ANDERSON SERANGOON JUNIOR COLLEGE

H2 MATHEMATICS

9758/02

3 hours

JC2 Preliminary Examination Paper 2
(100 marks)

Additional Material(s): List of Formulae (MF26)

CANDIDATE NAME

CLASS /

READ THESE INSTRUCTIONS FIRST

Write your name and class in the boxes above.
Please write clearly and use capital letters.
Write in dark blue or black pen. HB pencil may be used for graphs and diagrams only.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions and write your answers in this booklet.
Do not tear out any part of this booklet.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

All work must be handed in at the end of the examination. If you have used any additional paper, please insert them inside this booklet.
The number of marks is given in brackets [ ] at the end of each question or part question.

Question number	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total	

Section A: Pure Mathematics [40 marks]

1(a)

Solution

$\text{p} = \lambda\text{a} + (1-\lambda)\mathbf{b}$

$\text{p} = \lambda\text{a} + \mathbf{b} - \lambda\mathbf{b}$

$\text{p} - \mathbf{b} = \lambda(\text{a} - \mathbf{b})$

$\overrightarrow{BP} = \lambda\overrightarrow{BA}$ and common point B

$\therefore$ the points A, B and P are collinear.

(b)

Solution

Let F be the foot of perpendicular from O to AB

$\overrightarrow{OF} = \mathbf{a} + \mu(\mathbf{b} - \mathbf{a})$ for some μ

$\overrightarrow{OF} \bullet \overrightarrow{AB} = 0$

$[\mathbf{a} + \mu(\mathbf{b} - \mathbf{a})] \bullet (\mathbf{b} - \mathbf{a}) = 0$

$\mathbf{a} \bullet \mathbf{b} + \mu|\mathbf{b} - \mathbf{a}|^2 - \mathbf{a} \bullet \mathbf{a} = 0$

$\mu = \dfrac{|\mathbf{a}|^2}{|\mathbf{b} - \mathbf{a}|^2}$

$\overrightarrow{OF} = \mathbf{a} + \dfrac{|\mathbf{a}|^2}{|\mathbf{a}|^2 + |\mathbf{b}|^2}(\mathbf{b} - \mathbf{a}) \quad \because |\mathbf{a}|^2 + |\mathbf{b}|^2 = |\mathbf{b} - \mathbf{a}|^2$

2

Solution

Let $\mathrm{P}(z) = z^4 - 4z^3 + az^2 + bz + 78$

Since $3+2\mathrm{i}$ is a root of $\mathrm{P}(z)=0$, $\mathrm{P}(3+2\mathrm{i})=0$.

$$(3+2\mathrm{i})^4 - 4(3+2\mathrm{i})^3 + a(3+2\mathrm{i})^2 + b(3+2\mathrm{i}) + 78 = 0$$

$$(-119+120\mathrm{i}) - 4(-9+46\mathrm{i}) + a(5+12\mathrm{i}) + b(3+2\mathrm{i}) + 78 = 0$$

Equating real parts: $5a + 3b = 5$ ----- (1)

Equating imaginary parts: $12a + 2b = 64$ ----- (2)

Solving (1) and (2), we get $a = 7$ and $b = -10$

Now $\mathrm{P}(z) = z^4 - 4z^3 + 7z^2 - 10z + 78$

Since coefficients of $\mathrm{P}(z)$ are all real, $3-2\mathrm{i}$ is also a root of $\mathrm{P}(z)=0$.

A quadratic factor of $\mathrm{P}(z)$ $= (z-3+2\mathrm{i})(z-3-2\mathrm{i})$

$= z^2 - 6z + 13$

Consider $\mathrm{P}(z) = z^4 - 4z^3 + 7z^2 - 10z + 78 = (z^2 - 6z + 13)(z^2 + cz + d)$

Comparing constants: $78 = 13d \Rightarrow d = 6$

Comparing coefficient of z: $-10 = -6d + 13c \Rightarrow c = 2$

Equating $z^2 + 2z + 6 = 0$, we have $z = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(6)}}{2(1)} = \dfrac{-2 \pm \sqrt{4(-5)}}{2}$

$= -1 \pm \sqrt{5}\mathrm{i}$

Therefore, the other roots are $3-2\mathrm{i}$, $-1+\sqrt{5}\mathrm{i}$ and $-1-\sqrt{5}\mathrm{i}$.

3 (a)

Solution

$$\int e^{2x}\sin x\,dx=\frac{1}{2}e^{2x}\sin x-\frac{1}{2}\int e^{2x}\cos x\,dx$$

$$u=\sin x \qquad \frac{dv}{dx}=e^{2x}$$
$$\frac{du}{dx}=\cos x \qquad v=\frac{e^{2x}}{2}$$

$$=\frac{1}{2}e^{2x}\sin x-\frac{1}{2}\left[\frac{1}{2}e^{2x}\cos x+\frac{1}{2}\int e^{2x}\sin x\,dx\right]$$

$$u=\cos x \qquad \frac{dv}{dx}=e^{2x}$$
$$\frac{du}{dx}=-\sin x \qquad v=\frac{e^{2x}}{2}$$

$$\int e^{2x}\sin x\,dx=\frac{1}{2}e^{2x}\sin x-\frac{1}{4}e^{2x}\cos x-\frac{1}{4}\int e^{2x}\sin x\,dx+C_1$$

$$\frac{5}{4}\int e^{2x}\sin x\,dx=\frac{1}{2}e^{2x}\sin x-\frac{1}{4}e^{2x}\cos x+C_1$$

$$\int e^{2x}\sin x\,dx=\frac{2}{5}e^{2x}\sin x-\frac{1}{5}e^{2x}\cos x+\frac{4C_1}{5}$$

$$\int e^{2x}\sin x\,dx=\frac{1}{5}e^{2x}(2\sin x-\cos x)+c \text{ where } c=\frac{4C_1}{5} \text{ (Shown)}$$

(b)

Solution

Required volume

$$=\pi\int_0^{\frac{\pi}{2}}\left(e^{x-\frac{\pi}{2}}\sqrt{\sin x}\right)^2dx-\frac{1}{3}\pi(1)^2\left(\frac{\pi}{4}\right)\text{pi} \text{ or}$$

$$=\pi\int_0^{\frac{\pi}{2}}\left(e^{x-\frac{\pi}{2}}\sqrt{\sin x}\right)^2dx-\pi\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\frac{4}{\pi}x-1\right)^2dx$$

$$=\pi\int_0^{\frac{\pi}{2}}e^{2x-\pi}\sin x\,dx-\frac{1}{12}\pi^2$$

$$=\frac{\pi}{e^{\pi}}\int_0^{\frac{\pi}{2}}e^{2x}\sin x\,dx-\frac{1}{12}\pi^2$$

$$=\frac{\pi}{e^{\pi}}\left[\frac{1}{5}e^{2x}(2\sin x-\cos x)\right]_0^{\frac{\pi}{2}}-\frac{1}{12}\pi^2$$

$$=\frac{\pi}{5e^{\pi}}\left[e^{\pi}(2\sin\frac{\pi}{2}-\cos\frac{\pi}{2})-e^{0}(2\sin 0-\cos 0)\right]-\frac{1}{12}\pi^2$$

$$= \frac{\pi}{5e^{\pi}}\left(2e^{\pi}+1\right)-\frac{1}{12}\pi^2$$
$$= \frac{\pi}{5}\left(2+e^{-\pi}\right)-\frac{1}{12}\pi^2$$

4 **(a)**

<u>Solution</u>

Let $y=-\left(e^{x-3}-1\right)=1-e^{x-3}$ $\quad\quad \because x\le 3$

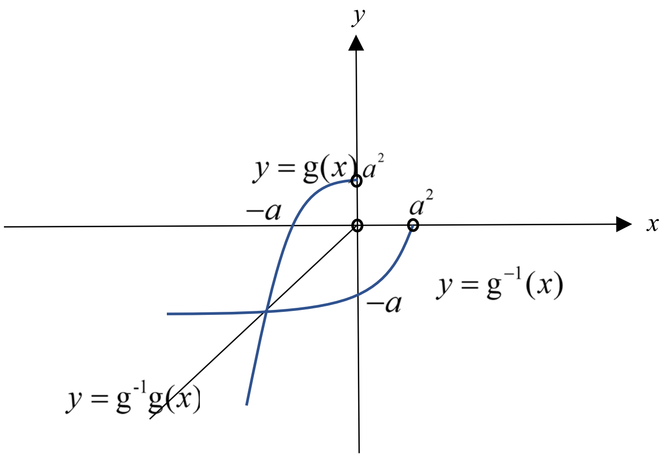
$e^{x-3}=1-y$

$x=3+\ln(1-y)$

$f^{-1}: x\mapsto 3+\ln(1-x),\quad 0\le x<1$

(b)

<u>Solution</u>



(c)

<u>Solution</u>

$g(x)=x$

$a^2-x^2=x$

$x^2+x-a^2=0$

$\left(x+\frac{1}{2}\right)^2-a^2-\frac{1}{4}=0$

$x=-\frac{1}{2}\pm\sqrt{\frac{1}{4}+a^2}$

$x=-\frac{1}{2}+\sqrt{\frac{1}{4}+a^2}$ $\quad$ or $\quad$ $x=-\frac{1}{2}-\sqrt{\frac{1}{4}+a^2}$

(rejected $\because x<0$)

(d)

Solution

$\mathrm{R_g} = \left(-\infty, a^2\right)$
$\mathrm{D_f} = \left(-\infty, 3\right]$

Since $\mathrm{R_g} \subseteq \mathrm{D_f}$ for $0 < a < 1$, therefore fg exist.

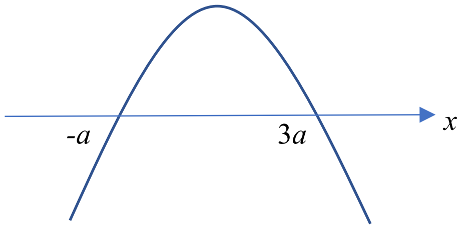
$$R_{\mathrm{g}} = \left(-\infty, a^2\right) \xrightarrow{\mathrm{f}} R_{\mathrm{fg}} = \left(1 - \mathrm{e}^{a^2-3}, 1\right)$$

5 (a)

Solution

$$\frac{y^2}{x^2}=\frac{3a-x}{a+x} \;,\; x\neq 0$$

Since $\frac{y^2}{x^2}\geq 0$ for all values of x and y,



$$\frac{3a-x}{a+x}\geq 0 \;,\; x\neq -a$$

$$(a+x)(3a-x)\geq 0$$

$$-a<x\leq 3a$$

Since $x\neq 0$,

$$\therefore -a<x<0 \text{ or } 0<x\leq 3a$$

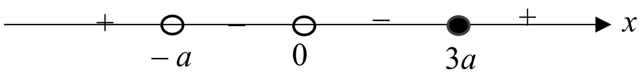
Alternative Solution

$$y^2=\frac{(3a-x)x^2}{a+x} \;,\; x\neq 0$$

Since $y^2\geq 0\ \forall y\in\mathbb{R}$, $\frac{(3a-x)x^2}{a+x}\geq 0$, $x\neq 0$

$$-\frac{(x-3a)x^2}{a+x}\geq 0$$

$$\frac{(x-3a)x^2}{x+a}\leq 0$$



$$\therefore -a<x<0 \text{ or } 0<x\leq 3a$$

(b)

Solution

Let $a=1$, we have $y^2(1+x)=x^2(3-x)$.

$$2y\frac{\mathrm{d}y}{\mathrm{d}x}(1+x)+y^2=2x(3-x)-x^2$$

When $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, $y^2=2x(3-x)-x^2$

$$=6x-3x^2$$

Substitute $y^2=6x-3x^2$ in $y^2(a+x)=x^2(3a-x)$,

$$(6x-3x^2)(1+x)=x^2(3-x)$$

$$6x+6x^2-3x^2-3x^3=3x^2-x^3$$

$$x^3-3x=0$$

$$x(x^2-3)=0$$

$$x = 0 \text{ or } x = \pm\sqrt{3}$$

Since $x \neq 0$ and $-a < x \leq 3a$, we have $x = \sqrt{3}$

When $x = \sqrt{3}$, $y^2 = 6\sqrt{3} - 3(3)$

$$y = \pm\sqrt{6\sqrt{3} - 9}$$

The coordinates of the points are $\left(\sqrt{3}, \sqrt{6\sqrt{3}-9}\right)$ and $\left(\sqrt{3}, -\sqrt{6\sqrt{3}-9}\right)$

(c)

Solution

When $x = 1$,

$$y^2(1+1) = (1)^2(3-1)$$

$$y = \pm 1$$

When $x = 1$, $y = 1$,

$$2\frac{\mathrm{d}y}{\mathrm{d}x}(1+1) + 1^2 = 2(3-1) - 1^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}$$

Equation of normal is $y - 1 = -2(x-1)$

$$\Rightarrow y = -2x + 3$$

When $x = 1$, $y = -1$,

$$-2\frac{\mathrm{d}y}{\mathrm{d}x}(1+1) + 1^2 = 2(3-1) - 1^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}$$

Equation of normal is $y - (-1) = 2(x-1)$

$$\Rightarrow y = 2x - 3$$

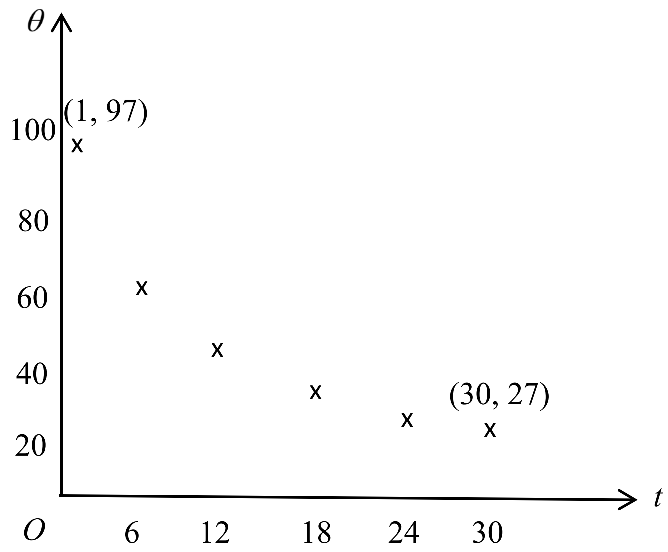
Solving $y = -2x + 3$ and $y = 2x - 3$, we get $x = \frac{3}{2}$ and $y = 0$

Therefore, the coordinates of N are $\left(\frac{3}{2}, 0\right)$

Section B: Probability and Statistics [60 marks]

6(a)

[Solution]



A student attempts to model the relationship between θ and t using two models:

(A) $\theta = a + bt$ or (B) $\theta = c + \dfrac{d}{t}$.

(b)
[Solution]

For model A, $r = -0.909$
For model B, $r = 0.933$
Model B is more appropriate since its $|r|$ is closer to 1 and the scatter diagram shows that a curve fits the data points better than a straight line.

(c)
[Solution]

$$\theta = 35.02439 + \frac{64.36079}{t}$$

$$\theta = 35.0 + \frac{64.4}{t}$$

When $t = 45$,

$$\theta = 35.02439 + \frac{64.36079}{45}$$

$$= 36.5\ ^{\circ}\text{C (3 sf)}$$

The prediction is not reliable as $t = 45$ lies outside the data range of $1 \le t \le 30$, hence extrapolation, and the linear relationship may not hold outside the data range.

When $t = 45$,

$$\theta = 35.0 + \frac{64.4}{45}$$

$$= 36.4\ ^{\circ}\text{C (3 sf)}$$

(d)

[Solution]

$$\theta = 35.02439 + \frac{64.36079}{t} \qquad T = \theta + 273.15$$

$$T = 308 + \left(\frac{64.4}{t}\right)$$

7(a)

[Solution]

$$\text{P(Score = 5)} = \frac{4}{5} \times \frac{\pi(10)^2}{\pi(30)^2} = \frac{4}{45}$$

$$\text{P(Score = 3)} = \frac{4}{5} \times \frac{\pi(20)^2 - \pi(10)^2}{\pi(30)^2} = \frac{4}{15}$$

$$\text{P(Score = 1)} = \frac{4}{5} \times \frac{\pi(30)^2 - \pi(20)^2}{\pi(30)^2} = \frac{4}{9} \quad \text{(Shown)}$$

(b)

[Solution]

(b) $\text{P}(X = 0) = \left(\frac{1}{5}\right)\left(\frac{1}{5}\right) = \frac{1}{25}$

$$\text{P}(X = 1) = \left(\frac{1}{5}\right)\left(\frac{4}{9}\right)(2) + \left(\frac{4}{9}\right)^2 = \frac{152}{405}$$

$$\text{P}(X = 3) = \left(\frac{1}{5} + \frac{4}{9}\right)\left(\frac{4}{15}\right)(2) + \left(\frac{4}{15}\right)^2 = \frac{56}{135}$$

$$\text{P}(X = 5) = \left(\frac{1}{5} + \frac{4}{9} + \frac{4}{15}\right)\left(\frac{4}{45}\right)(2) + \left(\frac{4}{45}\right)^2 = \frac{344}{2025}$$

x	0	1	3	5
$\text{P}(X = x)$	$\frac{1}{25}$	$\frac{152}{405}$	$\frac{56}{135}$	$\frac{344}{2025}$

(c)
[Solution]

$$\mathrm{E}(X) = 0\left(\frac{1}{25}\right) + 1\left(\frac{152}{405}\right) + 3\left(\frac{56}{135}\right) + 5\left(\frac{344}{2025}\right)$$

$$= \frac{200}{81}$$

8 (a)

[Solution]

No. of ways $= 4! \times 8! = 967680$

(b) (i)

[Solution]

<u>Method 1: Slotting technique</u>

No. of ways $= {}^{10}C_4 \times 4! \times {}^{5}C_2 \times 2! \times 6! \times 2 = 145152000$

Select and arrange 4 passengers to be seated with the 2 particular girls.

No. of ways to slot in the 2 particular girls.

No. of ways to arrange the remaining 6 passengers.

Each row

<u>Method 2</u>

No. of ways $= ({}^{6}C_2 - 5) \times 2! \times 10! \times 2 = 145152000$

No. of ways to arrange 2 particular girls in a row and excluding 5 possible ways (together).

Permutation between the 2 particular girls.

Arranging 10 other passengers

Each row

(ii)

[Solution]

Case 1:The 4 girls and the remaining girl are seated on the same side.

No of ways $= {}^{5}C_4 \times 4! \times {}^{7}C_1 \times 2! \times 6! \times 2 = 2419200$

No. of ways to choose 4 girls to be together and permutate them

Choose the boy to sit in between the 4 girls and 1 girl

Swap the position of 4 girls and 1 girl

Permutate the 6 boys

Swap top and bottom position

Case 2:The 4 girls and the remaining girl are seated on opposite sides.

No of ways = $^5C_4 \times 4! \times \; ^7C_2 \times \; 3! \times \; 6! \times \; 2 = 21772800$

No. of ways to choose 4 girls to be together and permutate them

Choose two boys to be at the same side as the 4 girls

Swap the position 2 boys and 4 girls

Permutate the 6 boys

Swap top and bottom position

Total no of ways = 2419200 + 21772800

= 24192000

<u>Alternative solution</u>

Case 1: Group of 4 girls in the middle.

No of ways = $^5C_4 \times 4! \times \; 6 \times \; 7! \times \; 2 \; = 7257600$

No. of ways to choose 4 girls to be together and permutate them

The 1 girl has 6 positions to sit across the 4 girls

The 7 boys can sit anywhere they want in the leftover seats

Swap top and bottom position

Case 2: Group of 4 girls at the side

No of ways = $^5C_4 \times 4!$ x 4 x $^7C_1$ x $7! = 16934400$

No. of ways to choose 4 girls to be together and permutate them

The group of 4 girls has 4 different sides to choose from

Choose a boy to be always next to the 4 girls

The leftover 6 boys and 1 girls has 7 positions to choose from.

Total no of ways = 7257600 + 16934400

= 24192000

9**(a)**

[Solution]

Let X be the random variable denoting the mass of a randomly chosen apple in grams.

i.e. $X \sim \mathrm{N}(260, 15^2)$

$\mathrm{P}(X \geq 250) = 0.748$

(b)

[Solution]

Let Y be the random variables denoting the mass of a randomly chosen pear in grams.

$Y \sim \mathrm{N}(210, 10^2)$

$X - Y \sim \mathrm{N}(260 - 210,\ 15^2 + 10^2)$
$X - Y \sim \mathrm{N}(50, 325)$

$\mathrm{P}(|X - Y| > 55)$

$= 1 - \mathrm{P}(-55 \leq X - Y \leq 55)$ or $\mathrm{P}(X - Y < -55) + \mathrm{P}(X - Y > 55)$

$= 0.39076 = 0.391$ (to 3 sig fig)

(c)
[Solution]

$\mathrm{T} = 0.95(X_1 + X_2 + \ldots + X_5) + 0.85(Y_1 + Y_2 + \ldots + Y_6)$
$\mathrm{T} \sim \mathrm{N}(0.95 \times 5 \times 260 + 0.85 \times 6 \times 210,\ 0.95^2 \times 5 \times 15^2 + 0.85^2 \times 6 \times 10^2)$
$\mathrm{T} \sim \mathrm{N}(2306, 1448.8125)$

$\mathrm{P}(\mathrm{T} \geq 2300)$
$= 0.56262$
$= 0.563$ (to 3 sig fig)

(d)

[Solution]

$W \sim \mathrm{N}(260, 15^2)$ and $V \sim \mathrm{N}(210, 10^2)$

$A = 3(0.7W) - 0.8(V_1 + V_2)$

$A \sim \mathrm{N}(0.7 \times 3 \times 260 - 0.8 \times 2 \times 210\ ,\ 0.7^2 \times 3^2 \times (15^2) + 0.8^2 \times 2 \times 10^2)$

$A \sim \mathrm{N}(210, 1120.25)$

$\mathrm{P}(A > 200)$

$= 0.61744$

$= 0.617$(to 3 sig fig)

$\mathrm{P}(3(0.7W) - 0.8(V_1 + V_2) > 200)$ refers to the probability that thrice the cost of a randomly chosen apple exceeds the total cost of two randomly chosen pears by more than \$2.

10 (a)

Solution

Let X be the number of defective phones out of 10 E-phones

$X \sim \mathrm{B}(10, 0.01)$

$\mathrm{P}(1 \le X < 3) = \mathrm{P}(X \le 2) - \mathrm{P}(X = 0)$ or $\mathrm{P}(X = 1) + \mathrm{P}(X = 2)$

$= 0.0955$

(b)

Solution

Let Y be the number of non-defective E-phones out of 9 E-phones

$Y \sim \mathrm{B}(9, 0.99)$

P(10^{th} E-phone is the 8^{th} phone non-defective)

= P(1^{st} 9 E-phones consist of 7 non-defective ones) × P(last one being non-defective)

$= \mathrm{P}(Y = 7) \times (0.99)$

$= (0.0033554)(0.99)$

= 0.00332188

= 0.00332

Alternative Method

P(10^{th} E-phone is the 8^{th} phone non-defective) = $({}^{9}C_{2} \times 0.99^{7} \times 0.01^{2}) \times 0.99$

= 0.00332

(c)

Solution

Let W be the number of defective E-phones out of 24 E-phones

$W \sim \mathrm{B}(24, 0.01)$

P(carton rejected) = $\mathrm{P}(W \ge 2)$

$= 1 - \mathrm{P}(W \le 1)$

= 0.0238544 = 0.0239

(d)

Solution

Let G be the number of defective phones out of 20 Galexy phones

$G \sim \mathrm{B}(20, 0.015)$

$\mathrm{E}(G) = 20 \times 0.015 = 0.3$ $\quad$ $\mathrm{Var}(G) = 20 \times 0.015 \times 0.985 = 0.2955$

$\mathrm{E}(W) = 24 \times 0.01 = 0.24$ $\quad$ $\mathrm{Var}(W) = 24 \times 0.01 \times 0.99 = 0.2376$

Let $S = G_1 + G_2 + \cdots + G_{500}$ and $T = W_1 + W_2 + \cdots + W_{250}$

$\mathrm{E}(S) = 500 \times 0.3 = 150$ $\quad$ $\mathrm{Var}(S) = 500 \times 0.2955 = 147.75$

$\mathrm{E}(T) = 250 \times 0.24 = 60$ $\quad$ $\mathrm{Var}(T) = 250 \times 0.2376 = 59.4$

Since 250 and 500 are large, by Central Limit Theorem,

$S \sim \mathrm{N}(150, 147.75)$ and $T \sim \mathrm{N}(60, 59.4)$ approximately.

Therefore, $S - T \sim \mathrm{N}(90, 207.15)$ approximately

P(Number of defective Galexy phones exceed that of E-phones by at least 100)

$= P(S - T \geq 100)$

$= 0.244$

Alternative method

Let S be the no of defective phones out of 10000 Galexy phones (500 cartons of 20 phones)

$S \sim \mathrm{B}(10000, 0.015)$

$\mathrm{E}(S) = 10000 \times 0.015 = 150$ $\quad$ $\mathrm{Var}(S) = 10000 \times 0.015 \times 0.985 = 147.75$

Let T be the no of E-phones out of 6000 E-phones (250 cartons of 24 phones)

$T \sim \mathrm{B}(6000, 0.01)$

$\mathrm{E}(T) = 6000 \times 0.01 = 60$ $\quad$ $\mathrm{Var}(T) = 6000 \times 0.01 \times 0.99 = 59.4$

Since 6000 and 10000 are large, by Central Limit Theorem,

$S \sim \mathrm{N}(150, 147.75)$ and $T \sim \mathrm{N}(60, 59.4)$ approximately.

Therefore, $S - T \sim \mathrm{N}(90, 207.15)$ approximately

P(Number of defective Galexy phones exceed that of E-phones by at least 100)

$= P(S - T \geq 100)$

$= 0.244$

11 (a)

Solution

Since not every student use the social media or follow Mr Kim on the social media platform, hence not every students of Junior colleges have an equal chance of being selected.

(b)

Solution

Unbiased estimate of population mean $\overline{x}$

$$= \frac{\sum x}{n}$$
$$= \frac{1707}{50}$$
$$= 34.14$$

Unbiased estimate of population variance s^2

$$= \frac{1}{n-1}\left[\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right]$$
$$= \frac{1}{50-1}\left[71695 - \frac{(1707\ )^2}{50}\right]$$
$$= 273.84$$
$$= 274 \text{ minutes}^2$$

(c)

Solution

Let X be the time JC student spent watching Kdrama in a day in minutes and μ be the mean time.

$\mathrm{H}_0: \mu = 37$

$\mathrm{H}_1: \mu < 37$

Left tailed test at 5% level of significance.

Under H_0, $\overline{X}$ ~N(37, $\frac{273.84}{50}$) approximately by Central Limit Theorem since $n = 50$ is large.
Test statistic: $\overline{x} = 34.14$
Using GC, p-value = $0.11084 \approx 0.111$ (3s.f.).
Since p-value = $0.111 > 0.05$, we do not reject H_0. There is insufficient evidence, at 5% level of significance, to conclude that Ms Shin has overstated the average time spent.

Alternatively

$Z = \dfrac{\overline{X} - 37}{\sqrt{273.84/50}}$ ~N(0,1) approximately
Test statistic: $z = -1.2221$
Critical value: $z_c = -1.6449$
Since $z = -1.2221 > z_c$, we do not reject H_0. There is insufficient evidence, at 5% level of significance, to conclude that Ms Shin has overstated the average time spent.

(d)

Solution

"5% level of significance" refers to the probability of 0.05 that the test wrongly concludes that the mean time JC student spent watching Kdrama in a day is less than 37 minutes, when in fact it is not.

(e)

Solution

$H_0: \mu = 37$
$H_1: \mu \neq 37$
Two-tailed test at 5% level of significance.
Under H_0, $\overline{X}$ ~N(37, $\frac{285}{50}$)
Using GC, the critical region: $\overline{x}_c < 32.321$ or $\overline{x}_c > 41.679$
Since there is sufficient evidence to say that the mean time spent is different from Ms Shin's claim, the test statistics k should fall within the critical region.

$\therefore 0 \leq k < 32.3$ or $41.7 < k \leq 1440$