Name:	Class:	Class Register Number:

中正中学
CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2023
SECONDARY 4

ADDITIONAL MATHEMATICS **4049/01**

Paper 1 **Thursday 24 August 2023**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
Total Marks	

This document consists of **19** printed pages and **1** blank page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ \ldots\ (n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The points R and S have coordinates $\left(\sqrt{3}, 2\sqrt{3}\right)$ and $\left(\sqrt{5}, 4\sqrt{5}\right)$ respectively. Show that the gradient of RS can be expressed in the form $a+b\sqrt{15}$, where a and b are integers to be found. [4]

2 Given that $\cos\theta = p$ and that θ is acute, express in terms of p,

(a) $\sin\theta$, [1]

(b) $\tan\left(90^\circ - \theta\right)$, [2]

(c) $\cos 2\theta$. [2]

3 Express $\dfrac{12x^2+32x+31}{(2x-1)(x+2)^2}$ in partial fractions. [5]

4 The expression $ax^3 + bx^2 + b$ leaves a remainder of R when divided by $(x+1)$ and a remainder of $5R-2$ when divided by $(x+2)$.

(a) Show that $b = \dfrac{2-3a}{5}$. [4]

(b) Given further that $ab = -8$ and $a > b$, find the value of a and of b. [3]

(c) Using the values of a and b found in **part (b)**, explain why the equation $ax^3 + bx^2 + b = 0$ has only one real root and state its value. [4]

5 Find the coordinates of the stationary points of the curve $y = \dfrac{(x-3)^2}{x}$ and determine the nature of each stationary point. [7]

6 The height of a ball above ground, h metres, released by a machine can be modelled by the equation $h = -0.2x^2 + 6x + 3$ where x is the horizontal distance travelled by the ball in metres.

(a) State the height above ground at which the ball is being released. [1]

(b) Express $h = -0.2x^2 + 6x + 3$ in the form $a\left(x-b\right)^2 + c$, where a, b and c are constants to be found. [2]

(c) Using your result from **part (b)**, explain why the height of the ball can never be more than 48 metres. [2]

(d) Hence, explain if this machine is safe for use in an indoor stadium with a ceiling height of 45 metres. [1]

7 **(a)** Solve the equation $3^x\left(18+3^x\right)=40$. [3]

(b) Solve the equation $\log_{\sqrt{2}} y = 3+\log_2\left(y+6\right)$. [5]

Continuation of working space for question 7(b)

(c) In order to obtain a graphical solution of the equation $x = 2\ln\left(4 - \dfrac{3x}{2}\right)$, a suitable straight line can be drawn on the same set of axes as the graph of $y = 4 - e^{\frac{x}{2}}$. Make $e^{\frac{x}{2}}$ the subject of $x = 2\ln\left(4 - \dfrac{3x}{2}\right)$ and hence find the equation of this line. [3]

8 A curve has the equation $y=\dfrac{x^2-x+1}{5x-5}$, $x\neq 1$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{x^2-2x}{5(x-1)^2}$. [3]

(b) Explain why the curve is increasing for $x>2$. [3]

9 **(a)** Given that $\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{3}{4}$, prove that $\cos\alpha\cos\beta=7\sin\alpha\sin\beta$. [2]

(b) Hence, deduce the relationship between $\tan\alpha$ and $\tan\beta$. [2]

(c) Given further that $\alpha+\beta=45°$, calculate the value of $\tan\alpha+\tan\beta$. [3]

10 A circle, whose equation is $x^2 + y^2 - 10x - 8y + 16 = 0$, has centre C and radius r.

(a) Find the coordinates of C and the value of r. [3]

(b) Explain whether the point $(9, 2)$ lies inside or outside the circle. [2]

The line $4y = 3x + 1$ meets the circle at the points *P* and *T*, and the *x*-axis at *S*.
T lies between *P* and *S*.

(c) Without finding the coordinates of *P* and of *T*, find the length *TS*. [4]

11 The table shows, to 1 decimal place, the mass, m, of a radioactive substance, in grams, after t days.

t	5	10	15	20	25
m	57.7	37.0	23.7	15.2	9.7

(a) On the grid opposite, plot $\ln m$ against t and draw a straight line graph. [2]

(b) Find the gradient of your straight line and hence express m in the form $m_0 e^{kt}$, where m_0 and k are constants. [4]

The half-life of a radioactive substance is the length of time it takes for half of the substance to decay.

(c) In order to determine the half-life of the radioactive substance, a suitable straight line can be drawn on the same set of axes as your graph. Find the equation of this line and hence determine the half-life of the radioactive substance. [3]

12 A particle travels in a straight line so that its velocity, v cm/s, t seconds after passing through a fixed point O, is given by $v = t^2 - kt + 5$, where k is a constant. The particle first comes to an instantaneous rest at the point P and then at the point Q.

(a) Given that the particle reaches a minimum velocity at $t = 3$, show that $k = 6$. [2]

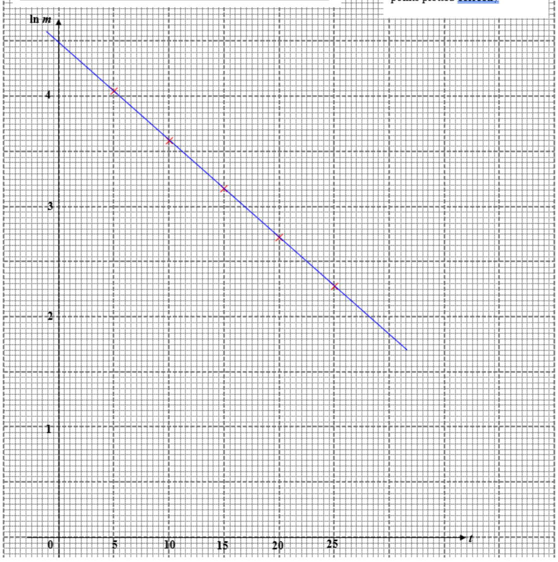
(b) Find the distance PQ. [6]

(c) With working clearly shown, explain whether the particle will pass by O again, after the first 7 seconds. [2]

BLANK PAGE

Answer Key

Qns	Ans
1	$7-3\sqrt{15}$
2(a)	$\sqrt{1-p^2}$
2(b)	$\dfrac{p}{\sqrt{1-p^2}}$
2(c)	$2p^2-1$
3	$\dfrac{8}{2x-1}+\dfrac{2}{x+2}-\dfrac{3}{(x+2)^2}$
4(a)	Show qns
4(b)	$a=4,\ b=-2$
4(c)	For $2x^2+x+1=0$, discriminant $=1^2-4(2)(1)=-7<0$ $\therefore 2x^2+x+1=0$ has no real roots. Only 1 real root of $x=1$.
5	$(3,0)$ is a minimum point $(-3,-12)$ is a maximum point
6(a)	3 metres
6(b)	$-0.2(x-15)^2+48$
6(c)	$-0.2(x-15)^2+48\le 48$ Since max height = 48m, it will never exceed 48m.
6(d)	Not safe, maximum height of 48 m > 45 m.
7(a)	x = 0.631 (3 s.f.)
7(b)	y = 6
7(c)	$y=\dfrac{3x}{2}$
8(a)	Show qns
8(b)	$x>2,\ x-2>0, x(x-2)>0$ since $(x-1)^2>0, \dfrac{x^2-2x}{5(x-1)^2}>0.$ Since $\dfrac{dy}{dx}>0,$ the curve is always increasing.
9(a)	Show qns
9(b)	$\tan\alpha\tan\beta=\dfrac{1}{7}$ or $\dfrac{1}{\tan\beta}=7\tan\alpha$
9(c)	$\tan\alpha+\tan\beta=\dfrac{6}{7}$

Qns	Ans
10(a)	$C(5,4)$, r = 5
10(b)	Since distance to centre = $\sqrt{20}<5$ (radius) ,$(9,2)$ lies inside the circle.
10(c)	$1\dfrac{2}{3}$ units
11(a)	t: 5, 10, 15, 20, 25 $\ln m$: 4.055, 3.61, 3.16, 2.72, 2.27 P1 – at least 3 points P2 – straight line drawn with all points plotted correctly 
11(b)	Gradient = -0.09 $m=90.0e^{-0.09t}$
11(c)	y = 3.8, half-life = 7.75 days
12(a)	Show qns
12(b)	PQ = $10\dfrac{2}{3}$ m
12(c)	At $t=7,\ s=2\dfrac{1}{3},\ v=12$ Since $s>0,\ v>0$ and there are no more turning points after $t=5,$ the particle will not return to O after 7 seconds.