TAMPINES MERIDIAN JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

CANDIDATE NAME: ____

CIVICS GROUP: ____

H2 MATHEMATICS 9758/02

Paper 2

16 SEPTEMBER 2024

3 hours

Candidates answer on the question paper.

Additional material: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and Civics Group on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

For Examiners' Use	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of 22 printed pages and 0 blank page.

Section A: Pure Mathematics [40 marks]

1 **(i)** Given that $y = \ln\left(1+3x+2x^2\right)$, show that $\left(1+3x+2x^2\right)\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}+\left(3+4x\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4$.

By further differentiating the above result, find the Maclaurin series for y, up to and including the term in x^3. [5]

(ii) Verify the correctness of your answer in part **(i)** by using the standard results given in the List of Formulae (MF26). [2]

(iii) Hence, by using $x = \frac{1}{2}$, estimate the value of $\ln 3$. [2]

2 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. The point C is such that O divides the line segment AC in the ratio $2:3$. The point D divides the line segment AB in the ratio $1:4$.

(i) Show that the area of triangle OCD is given by $\lambda|\mathbf{a}\times\mathbf{b}|$, where λ is a constant to be determined. [5]

(ii) It is now given that **a** is perpendicular to **b**. Given that $\overrightarrow{AP}$ is the projection vector of $\overrightarrow{AO}$ onto $\overrightarrow{AB}$, show that $\overrightarrow{AP} = \mu(\mathbf{b} - \mathbf{a})$, where μ is a constant to be determined in terms of $|\mathbf{a}|$ and $|\mathbf{b}|$. [3]

3 **Do not use a graphing calculator when answering this question.**

(a) The equation $z^3 - 8z^2 + 23z + k = 0$, where k is a real constant, has a root $2 + \sqrt{3}\text{i}$.

(i) Find the value of k and solve the equation. [5]

(ii) Hence, solve the equation $z^3 + 8\text{i}z^2 - 23z + k\text{i} = 0$ [2]

(b) The complex number w is given by $w = \cos\theta + \mathrm{i}\sin\theta$, where $0 < \theta < \frac{\pi}{2}$. Find, in terms of θ, the modulus and argument of $\frac{w^*}{w^2 + 1}$, where w^* is the complex conjugate of w. [4]

4 **(a)** **(i)** Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{\cos x}\right)$. [1]

(ii) Hence, show that

$$\int \sin 2x \; \mathrm{e}^{\cos x} \; \mathrm{d}x = 2\mathrm{e}^{\cos x}\left(1-\cos x\right)+C,$$

where C is an arbitrary constant. [3]

(iii) Find the exact value of $\int_0^{\pi} \left|\sin 2x \; \mathrm{e}^{\cos x}\right| \mathrm{d}x$. [3]

(b) Using the substitution $x = \sec\theta$, where $0 < \theta < \frac{\pi}{2}$, find $\int \frac{1}{\sqrt{x^2 - 1}}\,\mathrm{d}x$. [5]

Section B: Probability and Statistics [60 marks]

5 For events A and B, it is given that $\mathrm{P}(A) = \frac{1}{8}$, $\mathrm{P}(A \cup B) = \frac{9}{16}$ and $\mathrm{P}(A \cap B') = \frac{1}{16}$.

(i) Find $\mathrm{P}(B)$. [1]

(ii) Determine, with a reason, whether the events A and B are independent. [2]

For a third event C, it is given that $\mathrm{P}(C) = \frac{3}{8}$, $\mathrm{P}(A \cap B \cap C) = \frac{1}{64}$ and C is independent of A.

(iii) Find $\mathrm{P}(A' \cap C)$. [1]

(iv) Find the range of values for $\mathrm{P}\left(A'\cap B'\cap C'\right)$. [3]

6 At a carnival, a game stall offers a game of throwing darts. The darts will land either in the bull's eye, inner ring or outer ring. There is a probability of p, $2p$ and $5p$ of landing in the bull's eye, inner ring and outer ring respectively.

A player gets five points if the dart lands in the bull's eye, three points if the dart lands in the inner ring and one point if the dart lands in the outer ring. A round consists of two throws. Let X be the total number of points for one round of two throws.

(i) Find the value of p and show that $\mathrm{P}(X=6)=\dfrac{7}{32}$. Obtain the probability distribution of X. [5]

(ii) Find $\mathrm{E}(X)$. [1]

(iii) A player wins \$1 for every two points earned. If a player pays \$$k$ to play one round of two throws, find the value of k for the game to be fair. [2]

7 The owner of a popsicles store would like to study the effect of daily average temperature (x) on the number of popsicles sold (y). A random sample of 10 days is taken and the results are shown in the table.

Average Temperature, $x\left(°C\right)$	22	23	24	26	27	28	29	31	33	36
Number of popsicles sold (y)	90	102	118	145	134	159	166	179	185	190

(i) Draw a scatter diagram for these values, labelling the axes clearly. Comment on the relationship between x and y. [3]

(ii) By calculating the product moment correlation coefficients, determine and explain whether the relationship between x and y is modelled better by $y = a + bx$ or $\ln(200 - y) = c + dx$. Find the equation of the suitable regression line. [4]

(iii) The store owner would like to estimate the number of popsicles sold when the daily average temperature reaches $37\,°C$. Find the estimate and explain whether the estimate is reliable. [2]

(iv) Give a possible interpretation, in context, of the value 200 in the model $\ln(200 - y) = c + dx$. [1]

8 **In this question you should state the parameters of any normal distribution(s) that you use.**

A fruit seller claims that his oranges have a mean mass of 365g. A consumer association would like to test whether the fruit seller has overstated the mean. To conduct this test, a random sample of 40 oranges is taken and the mean mass is found to be 364.2g and has standard deviation 4g.

(i) Test, at the 5% level of significance, whether the fruit seller has overstated the mean. [5]

The diameter of oranges in centimetres is a normally distributed continuous random variable D. The standard deviation of D is 0.3 cm and under ordinary conditions the expected value of D is 9 cm. A test is carried out, at the 5% significance level, to determine whether the mean diameter exceeds 9 cm.

(ii) Given that the sample size is 50, find the range of values within which the mean diameter of this sample must lie, such that there is sufficient evidence from the sample to conclude that the mean diameter exceeds 9 cm. [4]

(iii) Another sample of fifty oranges is taken and the mean diameter of the sample is found to be 8.9 cm. Without any further calculations, explain, with justification, the conclusion that can be made at the 5% significance level. [1]

9 A carnival game involves one round of tossing 5 balls into a container, one at a time. It is known that, on average, 35% of the balls tossed will go into the container. The participant will win a prize if he tosses at least 3 balls into the container. The number of balls that a participant tosses into the container in one round is denoted by X.

(i) State, in the context of the question, two assumptions needed to model X by a binomial distribution. Explain why one of the assumptions may not hold. [3]

You are now given that X can be modelled by a binomial distribution.

(ii) Find the probability that a randomly chosen participant wins a prize. [2]

Ten randomly chosen participants played the carnival game once.

(iii) Find the probability that at most 6 of them did not win a prize. [2]

The game is modified to allow more participants to win a prize, where participants may play one or two rounds of the game.

- If the participant tosses at least 3 balls into the container in the first round, he wins a prize.
- If the participant tosses fewer than 2 balls into the container in the first round, he will not win any prize.
- If the participant tosses exactly 2 balls into the container in the first round, he will play a second round. If he tosses at least 4 balls into the container in the second round, he wins a prize.

(iv) Find the probability that a randomly chosen participant wins a prize in this modified game. [2]

(v) Given that a randomly chosen participant wins a prize in a modified game, find the probability that he tossed fewer than 7 balls into the container. [3]

10 **In this question you should state clearly the values of the parameters of any normal distribution(s) that you use.**

Papayas and watermelons are sold by weight. The masses, in kg, of papayas and watermelons are modelled as having normal distributions with means and standard deviations as shown in the table.

	Mean mass	Standard deviation
Papayas	1.85	0.12
Watermelons	6.50	0.72

(i) Sketch the distribution for the masses of papayas between 1.4 kg and 2.3 kg. [2]

(ii) Find the probability that the mass of a randomly chosen papaya is less than 1.7 kg. [1]

(iii) Find the probability that the mass of a randomly chosen watermelon exceeds the mass of a randomly chosen papaya by more than 4.5 kg. [3]

(iv) The mean mass of n randomly chosen watermelons is denoted by $\overline{Y}$ kg. Given that $\mathrm{P}\left(\overline{Y} > k\right) = 0.2,$ express k in terms of n. [3]

Papayas are sold at \$2.20 per kg and watermelons at \$1.45 per kg.

(v) Calculate the probability that the total selling price of 3 randomly chosen papayas and 4 randomly chosen watermelons exceed \$50. [3]

(vi) State an assumption for your calculations in parts **(iii), (iv)** and **(v)**. [1]

End of Paper