Class	Index Number

Name : ________________________________

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2023
Secondary 4

Friday	**ADDITIONAL MATHEMATICS**	**4049/01**
18 August 2023	**Paper 1**	2 h 15 min

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

/ 90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

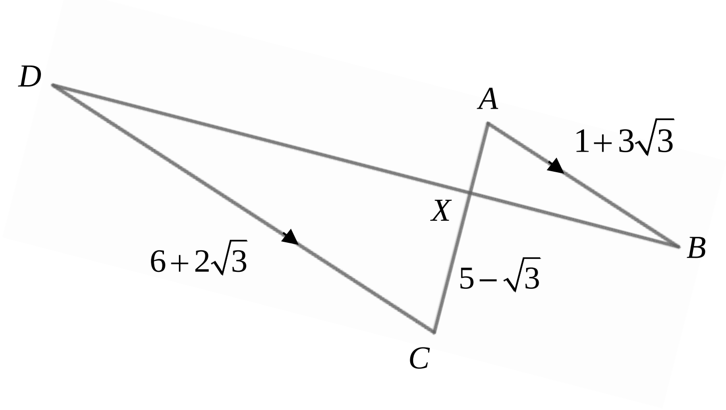
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **A calculator must not be used for this question.**



In the diagram, *AB* is parallel to *DC*, and *AC* meets *BD* at *X*.
Given that $AB = \left(1+3\sqrt{3}\right)$ cm, $CX = \left(5-\sqrt{3}\right)$ cm and $CD = \left(6+2\sqrt{3}\right)$ cm, find the exact length of *AX*. [5]

2 A curve has the equation $y = \dfrac{3-x}{e^{1-2x}}$.

(i) Show that the curve is an increasing function of x for $x < \dfrac{5}{2}$. [5]

(ii) Given that y is decreasing at a constant rate of 2 units per second, find the rate of change of x when $x = 0$. [2]

3 The function $\text{f}(x) = 2x^3 + ax^2 + bx + 6$, where a and b are constants, is exactly divisible by $x + 2$. When f (x) is divided by $x – 2$, the remainder is –12.

(i) Find the value of a and of b. [4]

(ii) Factorise f (x) completely and hence solve f (x) = 0. [3]

4 It is given that $y = 1 - 3\sin 4x$, for $0^\circ \leq x \leq 180^\circ$.

(i) State the amplitude and period of y. [2]

(ii) Sketch the graph of $y = 1 - 3\sin 4x$. [3]

(iii) Hence, state the number of solutions for $3\sin 4x = 2$. [1]

5 **(a)** The equation of a curve is $y = \frac{1}{3}\ln(px+3)$, where p is a constant to be determined. The gradient of the tangent to the curve at $x = -\frac{1}{2}$ is parallel to $3y = x$. Find the equation of the normal to the curve at $x = -\frac{1}{2}$.

[4]

(b) **(i)** Express $-2x^2 + 4x - 5$ in the form $a(x+p)^2 + q$. [2]

(ii) Hence, determine, with explanation, if the graph of $y = -2x^2 + 4x - 5$ will intersect the x–axis. [2]

6 **(a)** In the expansion of $(2+x)^n$, where $n > 0$, the coefficient of x^2 is twice the coefficient of x. Find the value of n. [4]

(b) Find the value of the term that is independent of x in the expansion of $\left(2x-\dfrac{1}{4x^4}\right)^{15}$. [3]

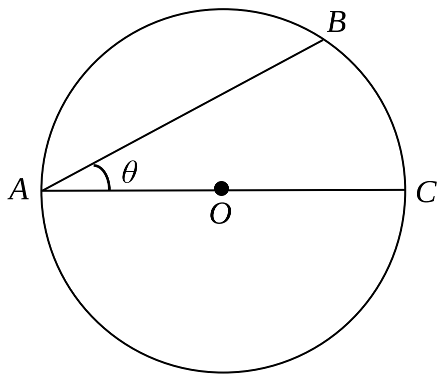
7 It is given that $y = \log_p (px) + 2\log_p (4x - 3) - 1$, where p is a positive integer.

(i) Write down the set of values of x for which y is defined. [1]

(ii) Show that y can be written as $\log_p (16x^3 - 24x^2 + 9x)$. [3]

(iii) Find the value of x for which $y = \dfrac{1}{\log_{9x} p}$. [3]

8 The figure shows a circular lake, centre O, of radius 2 km. A duck swims across the lake from A to B at 3 km/h and then walks clockwise around the edge of the lake from B to C at 4 km/h. If angle $BAC = \theta$ radians and the total time taken is T hours,



(i) show that $T = \frac{1}{3}\left(4\cos\theta + 3\theta\right)$, [4]

8 **(ii)** find the maximum value of *T*. [5]

9 **(a)** It is given that f $(x) = (2x-3)^5+4$.

Find $f'(x)$ and hence, determine the nature of the stationary point for $y = f(x)$.

[5]

9 **(b)** The gradient function of a curve varies directly with the square of $(x-2)$. Given that the gradient of the curve at $(1,9)$ is 6, find the coordinates of the point at which the curve meets the *y*–axis. [5]

10 **(a)** If $\cos 56^o = p$, express $\sin 28^o$ in terms of p. [2]

(b) **(i)** Prove the identity $\dfrac{\cos x + \sin x}{\cos x - \sin x} - \dfrac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$. [4]

(ii) Determine the values of x in the interval $180^\circ < x < 360^\circ$ for which the identity is not valid. [2]

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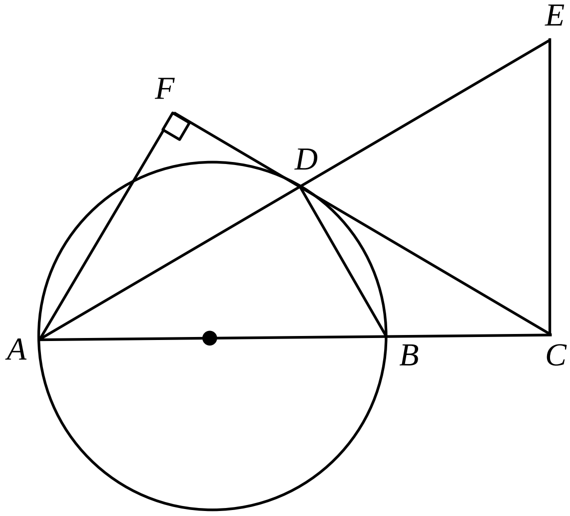


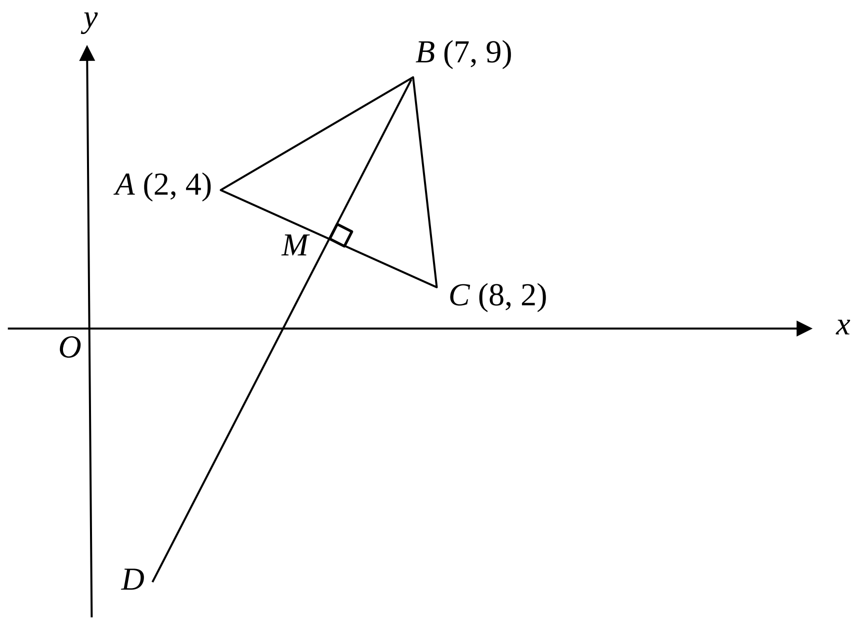
In the diagram, the diameter *AB* is produced to *C* and the line *CD* is the tangent to the circle at *D*. The line *AD* is produced to *E* such that a circle can be drawn passing through *B*, *C*, *E* and *D*. The foot of the perpendicular from *A* to *CD* produced is *F*.

Prove that

(i) triangle *CDE* is isosceles, [4]

(ii) *AE* bisects angle *FAB*. [3]

12 In the diagram, the points A, B and C have coordinates $(2,4)$, $(7,9)$ and $(8,2)$ respectively. The point M lies on AC such that the line BMD is perpendicular to AC.



(i) Show that $AB = BC$. [2]

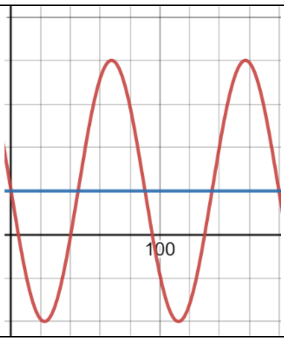
(ii) Find the equation of BD. [3]

(iii) Given that the ratio of $DM : MB = 2 : 1$, find the coordinates of *D*. [2]

(iv) Find the area of quadrilateral *ABCD*. [2]

End of Paper.

Answer Key

1	$\dfrac{23\sqrt{3}-27}{6}$		
2	**(ii)**	-1.09	
3	**(i)**	$a=-3$, $b=-11$	
	(ii)	$x=-2,\ \dfrac{1}{2}$ or 3	
4	**(i)**	Amplitude = 3 Period = 90°	
	(ii)		
	(iii)	4	
5	**(a)**	$y=-3x-\dfrac{3}{2}+\dfrac{1}{3}\ln 2$	
	(b)	**(i)**	$-2(x-1)^2-3$
		(ii)	Since coefficient of $x^2<0$, graph has maximum turning point at $(1,\ -3)$. Maximum value of y is -3 which is less than 0.
6	**(a)**	9	
	(b)	-29120	
7	**(i)**	$x>\dfrac{3}{4}$	
	(iii)	$x=\dfrac{3}{2}$	
8	**(ii)**	1.73 h	
9	**(a)**	$10(2x-3)^4$, point of inflexion	
	(b)	$(0,-5)$	
10	**(a)**	**(i)**	$\sqrt{\dfrac{1-p}{2}}$
		(ii)	$225^\circ, 315^\circ$
12	**(ii)**	$y=3x-12$	
	(iii)	$(1,-9)$	
	(iii)	60 units^2	