=1}^{n} r\right)$ $=\frac{1}{6}(2n)(2n+1)(4n+1)-\frac{1}{6}(n)(n+1)(2n+1)-\frac{1}{2}(2n)(2n+1)$ $+\frac{1}{2}n(n+1)$ $=\frac{1}{6}n\left[2(2n+1)(4n+1)-(n+1)(2n+1)-6(2n+1)+3(n+1)\right]$ $=\frac{1}{6}n\left(14n^2-2\right)$ $=\frac{1}{3}n\left(7n^2-1\right)$	
(b) (i) [3]	$\sum_{r=1}^{n}\left(\frac{(r+1)-\mathrm{e}r}{\mathrm{e}^r}\right)=\sum_{r=1}^{n}\left(\frac{(r+1)}{\mathrm{e}^r}-\frac{\mathrm{e}r}{\mathrm{e}^r}\right)$ $=\sum_{r=1}^{n}\left(\frac{(r+1)}{\mathrm{e}^r}-\frac{r}{\mathrm{e}^{r-1}}\right)$ $=\frac{2}{\mathrm{e}^1}-\frac{1}{\mathrm{e}^0}$ $+\frac{3}{\mathrm{e}^2}-\frac{2}{\mathrm{e}^1}$ $+\frac{4}{\mathrm{e}^3}-\frac{3}{\mathrm{e}^2}$ $\vdots$ $+\frac{n}{\mathrm{e}^{n-1}}-\frac{n-1}{\mathrm{e}^{n-2}}$ $+\frac{n+1}{\mathrm{e}^n}-\frac{n}{\mathrm{e}^{n-1}}$ $=\frac{n+1}{\mathrm{e}^n}-\frac{1}{\mathrm{e}^0}$ $=\frac{n+1}{\mathrm{e}^n}-1$	This part was well done. Some common mistakes seen were: • Missing out the $-$ sign in -1 • Writing the last term wrongly as $\frac{n+1}{\mathrm{e}^{n+1}}$
(b) (ii) [2]	$\sum_{r=2}^{n}\left(\frac{\mathrm{e}+r-\mathrm{e}r}{\mathrm{e}^{r-1}}\right)=\sum_{r+1=2}^{n}\left(\frac{\mathrm{e}+(r+1)-\mathrm{e}(r+1)}{\mathrm{e}^{(r+1)-1}}\right)$ $=\sum_{r=1}^{n-1}\left(\frac{(r+1)-\mathrm{e}r}{\mathrm{e}^r}\right)$ $=\frac{n}{\mathrm{e}^{n-1}}-1$	Take note of the "Hence" method required in this part. Many students failed to show proper working of using answer to **(b)(i)**. There were also a lot of mistakes in the change of lower and upper index of the summation.

	SOLUTION	COMMENTS
7 (a) [4]	$\frac{z}{3+2\text{i}} = \frac{4-6\text{i}}{z}$ $\Rightarrow z^2 = (4-6\text{i})(3+2\text{i}) = 24-10\text{i}$ Let $z = a+b\text{i}, \quad a,b \in \mathbb{R}$ $(a+b\text{i})^2 = 24-10\text{i}$ $\Rightarrow a^2 - b^2 + 2ab\text{i} = 24-10\text{i}$ Comparing Real and Imaginary parts, $a^2 - b^2 = 24$ $2ab = -10 \Rightarrow ab = -5$ $\Rightarrow \frac{25}{b^2} - b^2 = 24$ $\Rightarrow b^4 + 24b^2 - 25 = 0$ ------- (*) $\Rightarrow (b^2+25)(b^2-1) = 0$ $\Rightarrow b = \pm 1$ $\Rightarrow a = \mp 5$ The two possible complex numbers are $-5+\text{i}$ or $5-\text{i}$.	Students need to understand that when a question states "without using a calculator", they need to show all working no matter how trival it seemed to them. Many lose marks for failing to show the proper factorization of the quartic equation (*).
(b) (i) [4]	$\frac{w^2}{w^*} = \frac{(a+\text{i}b)^2}{a-\text{i}b}$ $= \frac{(a+\text{i}b)^3}{a^2+b^2}$ $= \frac{a^3 + 3\text{i}a^2b - 3ab^2 - \text{i}b^3}{a^2+b^2}$ $= \frac{(a^3-3ab^2) + \text{i}(3a^2b - b^3)}{a^2+b^2}$ $\frac{w^2}{w^*}$ is purely real $\text{Im}\left(\frac{w^2}{w^*}\right) = 0 \Rightarrow \frac{3a^2b - b^3}{a^2+b^2} = 0$ $\Rightarrow b(3a^2 - b^2) = 0$ $\Rightarrow b = 0$ (rejected, since $b \neq 0$) or $b = \pm a\sqrt{3}$ Thus, $w = a + \text{i}a\sqrt{3}$ or $w = a - \text{i}a\sqrt{3}$.	Generally well done for students who use the first method, except for a few who forgot to reject $b = 0$.

	Alternative method 1: $\arg\left(\frac{w^2}{w^*}\right)=k\pi, \quad \text{where } k\in\mathbb{Z}$ $3\arg w=k\pi$ $\arg w=\frac{k\pi}{3}$ Since a and b are non-zero, $\tan^{-1}\left(\left\lvert\frac{b}{a}\right\rvert\right)=\frac{\pi}{3}$ $\lvert b\rvert=\lvert a\rvert\sqrt{3}$ $b=a\sqrt{3}$ or $-a\sqrt{3}$ Thus, $w=a+\mathrm{i}a\sqrt{3}$ or $w=a-\mathrm{i}a\sqrt{3}$. **Alternative method 2:** Let $\frac{w^2}{w^*}=k$, where $k\in\mathbb{R}$ $\frac{(a+\mathrm{i}b)^2}{a-\mathrm{i}b}=k$ $a^2-b^2+\mathrm{i}(2ab)=ka-\mathrm{i}(kb)$ Comparing real and imaginary parts, $a^2-b^2=ka$ ----------- (1) $2ab=-kb$ ------------ (2) From (2), $k=-2a$ Substitute into (1): $a^2-b^2=-2a^2$ $b^2=3a^2$ $b=a\sqrt{3}$ or $-a\sqrt{3}$	Students who use the argument method (Alternative Method 1) mostly wrote $\arg w=\tan^{-1}\left(\frac{b}{a}\right)$ Students who wrote this were not penalized in this exam, but please do note that this is not correct.
(b) (ii) [2]	Basic angle $=\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)=\frac{\pi}{3}$ Thus, $\arg(w)=\frac{\pi}{3}$ or $\arg(w)=-\frac{\pi}{3}$.	

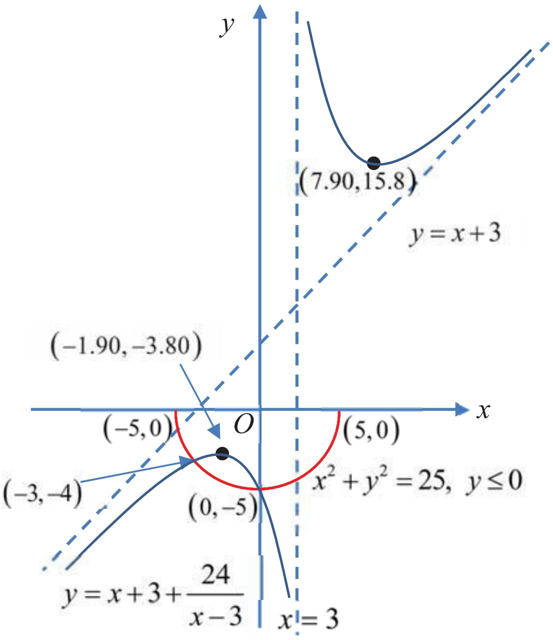
SOLUTION		COMMENTS
8(a) **[5]**	$\int_0^3 \frac{1}{9+x^2}\,dx = \left[\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\right]_0^3$ $= \frac{1}{3}\left(\frac{\pi}{4}\right)$ $= \frac{\pi}{12}.$ $\int_0^{\frac{1}{2m}} \frac{1}{\sqrt{1-m^2x^2}}\,dx = \frac{1}{m}\int_0^{\frac{1}{2m}} \frac{m}{\sqrt{1-(mx)^2}}\,dx$ $= \frac{1}{m}\left[\sin^{-1}(mx)\right]_0^{\frac{1}{2m}}$ $= \frac{1}{m}\left[\sin^{-1}\left(\frac{1}{2}\right)-0\right]$ $= \frac{\pi}{6m}.$ So $\int_0^{\frac{1}{2m}} \frac{1}{\sqrt{1-m^2x^2}}\,dx = \int_0^3 \frac{1}{9+x^2}\,dx$ $\frac{\pi}{6m} = \frac{\pi}{12}$ $m = 2$	Common mistakes: a) Exclude the $\frac{1}{m}$ in the $\frac{1}{m}\sin^{-1}(mx)$; b) Wrong value for $\tan^{-1}\left(\frac{3}{3}\right)$ or $\sin^{-1}\left(\frac{1}{2}\right)$. Some students resorted to using calculator to compute $\frac{\sin^{-1}\left(\frac{1}{2}\right)}{\tan^{-1}\left(\frac{3}{3}\right)}$, which illustrated that they do not have knowledge of the special angles. Note that since question asks for exact value of m, calculator should not be used in the computation.
(b) **[6]**	$u = \sin 2x \Rightarrow \frac{du}{dx} = 2\cos 2x$ When $x = 0,\ u = 0.$ When $x = \frac{\pi}{4},\ u = 1.$ $\int_0^{\frac{\pi}{4}} \sin^3 2x\cos^3 2x\ dx = \int_0^{\frac{\pi}{4}} \sin^3 2x\cos^2 2x\cos 2x dx$ $= \frac{1}{2}\int_0^{\frac{\pi}{4}} (\sin^3 2x)(1-\sin^2 2x)(2\cos 2x)dx$ $= \frac{1}{2}\int_0^1 u^3(1-u^2)\ du$ $= \frac{1}{2}\int_0^1 (u^3-u^5)\ du$	Some students did this: $\frac{du}{dx} = -2\cos 2x$ which is less than desirable. Very common presentation error: $\frac{1}{2}\int_0^1 (u^3)(\cos^3 2x)\frac{du}{\cos 2x}$ and the variations of mixing x and u in the integrand, or even integrating wrt to u but the limits are still in x. Some students failed to use the "hence" for the last numerical

	$\int_0^{\frac{\pi}{4}} \sin^3 2x\cos^3 2x \ \mathrm{d}x = \frac{1}{2}\int_0^1 \left(u^3 - u^5\right) \ \mathrm{d}u$ $= \frac{1}{2}\left[\frac{u^4}{4} - \frac{u^6}{6}\right]_0^1$ $= \frac{1}{2}\left[\frac{1}{4} - \frac{1}{6}\right]$ $= \frac{1}{24}$	computation and reworked from the start.

SOLUTION		COMMENTS
9(i) **[1]**	$\int \frac{1}{a^2 - v^2} \ \mathrm{d}v = \frac{1}{2a}\ln\left\|\frac{a+v}{a-v}\right\| + c$	This part was very poorly done. Most students either missed out on the modulus sign or the +c.
(ii) **(a)** **[8]**	$\frac{a^2}{10}\frac{\mathrm{d}v}{\mathrm{d}t} = a^2 - v^2$ $\frac{1}{a^2 - v^2}\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{10}{a^2}$ $\int \frac{1}{a^2 - v^2} \ \mathrm{d}v = \int \frac{10}{a^2}\mathrm{d}t$ $\frac{1}{2a}\ln\left\|\frac{a+v}{a-v}\right\| = \frac{10}{a^2}t + C$, where C is an arbitrary constant $\ln\left\|\frac{a+v}{a-v}\right\| = \frac{20t}{a} + D$, where $D = 2aC$ $\frac{a+v}{a-v} = Ae^{\frac{20t}{a}}$, where $A = \pm e^D$ Given that $v = 0$ when $t = 0$, $A = 1$.	Most students were able to do this part relatively well. A few points to take note 1. There should be a modulus in the natural log. 2. The modulus will be taken care of when we take $A = \pm e^D$ and then substitute initial conditions to calculate for A. 3. Some students substituted the initial conditions too early to calculate D, obtaining D=0. This is insufficient because upon simplifying the equation by removing the modulus sign, you will still get $\pm$. In this case you will need to substitute the initial conditions again to decide that the positive form is to be retained. Otherwise, students generally have no problems expanding

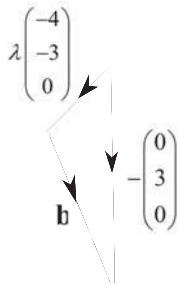
	$\dfrac{a+v}{a-v}=\mathrm{e}^{\frac{20t}{a}}$ $a+v=(a-v)\mathrm{e}^{\frac{20t}{a}}$ $\left(1+\mathrm{e}^{\frac{20t}{a}}\right)v=a\left(\mathrm{e}^{\frac{20t}{a}}-1\right)$ $v=a\left(\dfrac{\mathrm{e}^{\frac{20t}{a}}-1}{\mathrm{e}^{\frac{20t}{a}}+1}\right)$ (shown)	and rearranging the terms to obtain the final answer.
(ii) **[3]**	Since $a=2$, $v=2\left(\dfrac{\mathrm{e}^{10t}-1}{\mathrm{e}^{10t}+1}\right)$. $x=2\displaystyle\int_0^1 \frac{\mathrm{e}^{10t}-1}{\mathrm{e}^{10t}+1}\,\mathrm{d}t$ $=1.723$ metres (3 d.p.)	In this part of the question, many students wrongly substituted $t=1$ to find the value of v at that point. Do note that we are looking for x, not v. Many students did the integration manually without using GC, which led to many mistakes. Some examples: 1. Wrong method of integration. 2. Applying indefinite integration and then forgetting or wrongly calculating the value of the arbitrary constant c. Some students left their answer in 3 significant figures instead of 3 decimal places.

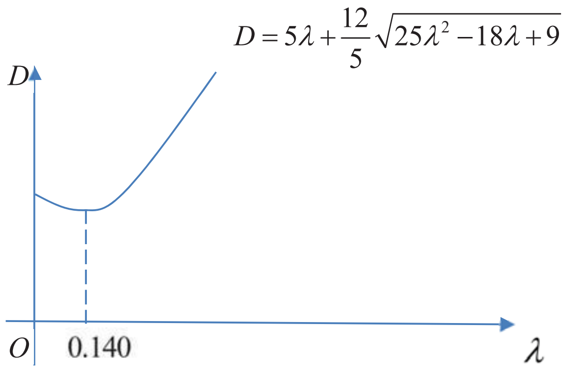
SOLUTION		COMMENTS
10 **(i)** **[1]**	For $x^2 + y^2 = 25, \quad y \le 0$ $(-3)^2 + (-4)^2 = 25.$ For $y = x + 3 + \dfrac{24}{x-3}$, $x = -3,\ y = -3 + 3 + \dfrac{24}{-3-3} = -4.$	This part is done well.

(ii) **[4]**	 Asymptotes : $y = x+3$ and $x = 3$	Graphs were generally sketched well. The only errors seen were related to inappropriate points i.e. (5, 0) being further to the right of (7.90, 15.8) or the curve C_2 not being sketched completely. A significant number of candidates used differentiation to compute the turning points when the graphing calculator could have been used.
(iii) **[6]**	Volume of solid obtained when R is rotated through 2π radians about the x-axis $= \pi \int_{-3}^{0} (25 - x^2) - \left(x + 3 + \frac{24}{x-3}\right)^2 \, dx$ $= \pi \int_{-3}^{0} \left[25 - x^2 - (x+3)^2 - 48\left(\frac{x+3}{x-3}\right) - \frac{24^2}{(x-3)^2} \right] dx$ $= \pi \int_{-3}^{0} \left[25 - x^2 - (x+3)^2 - 48\left(1 + \frac{6}{x-3}\right) - \frac{24^2}{(x-3)^2} \right] dx$ $= \pi \left[25x - \frac{x^3}{3} - \frac{(x+3)^3}{3} - 48\left(x + 6\ln\lvert x-3 \rvert\right) + \frac{24^2}{(x-3)} \right]_{-3}^{0}$ $= \pi \left(-\frac{27}{3} - 48(6\ln 3) + \frac{24^2}{-3} - 25(-3) + \frac{(-3)^3}{3} + 48(-3 + 6\ln 6) - \frac{24^2}{-6} \right)$ $= \pi(-9 - 192 + 75 - 9 - 144 + 96 - 288\ln 3 + 288\ln 6)$ $= \pi(288\ln 2 - 183)$	The instruction "Find the exact volume" was ignored by some candidates who had set up the integrals correctly but went on to use the calculator to compute the volume. It was necessary to realize that the points of intersection (–3, –4) and (0, –5) were already verified and found in parts **(i)** and **(ii)** respectively. Note $x + 3 + \frac{24}{x-3} \neq \frac{x^2 - 15}{x-3}$.

(iv) **[4]**	Translating the graph of $x^2+y^2=25$ 3 units in the negative x-direction, we have $(x+3)^2+y^2=25$. $(x+3)^2+y^2=25$ $(x+3)^2=25-y^2$ $x=-3\pm\sqrt{25-y^2}$ $x=-3+\sqrt{25-y^2}$ for $x\geq 3$ Volume of solid obtained when S is rotated through 2π radians about the line $x=3$ $=\pi\int_{-4}^{0}\left(-3+\sqrt{25-y^2}\right)^2\,\mathrm{d}y$ $=28.6$ (3 s.f.)	Most candidates were able to write down the equation for the translated graph but many went on to use the original equation and make y the subject to find the volume generated using integration of the region rotated about the x-axis when the question stated "rotated … about the vertical asymptote" which meant that the y-axis would be a more likely option.

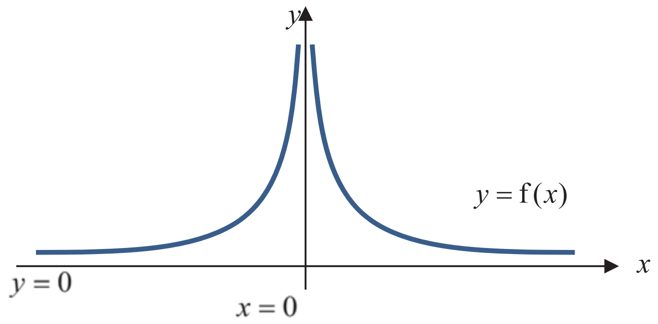
	SOLUTION	COMMENTS
11(i) **[3]**	$\overrightarrow{OA}=6\left[\frac{1}{3}\begin{pmatrix}-1\\2\\-2\end{pmatrix}\right]+15\left[\frac{1}{5}\begin{pmatrix}3\\0\\-4\end{pmatrix}\right]=\begin{pmatrix}-2\\4\\-4\end{pmatrix}+\begin{pmatrix}9\\0\\-12\end{pmatrix}=\begin{pmatrix}7\\4\\-16\end{pmatrix}$ Homebound global vector = $\overrightarrow{AO}=\begin{pmatrix}-7\\-4\\16\end{pmatrix}$ (shown) Distance from the hive $=\left\|\begin{pmatrix}-7\\-4\\16\end{pmatrix}\right\|=\sqrt{7^2+4^2+16^2}=\sqrt{321}$	A significant minority of students forgot to scale the directions to unit vectors. Some students missed this part.
(ii) **[1]**	The homebound vector may be blocked by obstacles along the path.	Any reasonable response is acceptable. However, anything blaming the poor bee is not, and neither are general statements that something can go wrong. Only a few students realized that (b) actually provided a plausible answer here.
(iii) **[4]**	Equation of line: $l:\mathbf{r}=\begin{pmatrix}3\\-2\\2\end{pmatrix}+\mu\begin{pmatrix}5\\1\\0\end{pmatrix},\mu\in\mathbb{R}$ Let N be the foot of perpendicular from A to the line. $\overrightarrow{ON}=\begin{pmatrix}3\\-2\\2\end{pmatrix}+\mu\begin{pmatrix}5\\1\\0\end{pmatrix}$, for some $\mu\in\mathbb{R}$ $\overrightarrow{AN}=\begin{pmatrix}3\\-2\\2\end{pmatrix}+\mu\begin{pmatrix}5\\1\\0\end{pmatrix}-\begin{pmatrix}7\\4\\-16\end{pmatrix}=\begin{pmatrix}-4+5\mu\\-6+\mu\\18\end{pmatrix}$ Since $\overrightarrow{AN}$ is perpendicular to the line,	This part is generally handled proficiently by most students.

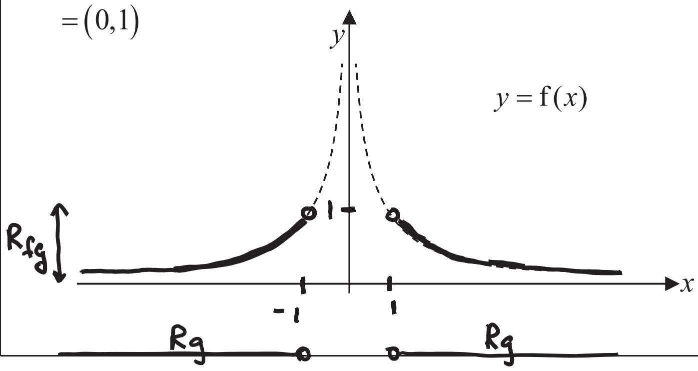
	$\overrightarrow{AN}\bullet\begin{pmatrix}5\\1\\0\end{pmatrix}=0$ $\begin{pmatrix}-4+5\mu\\-6+\mu\\18\end{pmatrix}\bullet\begin{pmatrix}5\\1\\0\end{pmatrix}=0$ $\Rightarrow -20+25\mu-6+\mu=0$ $\Rightarrow \mu=1$ $\Rightarrow \overrightarrow{ON}=\begin{pmatrix}8\\-1\\2\end{pmatrix}$	
(b) **[4]**	Let **b** be the direction vector back to the nest from any point along the homebound vector.  $\lambda\begin{pmatrix}-4\\-3\\0\end{pmatrix}+\mathbf{b}=-\begin{pmatrix}0\\3\\0\end{pmatrix}$ $\mathbf{b}=\lambda\begin{pmatrix}4\\3\\0\end{pmatrix}-\begin{pmatrix}0\\3\\0\end{pmatrix}=\begin{pmatrix}4\lambda\\3\lambda-3\\0\end{pmatrix}$ Distance to travel from the position vector $3\mathbf{j}$ back to the origin, D = Distance travelled via homebound vector + Distance travelled via searching process $=\left\|\begin{pmatrix}-4\lambda\\-3\lambda\\0\end{pmatrix}\right\|+2.4\left\|\begin{pmatrix}4\lambda\\3\lambda-3\\0\end{pmatrix}\right\|$ $=\sqrt{16\lambda^2+9\lambda^2}+\frac{12}{5}\sqrt{16\lambda^2+(3\lambda-3)^2}$ $=5\lambda+\frac{12}{5}\sqrt{25\lambda^2-18\lambda+9}$ Differentiating with respect to λ, $\frac{dD}{d\lambda}=5+\frac{6}{5}\left(16\lambda^2+(3\lambda-3)^2\right)^{-1/2}\left(32\lambda+6(3\lambda-3)\right)$ $5+\frac{6}{5}\left(16\lambda^2+(3\lambda-3)^2\right)^{-1/2}\left(32\lambda+6(3\lambda-3)\right)=0$ From G.C., $\lambda\approx 0.140$	Students' success in this part mostly relies on interpreting the question correctly and quality of responses various greatly. Many students who got here squared to remove the radical. This is not only unnecessary, but also introduced a spurious solution that needs to be rejected.

	OR  From G.C., $\lambda \approx 0.140$	

RAFFLES INSTITUTION
2019 YEAR 6 PRELIMINARY EXAMINATION

MATHEMATICS 9758/02 Suggested Solution

SOLUTION		COMMENTS
1(i) **[1]**	 	Students should be careful in showing asymptotic behavior and in the labeling of the two asymptotes $x=0$ (y-axis), $y=0$ (x-axis).
1(ii) **[2]**	Least value of $k=0$. If the domain of f is restricted to $x>0$, then for any horizontal line $y=h,\ h\in\mathbb{R}^+$, it cuts the graph of f at one and only one point, therefore f is a 1-1 function. Hence funcion f^{-1} exists.	For the horizontal line test to show that f is 1-1, many did not state that it is for any $y=h$, $h\in\mathbb{R}^+$ (which is the range of f) that cuts the graph of f at one and only one point.
1(iii) **[2]**	Consider $g(-x)=-g(x)$, with $x=1$. $\frac{2}{3^{-1}-1}+m=-\left(\frac{2}{3-1}+m\right)$ $2m=3-1$ $m=1$ NOTE : We may attempt this part with any specific value of x (in the domain of g).	Many did not see that since it is given that g is an odd function, then $g(-x)=-g(x)$ holds for any x in the domain. For those who tried to solve it for a general value of x, some did not simplify m to $m=\frac{-1}{3^x-1}-\frac{1}{3^{-x}-1}$ $=\frac{-1}{3^x-1}-\frac{3^x}{1-3^x}$. $=\frac{3^x-1}{3^x-1}=1$
1(iv) **[2]**	$R_g=(-\infty,-1)\cup(1,\infty)$.	R_g is found by looking at the asymptotic behavior of g as

	$R_{fg} = R_f$ with domain restricted to $(-\infty,-1)\cup(1,\infty)$ $=(0,1)$  	$x \to \pm\infty$, which gives the y-values of g as $y<-1$ or $y>1$. Note that equivalent set notations for $(-\infty,-1)\cup(1,\infty)$ are $\mathbb{R}\setminus[-1,1]$, or $(-\infty,\infty)\setminus[-1,1]$. Note also that $\cup$ should not be written as "or", "and", "$\cap$", ",".

SOLUTION		COMMENTS
2(a) **[4]**	When $x=0$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\left(\frac{81}{3}-15(0)\right)^{\frac{1}{3}}=3$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{3}\left(\frac{1}{3}y-15x\right)^{-\frac{2}{3}}\left(\frac{1}{3}\frac{\mathrm{d}y}{\mathrm{d}x}-15\right)$ When $x=0$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{3}\left(\frac{81}{3}-15(0)\right)^{-\frac{2}{3}}\left(\frac{1}{3}(3)-15\right)=-\frac{14}{27}$ Hence $y=81+3x-\frac{7}{27}x^2+\ldots$	Students should not present the following: $\frac{\mathrm{d}y}{\mathrm{d}x}=\left(\frac{y}{3}-15x\right)^{\frac{1}{3}}$ $=\left(\frac{81}{3}-15(0)\right)^{\frac{1}{3}}\cdots(*)$ (*) is only true when $\frac{\mathrm{d}y}{\mathrm{d}x}$ is evaluated at (0, 81). The presentation of $y=81+3x-\frac{7}{27}x^2+\ldots$ is not ideal for a significant number of students. Students should use $\approx$ or include $+\ldots$ in their answer.
2(b) **(i)** **[2]**	$\frac{4-3x+x^2}{(1+x)(1-x)^2}=\frac{A}{1+x}+\frac{1}{1-x}+\frac{B}{(1-x)^2}$ Therefore, $4-3x+x^2=A(1-x)^2+(1+x)(1-x)+B(1+x)$ When $x=1$: $2=2B \Rightarrow B=1$ When $x=-1$: $8=4A \Rightarrow A=2$	Quite a number of students tried to solve partial fractions in the form $\frac{A}{1+x}+\frac{C}{1-x}+\frac{B}{(1-x)^2}$, which involves solving 3 unknowns and is not necessary at all. A handful of students used

		$\frac{A}{1+x}+\frac{B}{1-x}+\frac{C}{(1-x)^2}$, i.e. the same unknowns as the ones given in the question, which is not accepted.
2(b) (ii) [3]	$g(x)=\frac{4-3x+x^2}{(1+x)(1-x)^2}$ $=\frac{2}{1+x}+\frac{1}{1-x}+\frac{1}{(1-x)^2}$ $=2(1-x+x^2-x^3+...)+(1+x+x^2+x^3+...)$ $+(1+2x+3x^2+4x^3+...)$ $=4+x+6x^2+3x^3+...$ Hence $c_0=4, c_1=1, c_2=6,$ and $c_3=3$.	A good number of students differentiated $g(x)$ repeatedly and obtained the following: $c_0=g(0), c_1=g'(0),$ $c_2=\frac{g''(0)}{2!}$ and $c_3=\frac{g'''(0)}{3!}$. They will arrive at the same answer, however, the working seems significantly longer. Please note that $(1+x)^{-1}$ and $(1-x)^{-1}$ could be obtained easily from MF26. Also note that, $\frac{1}{(1-x)^2}=\frac{d}{dx}\left(\frac{1}{1-x}\right)$.
2(b) (iii) [1]	From above, $c_r=2(-1)^r+1+(r+1)=2(-1)^r+r+2$.	Students should note that c_r only refers to the coefficient of the x^r term.

<table>
<tr><th colspan="2">SOLUTION</th><th>COMMENTS</th></tr>
<tr><td colspan="3">General Comments:

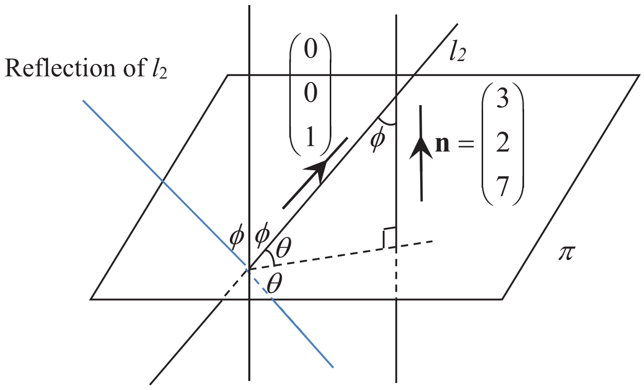
The question is well done in general, but many parts (ii)(b), (iii)(a), (iv) require detailed and clear explanations as they are show questions or the question explicitly required clear working. For example, in such show questions, scalar products should be expanded in full.

For (iv), there are many correct numerical answers but not all obtained full credit. For students who obtained $2\cos^{-1}\left(\frac{7}{\sqrt{62}}\right)=54.5^\circ$, the angle in question must be clearly defined or labelled in a diagram for full credit to be awarded.</td></tr>
<tr><td>3(i)
[1]</td><td>$(\mathbf{a}-\mathbf{b})\times(\mathbf{c}-\mathbf{b})=\begin{pmatrix}-3\\1\\1\end{pmatrix}\times\begin{pmatrix}-1\\-2\\1\end{pmatrix}=\begin{pmatrix}3\\2\\7\end{pmatrix}$.</td><td>When the position vectors/coordinates are given, there is no need to evaluate using properties of cross product and end up doing more work.</td></tr>
<tr><td>3(ii)
(a)
[1]</td><td>Hence exact area of triangle ABC is $\frac{1}{2}\left|\begin{pmatrix}3\\2\\7\end{pmatrix}\right|=\frac{\sqrt{62}}{2}$.</td><td>Do not forget the $\frac{1}{2}$.</td></tr>
<tr><td>3(ii)
(b)
[1]</td><td>Hence the equation of the plane ABC is given by
$\mathbf{r}\cdot\begin{pmatrix}3\\2\\7\end{pmatrix}=\begin{pmatrix}0\\1\\2\end{pmatrix}\cdot\begin{pmatrix}3\\2\\7\end{pmatrix}=2+14=16$.
Hence the cartesian equation of the plane is $3x+2y+7z=16$.</td><td>Choose a point on the plane with more 0's in the coordinates– the corresponding scalar product is easier to evaluate.</td></tr>
<tr><td>3(iii)
(a)
[3]</td><td>$\begin{pmatrix}1\\2\\-1\end{pmatrix}\cdot\begin{pmatrix}3\\2\\7\end{pmatrix}=3+4-7=0$
Since the line is perpendicular to the normal vector of the plane, the line and the plane are parallel.

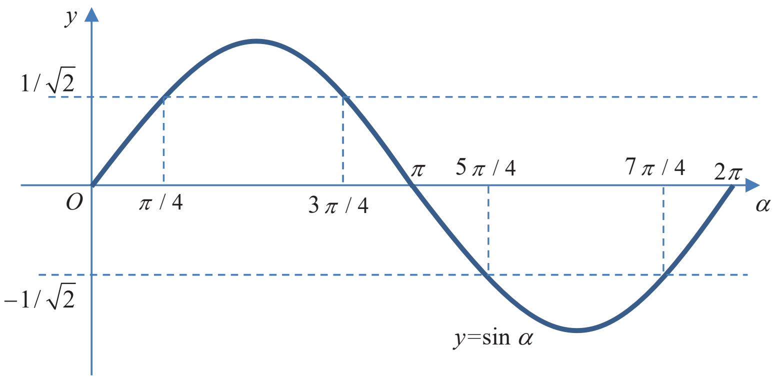
Since $\begin{pmatrix}1\\1\\6\end{pmatrix}\cdot\begin{pmatrix}3\\2\\7\end{pmatrix}=3+2+42=47\neq16$, a point on the line does not lie on the plane, and since the line and the plane are parallel, the line does not lie on the plane.</td><td>The scalar product being 0 only shows that the line and the normal vector of the plane are perpendicular. This relation has to be clearly stated.

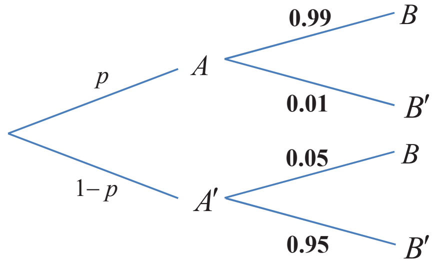
$\begin{pmatrix}1\\1\\6\end{pmatrix}\cdot\begin{pmatrix}3\\2\\7\end{pmatrix}\neq16$ only shows the point does not lie on the plane.</td></tr>
</table>

	Alternatively, $\left[\begin{pmatrix}1\\1\\6\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\-1\end{pmatrix}\right]\cdot\begin{pmatrix}3\\2\\7\end{pmatrix}=47\neq 16$ for all real λ so there is no intersection between the line and the plane. This means the line and the plane have to be parallel, and the line does not lie on the plane.	You need to show that the line and plane are parallel first before concluding that the line is not contained in the plane.
3(iii) (b) [2]	The distance between the line and the plane is $\dfrac{\left\|\left[\begin{pmatrix}1\\1\\6\end{pmatrix}-\begin{pmatrix}0\\1\\2\end{pmatrix}\right]\cdot\begin{pmatrix}3\\2\\7\end{pmatrix}\right\|}{\sqrt{7^2+3^2+2^2}}=\dfrac{31}{\sqrt{62}}=\sqrt{\dfrac{31}{2}}=\dfrac{\sqrt{62}}{2}.$ **Alternate method** (more tedious and not encouraged since question did not ask for foot of perpendicular) Let F be the foot of perpendicular from $P(1, 1, 6)$ to the plane. Then $\overrightarrow{OF}=\begin{pmatrix}1\\1\\6\end{pmatrix}+\mu\begin{pmatrix}3\\2\\7\end{pmatrix}$ for some $\mu\in\mathbb{R}$. Then since F lies on the plane, $\left[\begin{pmatrix}1\\1\\6\end{pmatrix}+\mu\begin{pmatrix}3\\2\\7\end{pmatrix}\right]\bullet\begin{pmatrix}3\\2\\7\end{pmatrix}=16\Rightarrow\mu=-\dfrac{1}{2}$. Therefore the distance is $\left\|\overrightarrow{PF}\right\|=\left\|\overrightarrow{OF}-\overrightarrow{OP}\right\|=\left\|\mu\begin{pmatrix}3\\2\\7\end{pmatrix}\right\|=\dfrac{1}{2}\sqrt{62}.$	Please be careful when transferring the values to your work. Many '6' morphed to '0' and the scalar products were not carefully evaluated. There are many students who went to find the foot of perpendicular from (1, 1, 6) to the plane. This is unnecessary and often showed conceptual errors such as letting $\overrightarrow{OF}$ be $\overrightarrow{PF}$.
3(iv) [3]	The angle θ between the line and the plane is given by $\sin\theta=\dfrac{\left\|\begin{pmatrix}0\\0\\1\end{pmatrix}\cdot\begin{pmatrix}3\\2\\7\end{pmatrix}\right\|}{\sqrt{62}}=\dfrac{7}{\sqrt{62}}\Rightarrow\theta=62.748^\circ$. Hence the angle between the line and its reflection in the plane is $2(62.748)=125.496^\circ$ and the required acute angle is thus $180^\circ-125.496^\circ=54.5^\circ$ (1 d.p) **Alternatively**, The angle ϕ between the line and the normal of the plane is given by	As mentioned earlier in the comments, you will need clearly defined angles or diagrams with angles indicated to gain full credit for this question even if the numerical answer is correct. Common errors include angle between line and plane is $\cos^{-1}\left(\dfrac{7}{\sqrt{62}}\right)$

	$\cos\phi = \dfrac{\left\lvert \begin{pmatrix}0\\0\\1\end{pmatrix} \cdot \begin{pmatrix}3\\2\\7\end{pmatrix} \right\rvert}{\sqrt{62}} = \dfrac{7}{\sqrt{62}} \Rightarrow \phi = 27.252^{\circ}$ The angle between the line and its reflection in the plane ABC is thus 2 times of the above angle, and since $2(27.252^{\circ}) = 54.5^{\circ} < 90^{\circ}$, this is the required angle.  	, when it should be $\sin^{-1}\left(\dfrac{7}{\sqrt{62}}\right)$ or $90^{\circ} - \cos^{-1}\left(\dfrac{7}{\sqrt{62}}\right)$. Again, there were a handful of students who went to calculate the foot of perpendicular from a point on the line, before finding the reflection of the point about the plane. This is totally inefficient and unnecessary, and often resulted in algebraic and conceptual errors mentioned in (iii).

	SOLUTION	COMMENTS
4(a) (i) [1]	$b=\dfrac{a+c}{2}$	Many students did this in a long-winded manner. It may be noted that a, b, c are also consecutive terms of an AP. Hence, $b-a=c-b$, and the answer can be quoted right away.
4(a) (ii) [1]	Common difference, $d=\dfrac{b-a}{2}$	
4(a) (iii) [2]	Sum of first 10 terms $=\dfrac{10}{2}\left(2a+(10-1)\left(\dfrac{b-a}{2}\right)\right)$ $=5\left(2a+\dfrac{9}{2}\left(\dfrac{a+c}{2}-a\right)\right)$ $=\dfrac{5}{4}(9c-a)$	Generally well done. Reminders ♦ To write 9 and a more distinctly different. ♦ Clear steps are expected (as the result is given in the question).
4(a) (iv) [2]	Let r be the common ratio. $r=\dfrac{4^{\text{th}}\text{ term}}{3^{\text{rd}}\text{ term}}=\dfrac{a}{b}$. Also, 3^{rd} term $=cr^2=b$. $\Rightarrow (2b-a)\left(\dfrac{a}{b}\right)^2=b$ $\Rightarrow (2b-a)a^2=b^3$	There are so many approaches to do this part. If your solution is more than 4 or 5 lines, please take a look at this suggested solution.
4(b) (i) [4]	Common ratio, $r=2\sin^2\alpha$ For the series to be convergent, $\|r\|<1$. That is, $\|2\sin^2\alpha\|<1$. $\Rightarrow \|\sin\alpha\|<\dfrac{1}{\sqrt{2}}$ $\Rightarrow -\dfrac{1}{\sqrt{2}}<\sin\alpha<\dfrac{1}{\sqrt{2}}$	Many students went on to consider u_n/u_{n-1} to find r. For this question, you don't have to, really. Other common **mistakes** : ♦ $\sin\alpha<\dfrac{1}{\sqrt{2}}$ (No lower bound.)

	 However, for this particular question, $r = 0$ makes the first term 0^0, which is not defined. Therefore, $2\sin^2\alpha \neq 0$. $\Rightarrow \quad \sin\alpha \neq 0$ $\Rightarrow \quad \alpha \neq 0,\ \pi,\ 2\pi$ Hence, $0 < \alpha < \frac{\pi}{4}$ or $\frac{3\pi}{4} < \alpha < \pi$ or $\pi < \alpha < \frac{5\pi}{4}$ or $\frac{7\pi}{4} < \alpha < 2\pi$.	♦ $0 < \sin\alpha < \frac{1}{\sqrt{2}}$ (Claiming $\sin\alpha$ cannot take negative values.) ♦ $\sin\alpha < \pm\frac{1}{\sqrt{2}}$ (This makes no sense.) ♦ $-\frac{1}{4} < \sin\alpha < \frac{1}{4}$ Many struggled to solve the inequality involving trigo expression. Advice : Sketch the graphs out! Very few students considered $r \neq 0$.
4(b) (ii) [2]	Sum to infinity $= \frac{1}{1-2\sin^2\alpha}$ $= \frac{1}{\cos 2\alpha}$ $= \sec 2\alpha$	Reminders : ♦ To write α and a more distinctly different. ♦ $\frac{1}{\cos 2\alpha} \neq \cos^{-1} 2\alpha$

	SOLUTION	COMMENTS
5(a) **[5]**	Let A be the event that a patient has a particular disease, and B be the event that a test for the disease gives a positive result.  $\mathrm{f}(p) = \mathrm{P}(A \mid B) = \dfrac{0.99p}{0.99p + 0.05(1-p)}$ $= \dfrac{99p}{94p+5}$ $\mathrm{f}'(p) = \dfrac{99(94p+5) - 99p(94)}{(94p+5)^2}$ $= \dfrac{495}{(94p+5)^2} > 0$, since $(94p+5)^2 > 0$ for $0 < p < 1$. Hence, f is an increasing function for $0 < p < 1$. The greater the prevalence of the disease in the population (hence the higher the possibility of being infected), the higher the chance that a positive test means that the patient actually has the disease. Examples of answers which ***were not accepted***: As the probability of a patient being infected with a particular disease increases, i) probability of patient being tested positive increases (No reference to f(p) being conditional probability) ii) probability of a patient being diagnosed correctly increases (not clear whether the given condition refers to patients who have tested positive or negative or patients who have disease or no disease)	A tree diagram is best suited for this type of question involving different conditional probabilities. Using the GC to sketch the graph of f is <u>not an acceptable method</u> to show f is an increasing function. Also, f(p) >0 does not imply f is an increasing function. The terms p and f(p) and their relationship has to be explained in <u>the context of the question.</u>
5(b) **(i)** **[1]**	Since A and B are independent events, A and B' are also independent events. $\mathrm{P}(A \cap B') = \mathrm{P}(A)P(B') = \dfrac{7(1-k)}{10}$	

5(b) (ii) [2]	Since A and B are mutually exclusive events $B \subseteq A'$, $0 \le k = \mathrm{P}(B) \le \mathrm{P}(A') = 0.3$ $\mathrm{P}(B \mid A') = \dfrac{\mathrm{P}(B \cap A')}{\mathrm{P}(A')} = \dfrac{\mathrm{P}(B)}{\mathrm{P}(A')} = \dfrac{k}{0.3} = \dfrac{10}{3}k$.	You are encouraged to draw your own Venn diagram to visualize the relation between A and B. The condition that A and B are independent is only applicable in (bi) and should not be assumed in (bii) (see the word " instead" used in the question for bii).

SOLUTION		**COMMENTS**
6(i) [2]	We require $a \to A,\ b \to C$ or D or E and no restriction on c, d and e. Number of ways $= 1 \times 3 \times 3! = 18$	Students who did not get the final answer but presented the correct "thinking process" were given a mark here. As such, for P&C questions, do provide some explanation instead of just writing a string of numbers.
6(ii) [3]	**Method 1** : Number of ways without restriction = 5! = 120 **Case 1** : Exactly one of a or b placed correctly. Number of ways = (1× 3 × 3!) × 2 = 36 (from part (i)) **Case 2** : both a and b placed correctly. Number of ways $= 1 \times 1 \times 3! = 6$ Total number of ways = 120 – 36 – 6 = 78 **Method 2** : Number of ways without restriction = 5! = 120 Number of ways $a \to A$ $= 1 \times 4! = 24$ Number of ways $b \to B$ $= 1 \times 4! = 24$ Number of ways $a \to A$ and $b \to B$ $= 1 \times 1 \times 3! = 6$ Total number of ways = 120 – 2(24) + 6 = 78	There are at least 10 ways to tackle this part. 4 have been out-lined here. Method 1 is "direct" as it makes use of the answer in (i). Quite a number of students were not careful and thought the complement was just Case 2. We can see (ii) as $n(A' \cap B') = n(\Omega) - n(A \cup B)$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

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<u>Method 3</u> :
<table>
<tr><th>Cases</th><th>Number of ways</th></tr>
<tr><td>$a \to B$, $b \to A, C, D, E$</td><td>$1 \times 4 \times 3! = 24$</td></tr>
<tr><td>$a \to C$, $b \to A, D, E$</td><td>$1 \times 3 \times 3! = 18$</td></tr>
<tr><td>$a \to D$, $b \to A, C, E$</td><td>18 (as above)</td></tr>
<tr><td>$a \to E$, $b \to A, C, D$,</td><td>18 (as above)</td></tr>
</table>
Total number of ways $= 24 + 18 + 18 + 18 = 78$

<u>Method 4</u> :
Case 1 : a is placed in B
Number of ways = $1 \times 4! = 24$

Case 2 : a is not placed in B
Number of ways = ${}^{3}C_{1} \times {}^{3}C_{1} \times 3! = 54$

Total number of ways $= 24 + 54 = 78$
</td>
<td>
A number of students did not realize that the 1st two cases are not mutually exclusive.

Method 3 is straightforward, although seemingly long. A simple table can help to make the solution less cumbersome however. When listing cases, it is important to ensure that you have considered all possibilities, and check if the cases are mutually exclusive.

Notice that Method 4 is similar to method 3. It considers cases 2 to 4 in method 3 as one case.
</td>
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<tr>
<td>6(iii)
[3]</td>
<td>
Case 1: 2 correct, 3 incorrect placings
Number of ways to choose 2 objects to be placed correctly $= {}^{5}C_{2} = 10$
Suppose $a \to A$ and $b \to B$.
Then $c \to D$, $d \to E$, $e \to C$
or $c \to E$, $d \to C$, $e \to D$
Number of ways = $10 \times 2 = 20$

Case 2: 3 correct, 2 incorrect placings
Number of ways to choose 3 objects to be placed correctly $= {}^{5}C_{3} = 10$
Suppose $a \to A$, $b \to B$ and $c \to C$.
Then $d \to E$, $e \to D$
Number of ways = $10 \times 1 = 10$

Case 3: 5 correct placings
Number of ways = 1

Total number of ways = $20 + 10 + 1 = 31$
</td>
<td>
Note that it is not possible to have 4 correct placings and 1 incorrect placing.

Many students thought that the answer is ${}^{5}C_{2} \times 3!$. This is actually the number of ways of getting at least 2 correct placings with repetitions. Try listing out the cases to convince yourself.

Although there are only 2 cases to consider if using the complement method, it is not so straightforward. The answer is
$5! - {}^{5}C_{1} \times {}^{3}C_{1} \times 3 - {}^{4}C_{1}\left(2 + 3 \times 3\right)$.
Try figuring it out if you are keen.
</td>
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</table>

SOLUTION		COMMENTS
7(i) **[2]**	Let X be the time in minutes of a medical consultation. If $X \sim \text{N}(15,10^2)$, then $\text{P}(X<0)=0.066807 \text{ (5 s.f.)} = 0.0668 \text{ (3 s.f.)}$ This means that about 7 patients out of every 100 take a "negative" amount of time which is not possible.	Most students did not mention that time cannot be negative. Proper justification needs to be given in order to get full credit.
[1]	$\text{H}_0: \ \mu=15$ vs $\text{H}_1: \ \mu>15$ where μ denotes the population mean consultation time in minutes.	
7(ii) **[3]**	Under H_0, since $n=30$ is large, $\bar{X} \sim \text{N}\left(15, \dfrac{10^2}{30}\right)$ approximately by Central Limit Theorem. We also assume that the population standard deviation is unchanged. $\text{P}(\bar{X}>18)=0.050174 \text{ (5 s.f.)} = 0.0502 \text{ (3 s.f.)}$ The smallest level of significance is 5.02%	Most students were able to pen down the sample mean distribution with correct use of CLT. Students must remember that to reject the null hypothesis, the level of significance must be greater or equals to the p-value.
7(iii) **[4]**	$\sum(y-15)=-50$, $\sum(y-15)^2=555$. An unbiased estimate of the population mean is, $\bar{y}=15+\dfrac{-50}{80}=\dfrac{115}{8}$ An unbiased estimate of the population variance is, $s^2=\dfrac{1}{79}\left(555-\dfrac{(50)^2}{80}\right)=\dfrac{2095}{316}$ $\text{H}_0: \ \mu=15$ vs $\text{H}_1: \ \mu<15$ Under H_0, since $n=80$ is large, $\bar{Y} \sim \text{N}\left(15, \dfrac{\frac{2095}{316}}{80}\right)$ approximately by Central Limit Theorem. Since p-value $= \text{P}\left(\bar{Y}<\dfrac{115}{8}\right)=0.0149623748=0.0150 \text{ (3 sf)}$ <0.05. We reject H_0, and conclude that there is sufficient evidence, at the 5% significance level, to support the administrator's claim that the average consultation time at private clinics is less than 15 minutes.	This part was very well done. Students who have studied this topic were able to find the unbiased estimate of the population mean and variance, set up the null and alternate hypothesis, correct sample mean distribution with CLT, and lastly, correct p-value. A handle of students were penalized for their tardy presentation and incomplete conclusion.

<table>
<tr><th colspan="2">SOLUTION</th><th>COMMENTS</th></tr>
<tr><td>8(i)
[3]</td><td>Sum of all probabilities = 1, thus,
$k+3k+5k+7k+11k+13k=1$
$40k=1$
$\therefore k=\frac{1}{40}$
P(X = 3) = 3k = $\frac{3}{40}$.</td><td>This is a 3-mark show question.
You need to show your detailed working.</td></tr>
<tr><td>8(ii)
[3]</td><td>$E(X)=k+3(3k)+5(5k)+7(7k)+11(11k)+13(13k)$
$=374k$
$=\frac{374}{40}$

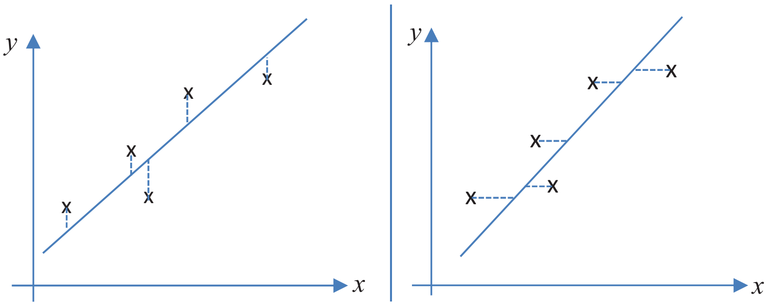
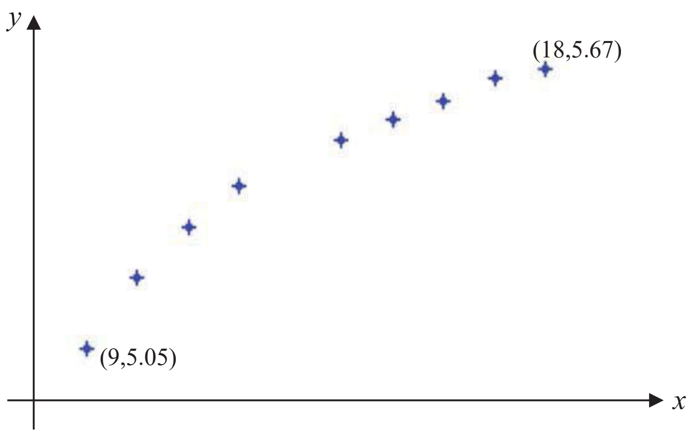
$E(X^2)=k(1+3^3+5^3+7^3+11^3+13^3)$
$=4024k$
$=\frac{4024}{40}$
$\text{Var}(X)=E(X^2)-[E(X)]^2$
$=\frac{4024}{40}-\left(\frac{374}{40}\right)^2$
$=\frac{5271}{400}$</td><td>GC can give you the answers for this part. However, Question asked for exact value.
You have to show the working on how you get the values of E(X) and E(X^2).</td></tr>
<tr><td>8(iii)
[2]</td><td>$R\ \sim \text{B}(15,0.4)$

$P(R\geq 5)=1-P(R\leq 4)$
$=0.783$</td><td>R is defined in the question. You NEED to state the distribution.</td></tr>
<tr><td>8(iv)
[2]</td><td>Let W be the random variable denoting the number of times that a score less than 10 is observed in 14 throws. Then $W\ \sim$ B(14, 0.4).
Required probability = $P(W=7)\times 0.4=0.0630$
(Explanation: "W = 7" represent 7 out of the first 14 throws have score less than 10 follow by the 15th throw having score less than 10)

Alternatively (without defining new random variable, BUT using the same argument AND the correct formula),
Required probability = ${}^{14}C_7\,(0.6)^7\,(0.4)^7\,(0.4)=0.0630$</td><td>Remember to define your random variable AND state its distribution.</td></tr>
</table>

SOLUTION		COMMENTS
9(i) **[1]**	Let R denote the working lifespan of Brand R tyre. $R \sim \mathrm{N}(64000, 8000^2)$ Required probability = P(R > 70000) = 0.227 (to 3 s.f.).	In general, the letter used for a new random variable should always be defined with the distribution explicitly written down, same for part (iii) This part was well done as it is only 1 mark so not penalized for presentation
9(ii) **[2]**	Consider P($R > t$) = 0.98. From G.C., t = 47570 (to nearest whole number).	It is a 2 mark question so student should have a simple line stating what they are working with, or a normal distribution graph with region shaded, before writing down the value of t
9(iii) **[2]**	Let S denote the working lifespan of Brand S tyre. $\bar{S} \sim \mathrm{N}\left(68000, \dfrac{7500^2}{50}\right)$. Required probability $= \mathrm{P}(\bar{S} > 70000)$ $= 0.0297$ (correct to 3 s.f.).	Some students chose to do by total sum of 50 tyres instead which is fine too. Do note that the question already gave the lifespan to be normally distributed so there is no reason to bring in C.L.T.
9(iv) **[3]**	$(R_1 + R_2 + R_3) - 3S \sim \mathrm{N}(-12000\ , 12(8000^2))$ Required probability $= \mathrm{P}((R_1 + R_2 + R_3) < 3S)$ $= \mathrm{P}((R_1 + R_2 + R_3) - 3S < 0)$ $= 0.667$ (to 3 s.f.)	Generally well done, usual careless mistake of not taking the square root of the variance when entering into the GC.
9(v) **[3]**	Consider $\mathrm{P}(S > 50000) > \mathrm{P}(R > 50000)$ $\mathrm{P}\left(Z > \dfrac{50000 - 68000}{\alpha}\right) > 0.9599408865$ $\mathrm{P}\left(Z > \dfrac{-18000}{\alpha}\right) > 0.9599408865$ $\dfrac{-18000}{\alpha} < -1.750000503$ $\therefore \quad 0 < \alpha < 10286$ (to nearest whole number)	Easy 1 mark for writing down this inquality. Subsequently, students who do not sketch the normal curve would have to be careful to get the

	[or $0 < \alpha \le 10285$]	correct direction for the inequality sign Unfortunately, many students could not get the last mark for this question, mostly either not stating the lower bound or stating wrongly (≤ instead of <)
9(vi) **[1]**	Assume that the working lifespan of **all** the tyres are independent.	This is another part where many students lost mark by not being clear enough. It is much simpler to say "all the tyres" but some students solution only suggested that Brand S needs to be independent with Brand R

SOLUTION		COMMENTS
10(a) **(i)** **[2]**	Regression line y on x : Regression line x on y :  The least squares regression line of y on x is the line with the minimum sum of the squared vertical distances for each point to the line while the the least squares regression line x on y is the line with the minimum sum of the squared horizontal distances from each point to the line.	Both the least squares regression line is related to the minimum sum of the squared distances and NOT the sum of the distances.
10(b) **(i)** **[2]**	 The scatter diagram shows that the rate of increase of y decreases as x increases. So the variables do not have a linear relation and hence the relationship is not likely to be well modelled by an equation of the form $y = a + bx$, where a and b are constants.	The points which defines the ranges of the values of x and y must be labelled. The scales of the axes should be carefully selected to "space out" the points so that the trend can be observed correctly. The scatter diagram should be used to explain the relationship and not using the product moment coefficient.
10(b) **(ii)** **[2]**	**(a)** product moment correlation coefficient between $\ln x$ and y, $r_1 = 0.986554$ (correct to 6 d.p.) **(b)** product moment correlation coefficient between x^2 and y, $r_2 = 0.945806$ (correct to 6 d.p.)	Note that the question has required the answers to be corrected to 6 decimal places and not 3 significant figures.

10(b) (iii) [1]	Since $\lvert r_2 \rvert < \lvert r_1 \rvert < 1$, the product moment correlation coefficient between ln x and y (= 0.986553) is closer to 1, hence $y = a + b\ln x$ is the better model.	It is must be noted that the better model is chosen based on the product moment coefficient being closer to 1 (or -1 in other cases) and NOT just being larger.
10(b) (iv) [3])	The equation of the suitable regression line is $y = 3.237876 + 0.854181\ln x$ $y = 3.24 + 0.854\ln x$ (3 s.f.) When $y = 6.5$, $x = 45.55899$ The population size first exceed 6.5 millions in the year 2045. Since the population size of 6.5 millions is not in the data range (from 5.05 to 5.67), the estimate is an extrapolation and so it is not reliable.	The regression line of ln x on y should NOT be used even though a similar answer is obtained. Need to observe carefully the required year is 2045 and NOT 2046 based on the context. It is not sufficient to mention that it is an extrapolation. It is necessary to state the given population size is not in the given data range.
10(b) (v) [1]	When $x = 13$, $y = 3.237876 + 0.854181\ln 13 = 5.428807$ The required estimate is 5.43 millions	It is necessary to state the estimate as 5.43 millions and not just 5.43.
10(b) (vi) [1]	5.31 millions is lower than 5.43 millions calculated in part (v) (and it does not follow the rising trend of the data given - outlier). There might be some form of deadly diseases spreading in part(s) of the communities in the country that caused the drop in actual figure that year. OR There might be a natural disaster in the country that caused the drop in actual figure that year.	Need to take note that the difference of estimated and actual population sizes is more than100,000 and the actual size is smaller than the previous year, so a decrease in birth rate is not enough to justify the difference.