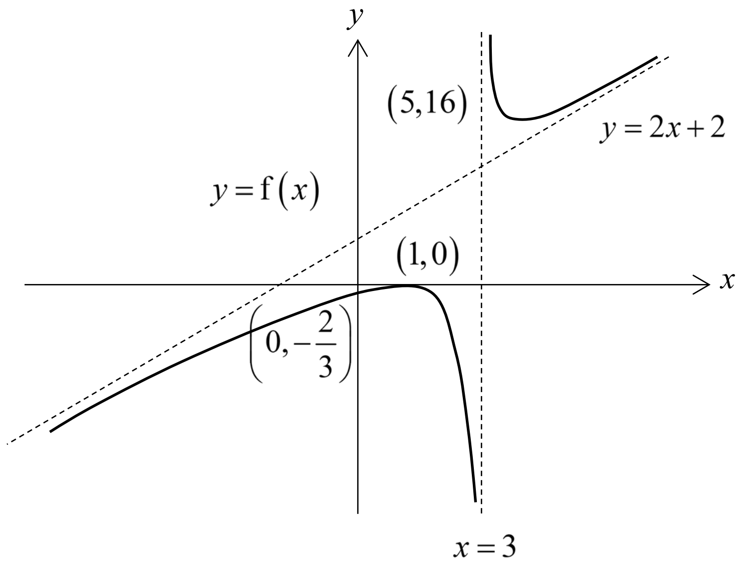
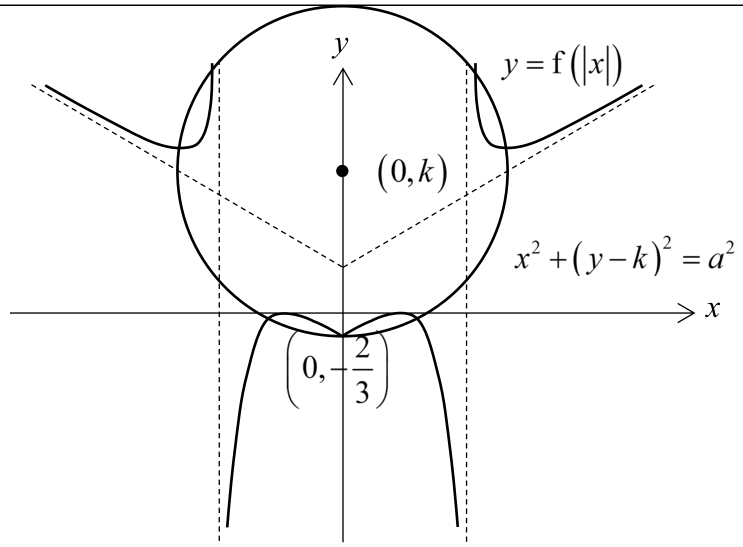
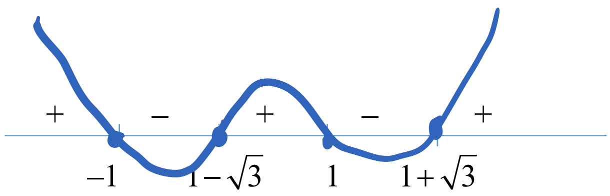
2020 HCI H2 Mathematics Preliminary Examination P1 Solutions

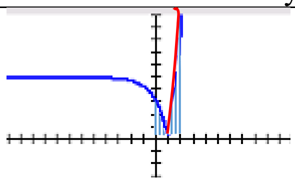
Qn	Solution
1	$\frac{dx}{dt} = 2 + \frac{1}{t^2}, \quad \frac{dy}{dt} = 1 - \frac{1}{t^2}$ $\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ $\frac{dy}{dx}$ $= \frac{1 - \frac{1}{t^2}}{2 + \frac{1}{t^2}}$ $= \frac{t^2 - 1}{2t^2 + 1}$ $= \frac{\frac{1}{2}(2t^2+1) - \frac{3}{2}}{2t^2+1}$ $= \frac{1}{2} - \frac{3}{2(2t^2+1)}$ $= \frac{1}{2} - \frac{3}{2(2t^2+1)}$ Long division: $2t^2+1 \overline{) t^2 - 1}$ gives quotient $\frac{1}{2}$; $t^2 + \frac{1}{2}$; $- - -$; remainder $-\frac{3}{2}$
(ii)	since $t \neq 0$ $2t^2 + 1 > 1$ $0 < \frac{1}{2(2t^2+1)} < \frac{1}{2}$ $-\frac{3}{2} < -\frac{3}{2(2t^2+1)} < 0$ $-1 < \frac{1}{2} - \frac{3}{2(2t^2+1)} < \frac{1}{2}$ $\therefore \quad -1 < \frac{dy}{dx} < \frac{1}{2}$

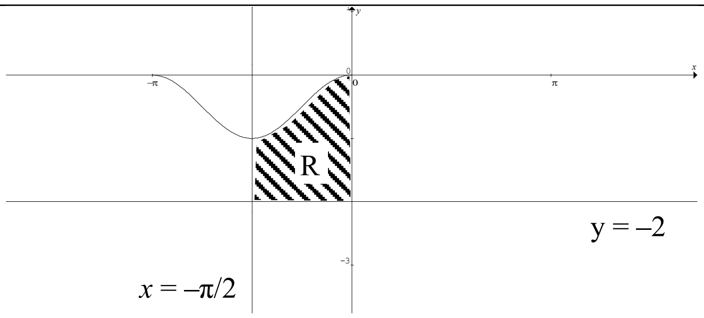
Qn	Solution
2	$\frac{1}{(2+ax)^5}$ $=(2+ax)^{-5}$ $=2^{-5}\left[1+\frac{ax}{2}\right]^{-5}$ $=\frac{1}{32}\left[1-\frac{5ax}{2}+\frac{(-5)(-6)}{2!}\left(\frac{ax}{2}\right)^2+\frac{(-5)(-6)(-7)}{3!}\left(\frac{ax}{2}\right)^3+...\right]$ Coefficient of x^3 = $\frac{1}{32}\times\frac{(-5)(-6)(-7)}{3!}\left(\frac{a}{2}\right)^3=-\frac{35}{256}a^3$ $\sqrt{4-ax}$ $=2\left(1-\frac{ax}{4}\right)^{\frac{1}{2}}$ $=2\left[1+\frac{1}{2}\left(-\frac{ax}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{ax}{4}\right)^2+...\right]$ Coefficient of x^2 = $-\frac{1}{64}a^2$ $-\frac{35}{256}a^3=5\left(-\frac{1}{64}a^2\right)$ $35a^3=20a^2$ $7a=4$, since $a\neq 0$ $a=\frac{4}{7}$

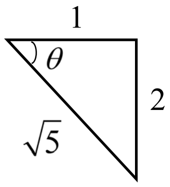
Qn	Solution
3(i)	
	$\begin{array}{r} 2x+2 \\ x-3\overline{)2x^2-4x+2} \\ 2x^2-6x \\ 2x+2 \\ 2x-6 \\ 8 \end{array}$ $\quad \mathrm{f}(x)=\dfrac{2(x-1)^2}{x-3}=2x+2+\dfrac{8}{x-3}$ The asymptotes are y = $2x$ + 2 and x = 3.
3(ii)	

Qn	Solution
	To have an odd number of roots, there are 7 points of intersection between the two curves, hence $a = k + \frac{2}{3}$
4(a)	**Algebraic Method:** $\frac{2x+1-(x^2-1)}{x^2-1} \le 0$ $\frac{-x^2+2x+2}{x^2-1} \le 0$ $\frac{x^2-2x-2}{x^2-1} \ge 0$ $\frac{(x-1)^2-3}{(x-1)(x+1)} \ge 0$ $\frac{\left[(x-1)-\sqrt{3}\right]\left[(x-1)+\sqrt{3}\right]}{(x-1)(x+1)} \ge 0$ $\left[(x-1)-\sqrt{3}\right]\left[(x-1)+\sqrt{3}\right](x-1)(x+1) \ge 0$  $x < -1$ or $1-\sqrt{3} \le x < 1$ or $x \ge 1+\sqrt{3}$
4(b)	$ay^2 + bx^3 + cx = 2$ Differentiate wrt x, $2ay\frac{dy}{dx} + 3bx^2 + c = 0 \ldots (*)$ Sub $(1, \sqrt{3})$ into eq C and obtain $3a + b + c = 2$ ..... (1) Sub $(-1, 1)$ into eq C and obtain $a - b - c = 2$ ..... (2) Sub $(-1, 1)$ into eq * and obtain $-3a + 3b + c = 0$ ..... (3)

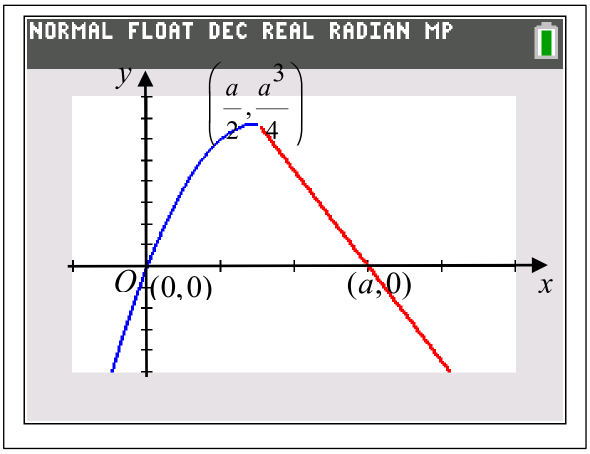
Qn	Solution
	Using GC, $a=1, b=2, c=-3$ Eq of C is $y^2+2x^3-3x=2$
5	Note that $l^2=h^2+r^2$. Total surface area of cone, $\pi r\sqrt{h^2+r^2}+\pi r^2=k\pi$ $\Rightarrow r\sqrt{h^2+r^2}=k-r^2$ $\Rightarrow r^2\left(h^2+r^2\right)=k^2-2kr^2+r^4$ $\Rightarrow r^2h^2+r^4=k^2-2kr^2+r^4$ $\Rightarrow r^2=\dfrac{k^2}{\left(h^2+2k\right)}$ (Shown) From above equation, $r^2=\dfrac{k^2}{h^2+2k}$ $V=\dfrac{1}{3}\pi r^2h=\dfrac{1}{3}\pi\left(\dfrac{k^2}{h^2+2k}\right)h=\dfrac{k^2\pi h}{3\left(h^2+2k\right)}$ $\dfrac{\mathrm{d}V}{\mathrm{d}h}=\dfrac{k^2\pi}{3}\dfrac{\left(h^2+2k\right)-h(2h)}{\left(h^2+2k\right)^2}$ $=\dfrac{k^2\pi\left(2k-h^2\right)}{3\left(h^2+2k\right)^2}$ At stationary point, $\dfrac{\mathrm{d}V}{\mathrm{d}h}=0$ $2k-h^2=0 \Rightarrow h=\sqrt{2k}$ $(\because h>0)$ <table><tr><td>h</td><td>$\left(\sqrt{2k}\right)^-$</td><td>$\left(\sqrt{2k}\right)$</td><td>$\left(\sqrt{2k}\right)^+$</td></tr><tr><td>$\dfrac{\mathrm{d}V}{\mathrm{d}h}$</td><td>+ve</td><td>0</td><td>-ve</td></tr></table>Alternatively, $\dfrac{\mathrm{d}^2V}{\mathrm{d}h^2}=\dfrac{k^2\pi}{3}\dfrac{\left(h^2+2k\right)^2(-2h)-\left(2k-h^2\right)2\left(h^2+2k\right)2h}{\left(h^2+2k\right)^4}$ $=\dfrac{2k^2\pi h}{3}\dfrac{-\left(h^2+2k\right)-2\left(2k-h^2\right)}{\left(h^2+2k\right)^3}$ $=\dfrac{2k^2\pi h}{3}\dfrac{h^2-6k}{\left(h^2+2k\right)^3}<0$ when $h=\sqrt{2k}$ Volume is maximum when $h=\sqrt{2k}$

Qn	Solution
	$V=\frac{k^2\pi\sqrt{2k}}{3(2k+2k)}=\frac{k\pi\sqrt{2k}}{12}=\frac{\sqrt{2}k^{\frac{3}{2}}\pi}{12}$ units^3
6(a)	$\int \frac{3e^x}{5-0.3e^x}\,dx$ $=-10\int \frac{-0.3e^x}{5-0.3e^x}\,dx$ $=-10\ln\left\|5-0.3e^x\right\|+C$ where C is an arbitrary constant.
6(b)	Let $I=\int \cos(\ln x)\,dx$ $u=\cos(\ln x)$, $u'=-\frac{1}{x}\sin(\ln x)$, $v'=1$, $v=x$ $I=x\cos(\ln x)-\int -\frac{1}{x}[x\sin(\ln x)]\,dx$ $=x\cos(\ln x)+\int \sin(\ln x)\,dx$ $u=\sin(\ln x)$, $u'=\frac{1}{x}\cos(\ln x)$, $v'=1$, $v=x$ $I=x\cos(\ln x)+x\sin(\ln x)-\int \frac{1}{x}[x\cos(\ln x)]\,dx$ $=x\cos(\ln x)+x\sin(\ln x)-\int \cos(\ln x)\,dx$ $2I=x\cos(\ln x)+x\sin(\ln x)$ $I=\frac{x}{2}[\cos(\ln x)+\sin(\ln x)]+C$ where C is an arbitrary constant.
6(c)	 When $2e^x-5=0$, $x=\ln 2.5$ $\left\|2e^x-5\right\|$ $\begin{cases} 2e^x-5 & ,x\geq \ln 2.5 \\ -(2e^x-5) & ,x<\ln 2.5 \end{cases}$ $\int_0^3 \left\|2e^x-5\right\|\,dx$

Qn	Solution
	$=-\int_0^{\ln 2.5} 2e^x-5\ \mathrm{d}x+\int_{\ln 2.5}^{3} 2e^x-5\ \mathrm{d}x$ $=\left[5x-2e^x\right]_0^{\ln 2.5}+\left[2e^x-5x\right]_{\ln 2.5}^{3}$ $=\left[\left(5\ln 2.5-2e^{\ln 2.5}\right)+2e^0\right]+\left[\left(2e^3-15\right)-\left(2e^{\ln 2.5}-5\ln 2.5\right)\right]$ $=10\ln 2.5-4e^{\ln 2.5}+2e^3-13$ $=10\ln 2.5-4(2.5)+2e^3-13$ $=10\ln 2.5+2e^3-23$
7(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin x\cos x$ $=-\sin 2x=0$ $2x=-\pi$ $x=-\frac{\pi}{2}$ $y=-1$ Minimum point = $\left(-\frac{\pi}{2},-1\right)$
7(ii)	 Area $=2\times\frac{\pi}{2}+\int_{-\pi/2}^{0}-\sin^2 x\ \mathrm{d}x$ $=\pi-\int_{-\pi/2}^{0}\frac{1-\cos 2x}{2}\ \mathrm{d}x$ $=\pi-\left[\frac{x}{2}-\frac{\sin 2x}{4}\right]_{-\pi/2}^{0}$ $=\pi-\left[0-\left(-\frac{\pi}{4}\right)\right]$ $=\frac{3\pi}{4}$ units^2
7(iii)	Volume $=\pi\left(\frac{\pi}{2}\right)^2(2-1)+\pi\int_{-1}^{0}\left(\sin^{-1}\left(-\sqrt{-y}\right)\right)^2\ \mathrm{d}y$ $=7.75156917+2.304987524$ $=10.05655669$ $=10.1$ units^3 (to 3 s.f.)

Qn	Solution
8(i)	$(z-w)(z-w^*)$ $=z^2-(w+w^*)z+ww^*$ $=z^2-2\,\mathrm{Re}(w)z+\|w\|^2$ Since $\mathrm{Re}(w),\|w\|^2\in\mathbb{R}$, therefore the product of the factors is a quadratic polynomial with real coefficients.
8(ii)	$w=\sqrt{5}\left[\cos\left(-\tan^{-1}(2)\right)+\mathrm{i}\sin\left(-\tan^{-1}(2)\right)\right]$ $=\sqrt{5}\left(\frac{1}{\sqrt{5}}-\mathrm{i}\frac{2}{\sqrt{5}}\right)$ $=1-2i$ 
8(iii)	$(z-w)(z-w^*)=z^2-2\,\mathrm{Re}(w)z+\|w\|^2$ $=z^2-2z+5$ **Method 1** $z^4+az^3+46z^2+bz+125=(z^2-2z+5)(z^2+kz+25)=0$ Compare coefficient of z^2: $46=25+5-2k\Rightarrow k=-8$ Compare coefficient of z^3: $a=k-2\Rightarrow a=-10$ Compare coefficient of z: $b=-50+5k\Rightarrow b=-90$ **Method 2** Substitute $z=1-2\mathrm{i}$ into the equation, $(1-2\mathrm{i})^4+a(1-2\mathrm{i})^3+46(1-2\mathrm{i})^2+b(1-2\mathrm{i})+125=0$ $-7+24\mathrm{i}+a(-11+2\mathrm{i})+46(-3-4\mathrm{i})+b(1-2\mathrm{i})+125=0$ $-20-11a+b+(-160+2a-2b)\mathrm{i}=0$ Comparing Real and Imaginary part respectively, $\begin{cases}11a-b=-20\\2a-2b=160\end{cases}\Rightarrow\begin{cases}a=-10\\b=-90\end{cases}$

Qn	Solution
	The remaining roots are $\therefore z^2-8z+25=0$ $(z-4)^2+9=0$ $z=4\pm 3\text{i}$ The roots to the equation are $4+3\text{i},\ 4-3\text{i},\ 1-2\text{i},\ 1+2\text{i}$.
9(i)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{a}{x^2}-bx$, where a & b are positive constants. When $x=0.5$, $\frac{\mathrm{d}x}{\mathrm{d}t}=0$ $4a=\frac{b}{2}$ $b=8a$ $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{a}{x^2}-8ax=\frac{a\left(1-8x^3\right)}{x^2}$ $\therefore\ \frac{\mathrm{d}x}{\mathrm{d}t}=\frac{k\left(1-8x^3\right)}{x^2}$
9(ii)	**<u>Method 1</u>:** $\frac{\mathrm{d}x}{\mathrm{d}t}=k\left(\frac{1-8x^3}{x^2}\right)$ $\frac{\mathrm{d}t}{\mathrm{d}x}=\left(\frac{x^2}{k\left(1-8x^3\right)}\right)$ $k\int 1\frac{\mathrm{d}t}{\mathrm{d}x}\,\mathrm{d}x=\int\frac{x^2}{\left(1-8x^3\right)}\,\mathrm{d}x$ $kt=-\frac{1}{24}\int\frac{-24x^2}{\left(1-8x^3\right)}\,\mathrm{d}x$ $kt=-\frac{1}{24}\ln\left\|1-8x^3\right\|+C$ $\ln\left\|1-8x^3\right\|=-24kt+24C$ $\left\|1-8x^3\right\|=\mathrm{e}^{24C}\mathrm{e}^{-24kt}$ $1-8x^3=A\mathrm{e}^{-24kt}$ where $A=\pm\mathrm{e}^{24C}$ When $t=0, x=3,\ A=-215$ $\therefore\ 1-8x^3=-215\mathrm{e}^{-24kt}$ When $t=1, x=2,\ \mathrm{e}^{-24k}=\frac{63}{215}$

Qn	Solution
	$\therefore\ 1-8x^3=-215\left(\frac{63}{215}\right)^t$ When $t=3$, $x=\frac{1}{2}\left(1+215\left(\frac{63}{215}\right)^3\right)^{\frac{1}{3}}=0.92877$ $\therefore$ Number of fish is 929 (nearest integer)
9(iii)	As $t\to\infty$, $\left(\frac{63}{215}\right)^t\to 0,\quad x\to 0.5$ In the long run, the number of fish decreases and tends to 500.
10(i)	
10 (ii)	When $x\le\frac{a}{2}$, every horizontal line $y=k$ cuts the graph of $y=\mathrm{f}(x)$ at most once. Hence f is one-one and therefore f^{-1} exists. The greatest value of k is $\frac{a}{2}$.
10(iii)	Let $y=a^2x-ax^2=-a\left(x-\frac{a}{2}\right)^2+\frac{a^3}{4}$ $\left(x-\frac{a}{2}\right)^2=\frac{1}{a}\left(\frac{a^3}{4}-y\right)$ $x-\frac{a}{2}=\pm\sqrt{\frac{1}{a}\left(\frac{a^3}{4}-y\right)}$ $x=\frac{a}{2}\pm\sqrt{\frac{1}{a}\left(\frac{a^3}{4}-y\right)}$

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<tr><th>Qn</th><th>Solution</th></tr>
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$\therefore x=\frac{a}{2}-\sqrt{\frac{1}{a}\left(\frac{a^3}{4}-y\right)}\quad (\because x\le \frac{a}{2})$

Hence, $\mathrm{f}^{-1}: x\to \frac{a}{2}-\sqrt{\frac{1}{a}\left(\frac{a^3}{4}-x\right)},\ x\in\mathbb{R},\ x\le\frac{a^3}{4}$

The graph of $y=\mathrm{f}^{-1}(x)$ is a reflection of the graph of $y=\mathrm{f}(x)$ in the line $y=x$.

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<tr><td>10(iv)</td><td>

Since $R_{\mathrm{f}}=\left(-\infty,\frac{a^3}{4}\right]\subseteq\left(-\infty,a^3\right]=D_{\mathrm{g}}$, the composite function gf exists.

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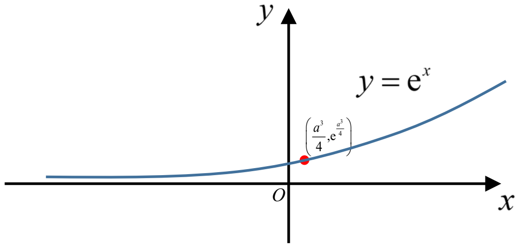
<tr><td>10(v)</td><td>

$D_{\mathrm{gf}}=D_{\mathrm{f}}\xrightarrow{\mathrm{f}}R_{\mathrm{f}}\xrightarrow{\mathrm{g}}\left(-\infty,a^3\right]$



$R_{\mathrm{gf}}=\left(0,\mathrm{e}^{\frac{a^3}{4}}\right]$

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<tr><td>11(i)</td><td>

Amount of drug before the 2nd dose = $\left(\frac{1}{2}\right)^3 D=\frac{1}{8}D$

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<tr><td>11(ii)</td><td>

<table>
<tr><th>n th dose</th><th>Amount of drug = U_n</th></tr>
<tr><td>1</td><td>D</td></tr>
<tr><td>2</td><td>$\left(\frac{1}{2}\right)^3 D+D=\frac{1}{8}D+D$</td></tr>
<tr><td>3</td><td>$\left(\frac{1}{8}D+D\right)\left(\frac{1}{2}\right)^3+D$
$=\left(\frac{1}{8}\right)^2 D+\left(\frac{1}{8}\right)D+D$</td></tr>
<tr><td>…</td><td></td></tr>
<tr><td>n</td><td>$\left(\frac{1}{8}\right)^{n-1} D+\left(\frac{1}{8}\right)^{n-2} D+\ldots+D$</td></tr>
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<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td></td><td>$U_n = \left(\frac{1}{8}\right)^{n-1} D + \left(\frac{1}{8}\right)^{n-2} D + \ldots + D$
$= \dfrac{D\left(1-\left(\frac{1}{8}\right)^n\right)}{1-\frac{1}{8}}$
$= \frac{8}{7}D\left(1-\left(\frac{1}{8}\right)^n\right)$ (Shown)</td></tr>
<tr><td>11(iii)</td><td>As $n \to \infty$, $\left(\frac{1}{8}\right)^n \to 0$. Hence $U_n \to \frac{8}{7}D$.
The amount of drug that is present in the bloodstream if the patient continues to take it over a long period of time is $\frac{8}{7}D$ milligrams.
Thus, $\frac{8}{7}D \le 60$
$D \le 52.5$</td></tr>
<tr><td>11(iv)</td><td><table><tr><td>n</td><td>$\left|\frac{8}{7}(40)\left(1-\left(\frac{1}{8}\right)^n\right)-50\right| \le 4.3$</td></tr><tr><td>3</td><td>4.375 > 4.3</td></tr><tr><td>4</td><td>4.2969 < 4.3</td></tr><tr><td>5</td><td>4.2871 < 4.3</td></tr></table>
The least number of dose is 4.
<u>Alternate method</u></td></tr>
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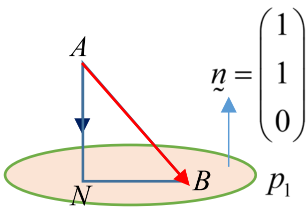
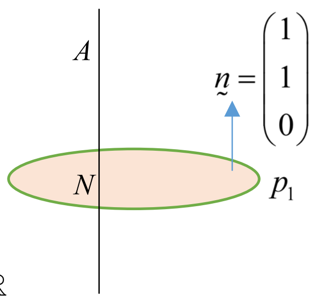
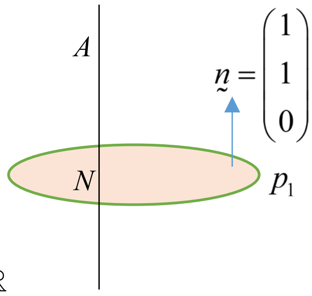
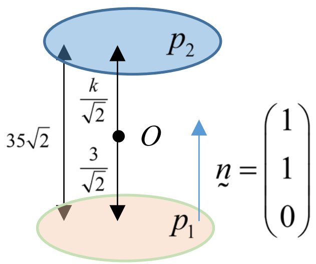
Qn	Solution
	$\left\|\frac{8}{7}(40)\left(1-\left(\frac{1}{8}\right)^n\right)-50\right\| \leq 4.3$ $\left\|\frac{320}{7}\left(1-\left(\frac{1}{8}\right)^n\right)-50\right\| \leq 4.3$ $\left\|-\frac{30}{7}-\frac{320}{7}\left(\frac{1}{8}\right)^n\right\| \leq 4.3$ $\frac{30}{7}+\frac{320}{7}\left(\frac{1}{8}\right)^n \leq 4.3$ $\left(\frac{1}{8}\right)^n \leq 3.125\times10^{-4}$ $n \geq \frac{\ln(3.125\times10^{-4})}{\ln\left(\frac{1}{8}\right)}$ $n \geq 3.88$ Least n is 4. Therefore, the least number of dose is 4.
11(v)	Let t_n mg be the amount of drug taken by the patient on the n^{th} day $t_n = 4+(n-1)(2) \leq 20$ $n \leq 9$ The 9^{th} day after the patient has started to take the medication is the first day in which his medication not exceeding the maximum dose. Total amount of drug taken $= \frac{9}{2}\left[2(4)+8(2)\right]+20\times5 = 208$ mg
12 (i)	Equation of p_1: $x+y=-3$ Hence scalar product form of p_1: $\underset{\sim}{r}\bullet\begin{pmatrix}1\\1\\0\end{pmatrix}=-3$ $\overrightarrow{OB}=\begin{pmatrix}-1\\-2\\-1\end{pmatrix}$ Since $\begin{pmatrix}-1\\-2\\-1\end{pmatrix}\bullet\begin{pmatrix}1\\1\\0\end{pmatrix}=-1-2=-3$

Qn	Solution
	Thus B lies on p_1
12 (ii)	$\overrightarrow{OA}=\begin{pmatrix}-3\\4\\-1\end{pmatrix}$ **<u>Method 1</u>**:  $\overrightarrow{AB}=\begin{pmatrix}-1\\-2\\-1\end{pmatrix}-\begin{pmatrix}-3\\4\\-1\end{pmatrix}=\begin{pmatrix}2\\-6\\0\end{pmatrix}$ Shortest distance from A to p_1 $=\left\|\overrightarrow{AB}\cdot\hat{\underset{\sim}{n}}\right\|=\left\|\frac{\begin{pmatrix}2\\-6\\0\end{pmatrix}\cdot\begin{pmatrix}1\\1\\0\end{pmatrix}}{\sqrt{2}}\right\|=\frac{4}{\sqrt{2}}=2\sqrt{2}$ metres **<u>Method 2</u>:** Let N be the foot of perpendicular of point A on plane p_1 $L_{AN}:\underset{\sim}{r}=\begin{pmatrix}-3\\4\\-1\end{pmatrix}+\mu\begin{pmatrix}1\\1\\0\end{pmatrix},\quad \mu\in\mathbb{R}$  Since N is a point on line L_{AN}, $\overrightarrow{ON}=\begin{pmatrix}-3+\mu\\4+\mu\\-1\end{pmatrix}$ for some $\mu\in\mathbb{R}$ Since N is also on plane p_1,

Qn	Solution
	$\begin{pmatrix}-3+\mu\\4+\mu\\-1\end{pmatrix}\bullet\begin{pmatrix}1\\1\\0\end{pmatrix}=-3$ $-3+\mu+4+\mu+0=-3$ $\mu=-2$ $\overrightarrow{AN}=\left\|-2\begin{pmatrix}1\\1\\0\end{pmatrix}\right\|=2\sqrt{2}$ units
12 (iii)	Let θ be the acute angle the path *AB* makes with p_1 $\theta=90°-\cos^{-1}\dfrac{\left\|\begin{pmatrix}2\\-6\\0\end{pmatrix}\bullet\begin{pmatrix}1\\1\\0\end{pmatrix}\right\|}{\sqrt{40}\sqrt{2}}$ $=90°-\cos^{-1}\dfrac{1}{\sqrt{5}}$ $=26.56505=26.6°$ (1 d.p) **OR** $\sin\theta=\dfrac{\left\|\begin{pmatrix}2\\-6\\0\end{pmatrix}\bullet\begin{pmatrix}1\\1\\0\end{pmatrix}\right\|}{\sqrt{40}\sqrt{2}}=\dfrac{4}{2\sqrt{20}}=\dfrac{1}{\sqrt{5}}$ $\theta=26.56505=26.6°$ (1 d.p)

Qn	Solution
12 (iv)	Let N be the foot of perpendicular of point A on plane p_1 $L_{AN}: \underset{\sim}{r} = \begin{pmatrix} -3 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \mu \in \mathbb{R}$  Since N is a point on line L_{AN}, $\overrightarrow{ON} = \begin{pmatrix} -3+\mu \\ 4+\mu \\ -1 \end{pmatrix}$ for some $\mu \in \mathbb{R}$ Since N is also on plane p_1, $\begin{pmatrix} -3+\mu \\ 4+\mu \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = -3$ $-3+\mu+4+\mu+0=-3$ $\mu=-2$ $\overrightarrow{ON} = \begin{pmatrix} -3 \\ 4 \\ -1 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix}$ $\overrightarrow{AN} = \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix}$ $2\overrightarrow{AN} = \overrightarrow{AA'}$ $\overrightarrow{OA'} = 2\overrightarrow{AN} + \overrightarrow{OA} = \begin{pmatrix} -7 \\ 0 \\ -1 \end{pmatrix}$ $\overrightarrow{BA'} = \begin{pmatrix} -7 \\ 0 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix}$ Vector equation of BA': $\underset{\sim}{r} = \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix} + \alpha \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}, \quad \alpha \in \mathbb{R}$

Qn	Solution
12 (v)	**<u>Method 1</u>**: $p_1: \underset{\sim}{r} \bullet \begin{pmatrix}1\\1\\0\end{pmatrix} = -3 \Rightarrow \underset{\sim}{r} \bullet \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix} = \frac{-3}{\sqrt{2}}$ $p_2: \underset{\sim}{r} \bullet \begin{pmatrix}1\\1\\0\end{pmatrix} = k \Rightarrow \underset{\sim}{r} \bullet \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix} = \frac{k}{\sqrt{2}}$ Since $\frac{k}{\sqrt{2}}$ is positive and $\frac{-3}{\sqrt{2}}$ is negative, the origin is in between p_1 and p_2, and the distance between the planes is $\frac{k}{\sqrt{2}} - \left(\frac{-3}{\sqrt{2}}\right) = \frac{k+3}{\sqrt{2}}$ Therefore we have $\frac{k+3}{\sqrt{2}} = 35\sqrt{2}$ $k+3 = 35\sqrt{2}\sqrt{2}$ $k+3 = 70$ $k = 67$  Hence $p_2: \underset{\sim}{r} \bullet \begin{pmatrix}1\\1\\0\end{pmatrix} = 67$ $L_{BC}: \underset{\sim}{r} = \begin{pmatrix}-1\\-2\\-1\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\2\end{pmatrix}, \quad \lambda \in \mathbb{R}$ Since C is on the line L_{BC}, we have $\overrightarrow{OC} = \begin{pmatrix}-1+3\lambda\\-2+\lambda\\-1+2\lambda\end{pmatrix}$ for some $\lambda \in \mathbb{R}$ Since C is also on the plane p_2, we have $\begin{pmatrix}-1+3\lambda\\-2+\lambda\\-1+2\lambda\end{pmatrix} \bullet \begin{pmatrix}1\\1\\0\end{pmatrix} = 67$ $-1+3\lambda-2+\lambda = 67$ $\lambda = \frac{35}{2}$

Qn	Solution
	Therefore $\overrightarrow{OC} = \begin{pmatrix} -1+3\left(\frac{35}{2}\right) \\ -2+\frac{35}{2} \\ -1+2\left(\frac{35}{2}\right) \end{pmatrix} = \begin{pmatrix} \frac{103}{2} \\ \frac{31}{2} \\ 34 \end{pmatrix}$, and hence the coordinates of C: $(51.5,\quad 15.5,\quad 34)$. Since C lies on p_2: $\begin{pmatrix} 51.5 \\ 15.5 \\ 34 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 67$ $\underset{\sim}{r} \bullet \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 67$ Cartesian equation: $x + y = 67 \Rightarrow k = 67$