Candidate Index Number

Anglo - Chinese School (Independent)

FINAL EXAMINATION 2021
YEAR 3 INTEGRATED PROGRAMME
CORE MATHEMATICS
PAPER 1

Friday **1st October 2021** **1 hour 30 minutes**

Candidates answer on the Question Paper.
No additional materials are required.

INSTRUCTIONS TO CANDIDATES

- Write your index number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.

For Examiner's Use

This paper consists of 15 printed pages and 1 blank page.

[Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

*Answer **all** the questions in the spaces provided.*

1. *[Maximum mark: 9]*

(a) Evaluate $4-\dfrac{3}{3-\dfrac{3}{3-\dfrac{3}{4}}}$. *[3 marks]*

(b) Make q the subject of the formula, $p=\sqrt{\dfrac{3q-2p}{q+5}}$. *[3 marks]*

(c) Factorise $12x^3-4x^2y-3xz^2+yz^2$ completely. *[3 marks]*

..........[*Working may be continued next page*]

[Continuation of working space for Question 1] ....

2. *[Maximum mark: 8]*

(a) Sketch the graph of $y = x^2 - 9$ clearly labelling the coordinates of the axes-intercepts and turning point. *[2 marks]*

(b) The curve $y = x^2 - 9$ meets the line $y = 2x + 6$ at points *P* and *Q*. Find the coordinates of *P* and *Q*. *[4 marks]*

(c) Hence, find the area of ΔPOQ where *O* is the origin. *[2 marks]*

3. *[Maximum mark: 8]*

(a) Solve $5y \geq y^2 + 6$. *[2 marks]*

(b) Solve $\dfrac{2x+1}{2} < -4(x+2) \leq 5x+19$ and hence state the integer values of *x* that satisfy the inequality.

[6 marks]

4. *[Maximum mark: 5]*

A cuboid has a square base of length $1+\sqrt{2}$ units and the volume is $7+5\sqrt{2}$ units^3. Express the height of the cuboid, H, in the form $a+b\sqrt{2}$, where a and b are constants. What do you notice about this cuboid?

[5 marks]

5. *[Maximum mark: 5]*

Given that A is an acute angle and $\cos A = \frac{1}{3}$,

(a) find the value of $\sin A$, *[2 marks]*

(b) hence, show that $\frac{2\tan A - 1}{3\sin A} = 2 - \frac{\sqrt{2}}{4}$. *[3 marks]*

6. *[Maximum mark: 9]*

Given that the roots of the quadratic equation $3x^2 - 2x - 3 = 0$ are α and β.

(a) State the value of $\alpha + \beta$ and of $\alpha\beta$. *[2 marks]*

(b) Show that $\alpha^2 + \beta^2 = \dfrac{22}{9}$. *[2 marks]*

(c) Find the quadratic equation with roots $\alpha + 2\beta$ and $2\alpha + \beta$.. *[5 marks]*

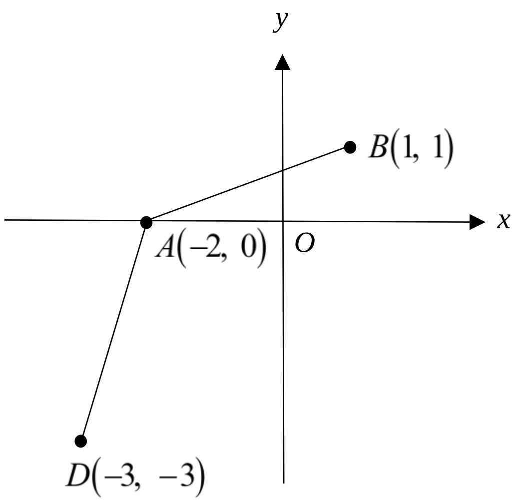
7. *[Maximum mark: 8]*

The points $A(-2,0)$, $B(1,\ 1)$ and $D(-3,-3)$ are the three vertices of a rhombus *ABCD*. *E* is a point at the foot of the perpendicular from *A* to *BD*.

(a) Find the equation of *CD*. *[3 marks]*

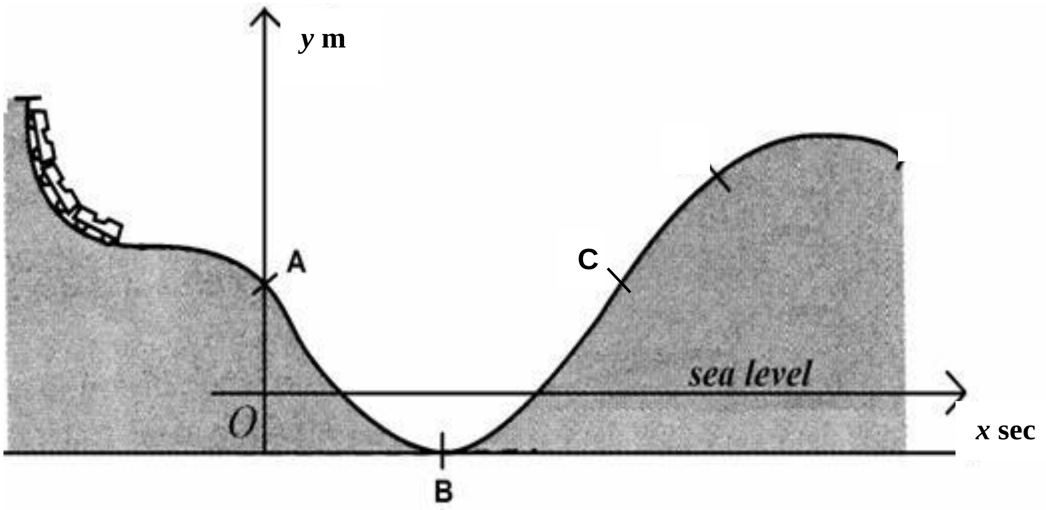
(b) Find the length of *AE*, leaving your answer in surd form. *[3 marks]*

(c) Find the area of the rhombus. *[2 marks]*



8. *[Maximum mark: 9]*

A roller coaster at an amusement park goes through an underwater tunnel. The track traveled by the roller coaster is shown in the diagram below. The height, y metres of the roller coaster above the sea level as it travels from point A to point C can be modelled by the equation $y = \frac{1}{10}x^2 - 4x + 5,$ where x is the time in seconds.



(a) Express $y = \frac{1}{10}x^2 - 4x + 5$ in the form of $y = a\left(x-h\right)^2 + k$. *[3 marks]*

(b) **(i)** State the height of the roller coaster above sea level at point A. *[1 mark]*

(ii) State the time when the roller coaster reaches the lowest point at B. *[1 mark]*

(c) Hence, solve $y = 0$, leaving your answers in the surd form and state what the answers represent. *[4 marks]*

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[Continuation of working space for Question 8]

9. *[Maximum mark: 10]*

(a) Find $\log_2 r$ if $r^x = 16$ and $3^x = 81$. *[3 marks]*

(b) Given that $3^p = 8$ and $8^q = 81$, find the value of pq. *[3 marks]*

(c) Solve for x if $(\ln x)^2 - 3(\ln 5)^2 = 2(\ln 5)(\ln x)$. *[4 marks]*

[*Working may be continued next page*]

[Continuation of working space for Question 9]

10. *[Maximum mark: 9]*

(a) Find the range of values of k such that the curve $y=-\left(x-1\right)^{2}-k^{2}-4k$ is always negative for all real values of *x*. *[4 marks]*

(b) Given that $a-b=22$ and $\sqrt{a}+\sqrt{b}=11,$ find the value of $\sqrt{ab}.$ *[5 marks]*

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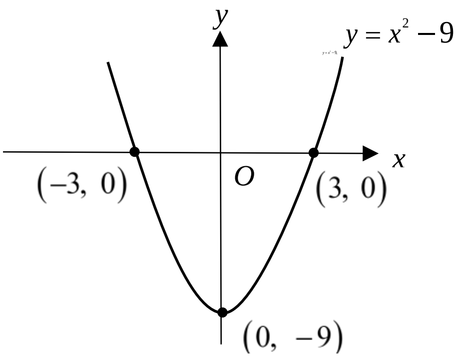
[Continuation of working space for Question 10]

End of Paper

Answer key	
1 (a)	$\frac{11}{5}$
(b)	$q=\frac{-2p-5p^2}{p^2-3}$
(c)	$(2x+z)(2x-z)(3x-y)$
2 (a)	
(b)	**The coordinates of P and Q are (5, 16) and (-3, 0).**
(c)	**Area of triangle *POQ* =** 24 units^2
3 (a)	$2\le y\le 3$
(b)	$\therefore -3\le x<-1.7$ **The integer values are -3 and -2.**
4	$H=1+\sqrt{2}$ units **The height of the cuboid is equal to the length of its square base. Hence it is a cube.**
5 (a)	$\sin A=\frac{\sqrt{8}}{3}\,/\,\frac{2\sqrt{2}}{3}$
(b)	$\frac{2\tan A-1}{3\sin A}=\frac{4\sqrt{2}-1}{3\left(\frac{2\sqrt{2}}{3}\right)}$ $=2-\frac{1}{4\sqrt{2}}$ or $2-\frac{\sqrt{2}}{4}$ (shown)
6 (a)	$\alpha+\beta=\frac{2}{3},\quad \alpha\beta=-1.$
(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=\left(\frac{2}{3}\right)^2-2(-1)$ $=\frac{22}{9}$ (shown)
(c)	**Quadratic equation,** $x^2-2x-\frac{1}{9}=0\,/\,9x^2-18x-1=0$
7 (a)	**Equation of CD,** $y=\frac{1}{3}x-2$
(b)	**Distance of AE** $=\sqrt{2}$ units
(c)	**Area of rhombus** $=8$ units^2

8 (a)	$y=\frac{1}{10}(x-20)^2-35$
(bi)	**The height of the roller coaster above sea level at point A is 5 m.**
(ii)	**The time when the roller coaster reaches the lowest point at B is 20 seconds.**
(c)	$x=20\pm\sqrt{350}$ or $x=20\pm5\sqrt{14}$ $20\pm\sqrt{350}$ or $20\pm5\sqrt{14}$ **seconds are the time when the roller coaster is at the sea level.**
9 (a)	$\log_2 r=1$
(b)	$pq=4$
(b)	$x=125$ or $x=\frac{1}{5}$
10 (a)	$k<-4$ or $k>0$
(b)	$\sqrt{ab}=\frac{117}{4}$