

First Law of Thermodynamics

Answer Key

1.	D	10. (a)	1.25 x 10-3 m ³	R1(b)	273.15 K
2.	D	10. (b)	1.37 x 105 Pa	R1(c)(i)	1.35x10 ⁶ m ³
3.	D	14.(a)	0.681 mol	R1(ii)	1.77x10⁵ N
4.	E	14.(b)	0.033 m ³	R1(iii)	1.77x10⁵ N
5.	В	14.(c)	2250 J	R1(iv)	3.99x10 ³
					Ν
6.	-6 J	14(d)	0	R1(v)	0.221 ms ⁻²
7(a)	3.78 x 10 ⁴ J	15.(a)	8.0x10⁵ Pa	R1(d)(ii)	4.72x10 ⁻²¹ J
7(b)	7.72 x 10 ⁴ J	15.(b)	296 K	R1(iii)	6.33x10 ⁵ mol
8(a)	1.66 x 10⁵ J	15.(c)	q _{ab} =60,000J	R1(iv)	1.80x10 ⁹ J
8(b)	2.03 x 10 ⁶ J		$q_{bc} = -36,000 \text{ J}$		
			q _{ca} = -11100 J	R2(b)(i)	0.295 mol
9(a)	-224 J	15(.d)	∆Uab = 36000 J	R2(c)((ii)	[3.71 x 10 ³ J]
9(b)	224 J		∆Ubc = 36000 J	R2(c)((iii)	1100 J
			$\Delta Uca = 0$		

6. N07/l/20

An ideal gas undergoes an expansion in volume from $1.3 \times 10^{-4} \text{ m}^3$ to $3.6 \times 10^{-4} \text{ m}^3$ at a constant pressure of 1.3×10^5 Pa. During this expansion, 24 J of heat is supplied to the gas.

What is the overall change in the internal energy of the gas? [-6 J]

Isobaric expansion: $w = -p\Delta V = -(1.3 \times 10^5) [(3.6 - 1.3) \times 10^{-4}] = -29.9 \text{ J}$ First law: $\Delta U = q + w = 24 + (-29.9) = -5.9 \text{ J}$

7. A gas in a cylinder expands from a volume of 0.110 m³ to 0.320 m³. Heat flows into the gas just rapidly enough to keep the pressure constant at 1.80 x 10^5 Pa during the expansion. The total heat added is 1.15 x 10^5 J.

(a)	Find the work done by the gas.	[3.78 x 10 ⁴ J]
	$w_g = p\Delta V = (1.80 \times 10^5)[0.320 - 0.110] = 3.78 \times 10^4 \text{ J}$	

(b) Find the change in internal energy of the gas. $[7.72 \times 10^4 \text{ J}]$

 $\Delta U = q + w = 1.15 \times 10^5 + (-3.78 \times 10^4) = 7.72 \times 10^4 \text{ J}$



8. When water is boiled under a pressure of 2.00 atm, the heat of vaporization is 2.20 MJ kg⁻¹ and the boiling point is 120 °C. At this pressure and temperature, 1.00 kg of water has a volume of 1.00 x 10⁻³ m³, and 1.00 kg of steam has a volume of 0.824 m³.

 $[1 \text{ atm} = 1.013 \text{ x} 10^5 \text{ Pa}]$

(a) Compute the work done when 1.00 kg of steam is formed at this temperature.

 $w_{a} = p\Delta V = (2 \times 1.013 \times 10^{5})(0.824 - 0.001) = 1.67 \times 10^{5} \text{ J}$

(b) Compute the increase in internal energy of the water.

 $\Delta U = q + w = 2.20 \times 10^6 + (-1.67 \times 10^5) = 2.03 \times 10^6 \text{ J}$

(c) The work done for 1.00 kg of steam to form at 100°C and 1 atm is 1.69 x 10⁵ J, suggest a reason why the work done against external pressure decreases when external pressure increases.

The work done has decreased at higher pressure because the volume occupied by the water vapour has decreased by a factor larger than the factor increase in pressure.

E.g. At 1 atm; volume occupied by 1 kg of steam is 1.671 m³.

$$\frac{Vol_{1 \text{ atm}}}{Vol_{2 \text{ atm}}} = \frac{1.671}{0.824} = 2.028$$

[1.67 x 10⁵ J, 2.03 x 10⁶ J]

- 9. During an adiabatic expansion, the temperature of 0.450 mol of argon (Ar) drops from 50.0 °C to 10.0 °C. Argon is monatomic and may be treated as an ideal gas.
 - (a) What is the change in internal energy of the gas?

$$\Delta U = \frac{3}{2} nR \Delta T = \frac{3}{2} (0.450) (8.31) (10.0 - 50.0) = -224 \text{ J}$$

(b) How much work is done by the gas? $\Delta U = q+w$ w = -224JWork done by gas, $w_g = -w = -224 J$

10(a) Using pV = nRT $n = \frac{pV}{RT} = \frac{1.1x10^{5}(1x10^{-3})}{8.31x300} = 0.0441 \text{ mol}$ Using pV = nRT, $V_{2} = \frac{nRT}{p} = \frac{0.0441(8.31)(375)}{1.1x10^{5}} = 1.25 \text{ x } 10^{-3} \text{ m}^{3}$ 10(b) Using pV = nRT, $p_{2} = \frac{nRT}{V} = \frac{0.0441x8.31x375}{1.0x10^{-3}} = 1.37 \text{ x } 10^{5} \text{ Pa}$ 11. (i) B to C: constant pressure: $\frac{V_B}{T_P} = \frac{V_C}{T_C}$

$$T_{C} = \frac{V_{C}T_{B}}{V_{B}} = \frac{6x660}{1.4} = 2830 \text{ K}$$

D to A: constant volume: $\frac{P_{D}}{T_{D}} = \frac{P_{A}}{T_{A}}$
 $T_{D} = \frac{P_{D}T_{A}}{P_{A}} = \frac{7.8x300}{1.0} = 2340 \text{ K}$

(ii) (a)

Work done on the gas is given by the force exerted by an exerted agent multiply the displacement.

When the gas expands, the displacement is opposite in direction to the force exerted by the external agent thereby giving rise to a negative value of work done on the system.

(b)

Section of cycle	Heat supplied to gas / J	Work done on gas/J	Increase in internal energy of gas/J
A to B	0	300	300
B to C	2580	-740	1840
C to D	0	-440	-440
D to A	-1700	0	-1700

12 N08/III/4(b)

An ideal gas undergoes a cycle of changes $A \rightarrow B \rightarrow C \rightarrow A$, as shown in Figure 4.1.





(i) Calculate the work done by the gas during the change $C \rightarrow A$.

[2]

 $w_{g} = p\Delta V = (1 \times 10^{5})(20 - 5) \times 10^{-6} = 1.5 \text{ J}$

(ii) Figure 4.2 is a table of energy changes during one cycle. Complete figure 4.2.

section of cycle	heating supplied to gas / J	work done on gas / J	increase in internal energy of gas / J
$A \rightarrow B$	zero	4.2	.4.2
$B \rightarrow C$	-8.5	0	-8.5
$C \rightarrow A$	5.8	-1.5	4.3

Figure 4.2

13.

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	Increase in Internal energy/ J	Heat supplied to gas/ J	Work done on gas/ J
A to B	1200	0	1200
B to C	- 1350	-1350	0
C to D	- 600	0	-600
D to A	750	750	0

14. In the process illustrated by the pV diagram in Figure 2, the temperature of the ideal gas remains constant at 85.0 °C.



(a) How many moles of gas are involved?

Choose point b: $pV = nRT \Rightarrow (0.200 \times 1.013 \times 10^5)(0.100) = n(8.31)(85 + 273.15)$ $\Rightarrow n = 0.681 \text{ mol}$

(b) What volume does this gas occupy at a?

$$P_a V_a = P_b V_b \Longrightarrow V_a = \left(\frac{0.200}{0.600}\right) (0.100) = 0.033 \text{ m}^3$$

(c) How much work was done by or on the gas from *a* to *b*? [2250 J]



[0.033 m³]

[0.681 mol]

Isothermal expansion:

$$w = -nRT \ln\left(\frac{V_f}{V_i}\right) = -(0.681)(8.31)(273.15 + 85.0) \ln\left(\frac{0.100}{0.033}\right) = -2250 \text{ J}$$
2250 J of work is done by the gas.

(d) By how much did the internal energy of the gas change during this process? [0 J]

$$\Delta U = \frac{3}{2} nR \Delta T \Longrightarrow \Delta U = 0 \because \Delta T = 0$$

15. The graph in Figure 3 shows a pV diagram for 3.25 moles of ideal helium (He) gas. Part *ca* of this process is isothermal.



(a) Find the pressure of the Heat point *a*.

$$P_a V_a = P_c V_c \Rightarrow P_a = \frac{0.040}{0.010} \times 2.0 \times 10^5 = 8.0 \times 10^5$$
 Pa

(b) Find the temperature of the He at points *a*, *b* and *c*.

$$T_c = \frac{P_c V_c}{nR} = \frac{2.0 \times 10^5 \times 0.040}{8.31 \times 3.25} = 296 \text{ K}$$

$$T_b = \frac{P_b V_b}{nR} = \frac{8.0 \times 10^5 \times 0.040}{8.31 \times 3.25} = 1185 \text{ K}$$

 $T_{a} = T_{c} = 296 \text{ K}$

(c) How much heat entered or left the He during segments *ab*, *bc* and *ca*? In each did heat enter or leave?

ab (isobaric):

$$w_g = (8.0 \times 10^5)(0.040 - 0.010) = 24000 \text{ J}$$

 $\Delta U = q + w$
 $\frac{3}{2}(3.25)(8.31)(1185 - 296) = q - 24000$
 $q_{ab} = \frac{3}{2}(3.25)(8.31)(1185 - 296) + 24000 = 60000 \text{ J}$
bc (isochoric):

$$q = \Delta U = \frac{3}{2} nR\Delta T = \frac{3}{2} (3.25)(8.31)(296 - 1185) = -36000 \text{ J}$$

ca (isothermal):
$$q = w = nRT \ln\left(\frac{V_f}{V_i}\right) = (3.25)(8.31)(296) \ln\left(\frac{0.010}{0.040}\right) = -11100 \text{ J}$$

(d) By how much did the internal energy of the He change from *a* to *b*, from *b* to *c* and from *c* to *a*? Indicate whether this energy has increased or decreased.

$$\Delta U_{ab} = \frac{3}{2} (3.25) (8.31) (1185 - 296) = 36000 \text{ J}$$
$$\Delta U_{bc} = \frac{3}{2} (3.25) (8.31) (296 - 1185) = -36000 \text{ J}$$
$$\Delta U_{ca} = 0$$

- R1 N05/III/02
 - (a) The ideal gas equation is pV = nRT. Explain why non-SI units may be used for *p* and *V* but the temperature cannot have the unit °C. [2]

[2]

Non-SI units may be used for p and V because we just need a corresponding change in the value for R. R has many possible values depending on the unit system.

 $T_{\kappa} = T_{c} + 273.15$ cannot be corrected in the equation by a simple multiplicative factor. i.e. $pV = nR(T_{\kappa} - 273.15)$ cannot be simplified to pV = nRT.

(b) Write down the exact temperature on the Kelvin scale of zero degrees Celsius. [1]

0 °C = 273.15 K

(c) In an attempt to beat the world altitude record for balloon, a helium balloon containing 15 000 m³ of helium at a temperature of 288 K was launched from sea level, where the pressure of the gas was 101 kPa. The balloon, carrying a payload, rose to an altitude of 32.0 km before reaching equilibrium. Data concerning atmospheric conditions are given in the table.

	sea level altitude = 0	equilibrium altitude = 32.0 km
pressure of helium	101 kPa	0.890 kPa
temperature	288 K	228 K
density of air	1.23 kg m ⁻³	0.0134 kg m ⁻³

Calculate

(i) the volume of helium at 32.0 km,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Longrightarrow \frac{(101000)(15000)}{288} = \frac{890 V_2}{228} \Longrightarrow V_2 = 1.35 \times 10^6 \text{ m}^3$$

(ii) the weight of air displaced by the balloon at equilibrium altitude,

[2]

[2]

[2]

 $W = \rho Vg = 0.0134 (1.35 \times 10^6) (9.81) = 1.77 \times 10^5 \text{ N}$

(iii) the total weight of balloon, helium and payload [1]

weight of balloon system = $1.77 \times 10^5 \text{ N}$

(iv) the resultant force on the balloon at sea level, [2]

$$F = U - mg = \rho Vg - 1.77 \times 10^5 = (1.23)(15000)(9.81) - 1.77 \times 10^5$$
$$= 3.99 \times 10^3 \text{ N}$$

(v) the acceleration of the balloon at take-off.

$$a = \frac{F}{m} = \frac{3.99 \times 10^3}{\left(1.77 \times 10^5 / 9.81\right)} = 0.221 \text{ ms}^{-2}$$

(d) (i) Using the equations $pV = \frac{1}{3}Nm\langle c^2 \rangle$ and pV = NkT, derive an expression for the relationship between the average translational kinetic energy of a helium atom and the temperature. [1]

$$\frac{1}{3}Nm\left\langle c^{2}\right\rangle = NkT \Longrightarrow \frac{1}{2}m\left\langle c^{2}\right\rangle = \frac{3}{2}kT$$

(ii) Hence find the average translational kinetic energy of one of the helium atoms in the balloon in (c), when the balloon is at an altitude of 32.0 km.

$$KE = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23})(228) = 4.72 \times 10^{-21} \text{ J}$$

(iii) Calculate the amount, in mol, of helium in the balloon. [2]

 $n = \frac{PV}{RT} = \frac{101000(15000)}{8.31(288)} = 6.33 \times 10^5 \text{ mol}$

(iv) Assuming that the gas behaves as an ideal gas, calculate the kinetic energy of all the helium at equilibrium altitude. [2]

$$KE = U = \frac{3}{2}nRT = \frac{3}{2}(6.33 \times 10^5)(8.31)(228) = 1.80 \times 10^9 \text{ J}$$

(e) Suggest why the change in the potential energy of the gas in the balloon as it rises does **not** change its internal energy. [2]

The increase in gravitational potential energy changes the total mechanical energy of the system and does not change its internal energy. The potential energy portion of the internal energy arises from



intermolecular forces and not gravitational force.

R2. (a) An ideal gas is a gas that obeys the <u>equation of state of an ideal gas for all values</u> of pressure, volume and temperature.

(b)(i) Using pV = nRT

$$\Delta n = \frac{\Delta PV}{RT} = \frac{0.012}{8.31(273.15 + 25)} (3.23 - 2.62) \times 10^5 = 0.295 \text{ mol}$$

(ii) The pressure in the portable supply cannot be lower than 3.23×10^5 Pa otherwise it will not be able to fill the tyre.

At a pressure of 8.72×10^5 Pa, the number of moles inside the portable air supply

 $n_{\text{supply}} = \frac{8.72x10^5(0.0108)}{8.31(273.15+25)} = 3.801 \,\text{mol}$

At the pressure of 3.23x10⁵, the number of moles available,

 $n = \frac{3.23 \times 10^5 (0.0108)}{8.31 (273.15 + 25)} = 1.408 \,\text{mol}$

Hence the number of moles available for filling the tyre = 3.801-1.408=2.39 mol which is greater than the required 1.18 mol (4x0.295).

(c)(i) Internal energy of one molecule = $3/2kT = 3/2(1.38x10^{-23})(273.15+25)$ = 6.17x10⁻²¹ J

(ii) Hence, the internal energy of one mole of gas = $6.02 \times 10^{23} (6.17 \times 10^{-21})$ = 3.71×10^3 J

Note: Student can't use 3/2nRT because of the word "hence" in the qn.

(iii)
$$\Delta U = 3.71 \times 10^3 (0.295) = 1095 J$$



First Law of Thermodynamics – Quick Revision Exercise





ii. . Calculate the change in the internal energy from state B to state C. Change in internal energy = $U_{\rm C} - U_{\rm B}$ $= 3/2 P_{\rm C}V_{\rm C} - 3/2 P_{\rm B}V_{\rm B}$ $= 3/2 (2X10^5 X 2X10^2 - 2X10^5 X 8X10^2)$ $= 3/2 (-12 \times 10^3)$ = -18,000 Jiii. State with reason whether heat is absorbed or given out during the process B to C? $\Delta U = q + w$ (-ve) = q + (+ve)The internal energy of the system has decreased while work done on the system is positive. Hence heat must be given out of the system. d i. Determine the work done in process C to A. Process C to A is isochoric, no change in volume. Hence work done is zero. ii. State with reason whether heat is absorbed or given out during the process C to A? $\Delta U = q + w$ Temperature of A is higher than C. Hence from C to A, the internal energy must increase. W is zero as it is an isochoric process. (+ve) = q + 0Hence heat is supplied to the system in order for the internal energy to increase. iii. Calculate the increase in the internal energy from state C to state A. Increase in internal energy = $U_A - U_C$ $= 3/2 P_A V_A - 3/2 P_C V_C$ = 3/2 (8X10⁵ X 2X10⁻² - 2X10⁵ X 2X10⁻²) $= 3/2 (12 \times 10^3)$ = 18,000 J



e Complete the thermal process table below:

Process	ΔU	q	w
A to B	0	+22.18 kJ	-22.18 kJ
B to C	-18,000 J	-30,000 J	+12,000 J
C to A	+18,000 J	+18,000 J	0
Cyclic process ABCA	0	Σq +10180	Σw -10180

g. For the entire cycle, state whether

i. net work done by the gas is positive?

Net work done by gas is positive.

ii. the heat is absorbed or given out?

Heat is absorbed by the gas in the cyclic process.



Assignment

A1 (a) The internal energy of a substance is sum of a random distribution of kinetic and potential energies associated with the molecules of a system.

- (b) When ice becomes water at 0°C, there is no change in the kinetic energy of the molecules because all the heat absorbed goes into increasing the potential energy of the molecules as the molecules breakaway from the lattice structure and acquire a higher degree of freedom.
- (c) From the table, specific latent heat of fusion of ice is 3.36 x 10⁵ Jkg⁻¹. This is because when ice is converted to water at 0°C, the (latent) heat supplied goes into increasing the potential energy of all the molecules. The change in volume is negligible hence we can ignore work done on the system.
- (d) Volume of 1 kg of water at 100°C, standard pressure = 0.00104 m³
 Volume of 1 kg of steam at 100°C, standard pressure = 1.67 m³

Work done on water in changing to steam at constant pressure

=-
$$\int p dV$$

= -P (ΔV) =- 1.01 x 10⁵ x (1.67 - 0.00104)
= -1.69 x 10⁵ J or - 0.169 MJ

Work done by water in changing to steam at constant pressure= 1.69 x 10⁵ J

(e) From 1st law of thermodynamics, $\Delta U = q + w$ The increase in internal energy = heat supplied to the system + work done on the system

The latent heat of vaporization is the heat required to convert 1 kg of water at 100°C to steam at 100°C (at constant pressure) and this is equal to q.

Hence latent heat of vaporization, $q = \Delta U - w$ = (26.9 - 5.99) x 10⁵ - (-1.69 x 10⁵) = **2.26 MJ** The specific latent heat of vaporization is 2.26 MJkg⁻¹

(f) The specific latent heat of vaporization of water is much greater than specific latent heat of fusion of ice because the thermal energy supplied to the water molecules is needed to free themselves away from the intermolecular forces of one another and at the same time do work against the environment or atmosphere.

(g) The surrounding pressure must be constant to ensure that the gas pressure to remain unchanged during the isobaric gas expansion.