Chapter 16

ELECTROMAGNETISM

Content

- Concept of a magnetic field
- Magnetic fields due to currents
- Force on a current-carrying conductor
- Force between current-carrying conductors
- Force on a moving charge

Learning Outcomes

Candidates should be able to:

(a) show an understanding that a magnetic field is an example of a field of force produced either by current carrying conductors or by permanent magnets ☐ ☐ ☐

(b) sketch flux patterns due to currents in a long straight wire, a flat circular coil and a long solenoid ☐ ☐ ☐

(c) use $B = \mu_0 I / 2\pi d$, $B = \mu_0 N I / 2r$ and $B = \mu_0 n I$ for the flux densities of the fields due to currents in a long straight wire, a flat circular coil and a long solenoid respectively ☐ ☐ ☐

(d) show an understanding that the magnetic field due to a solenoid may be influenced by the presence of a ferrous core ☐ ☐ ☐

(e) show an understanding that a current-carrying conductor placed in a magnetic field might experience a force ☐ ☐ ☐

(f) recall and solve problems using the equation $F = BIl \sin\theta$, with directions as interpreted by Fleming's left-hand rule ☐ ☐ ☐

(g) define magnetic flux density and the tesla ☐ ☐ ☐

(h) show an understanding of how the force on a current-carrying conductor can be used to measure the flux density of a magnetic field using a current balance ☐ ☐ ☐

(i) explain the forces between current-carrying conductors and predict the direction of the forces ☐ ☐ ☐

(j) predict the direction of the force on a charge moving in a magnetic field ☐ ☐ ☐

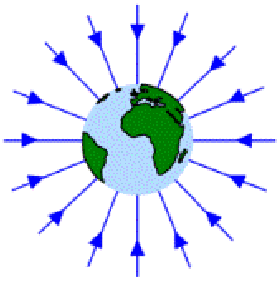
(k) recall and solve problems using the equation $F = BQv\sin\theta$ ☐ ☐ ☐

(l) describe and analyse deflections of beams of charged particles by uniform electric and uniform magnetic fields ☐ ☐ ☐

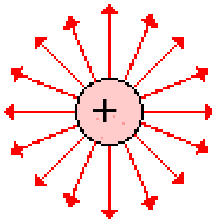
(m) explain how electric and magnetic fields can be used in velocity selection for charged particles ☐ ☐ ☐

1 Magnetic Field

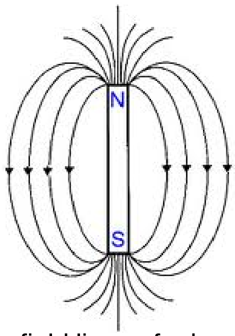
The idea of a field is a theoretical concept used by scientists to explain how objects interact with one another without physically touching each other. Field lines (flux patterns) are imaginary lines which aid visualization and quantification of the field.



Gravitational field lines of the Earth



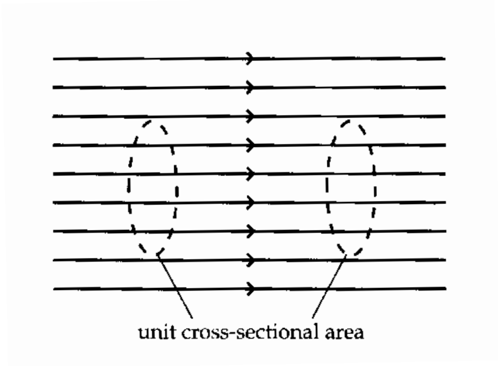
Electric field lines of a positive charge



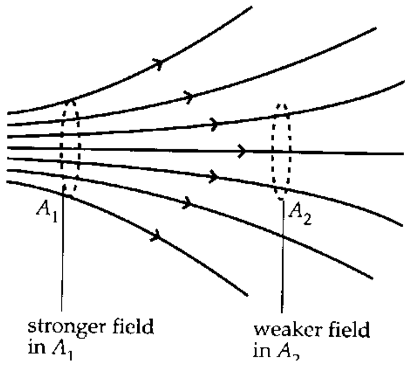
Magnetic field lines of a bar magnet

A magnetic field is a region of space in which a magnetic material, a current-carrying conductor or a moving charge located in it may experience a force.

Magnetic field lines are lines of force as they indicate the direction of the force acting on a magnetic North pole. If the lines of force are parallel, the field is uniform. This means that the number of lines passing perpendicularly through unit area at all cross-sections in a magnetic field is the same. If the lines of force are closer together, the field is said to be stronger.



Uniform magnetic field



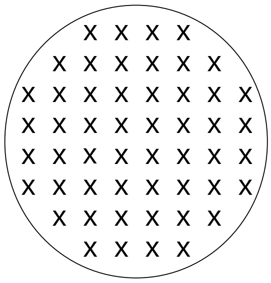
Non-uniform magnetic field

Magnetic field lines do not intersect as there can only be one direction for the field at any point.

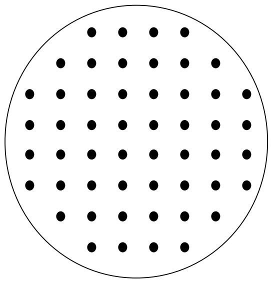
Convention in Representing Field Directions:

1) Field lines around a magnet is represented by lines of force leaving the North pole and entering the South pole
2) Field lines directed in and out of a page are represented by dot and cross diagrams.

B pointing into paper

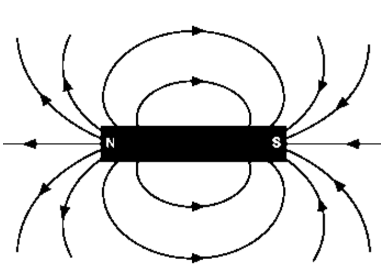


B pointing out of paper

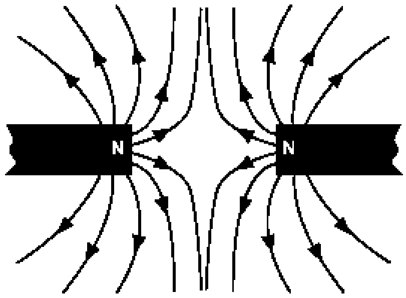


Cross and dot notation

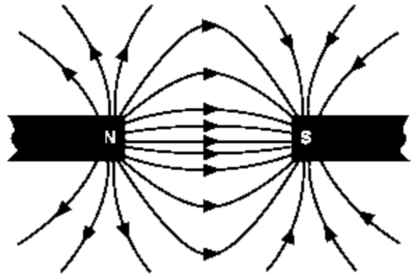
1.1 Magnetic Flux Pattern of Magnets



Resultant lines of force of a bar magnet



Resultant lines of force between two North poles

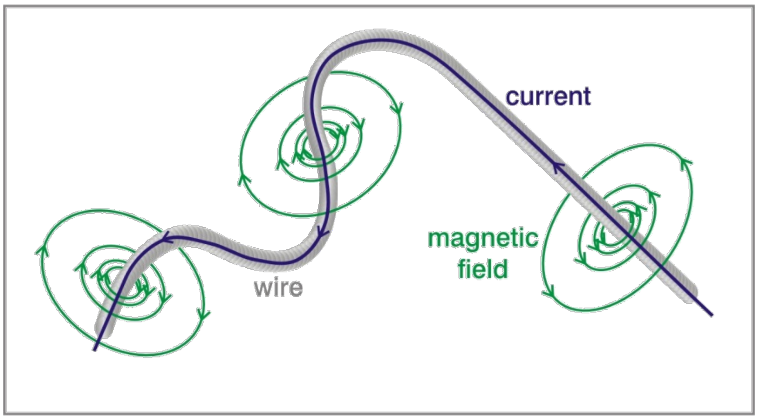


Resultant lines of force between North and South poles

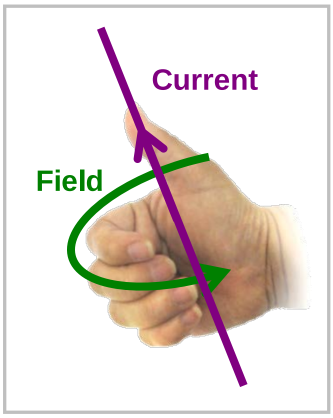
Since magnetic field lines do not intersect, they interact to produce a resultant line of force at each point. Field lines are not rigid but can be deformed and give way to each other as shown in the diagrams above.

1.2 Magnetic Flux Pattern due to a Current-carrying Conductor

When there is a current in a conductor, a magnetic field will be set up around it. The magnetic flux pattern consists of concentric circles, radiating outwards from the conductor.



Magnetic flux pattern of a piece of wire

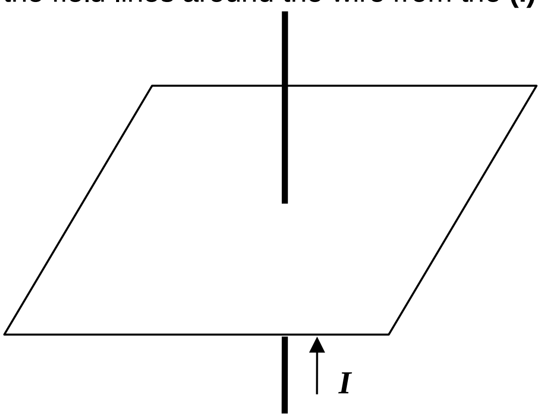


Right Hand Grip Rule

Right Hand Grip Rule is useful in determining the direction of magnetic field lines. For a current-carrying conductor, the thumb points along the direction of the current while the fingers point in the direction of the magnetic field around the conductor.

<u>Example 1</u>
Draw the field lines around the wire from the **(i)** 3-D view and **(ii)** side view.

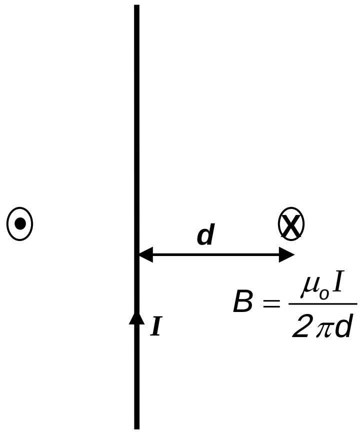
(i)



(ii)

The magnetic flux density *B* at a perpendicular distance *d* due to current I in a long straight wire placed in vacuum of permeability μ_0 is given by the expression

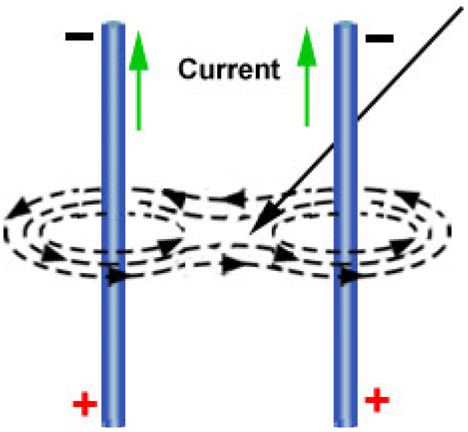
$$B = \frac{\mu_o I}{2\pi d}$$



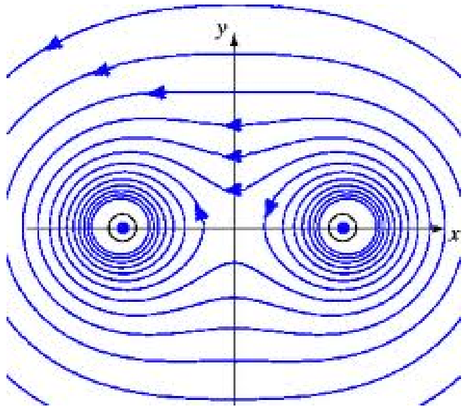
1.3 Magnetic Flux Pattern due to Two Current-carrying Conductors

Two straight current-carrying conductors can be placed near to one another. Since each conductor creates its own magnetic field, the two magnetic fields will overlap each other and form a resultant magnetic field. The resultant magnetic field can be obtained by the vector addition of individual magnetic field lines. Regions with no resultant magnetic field (neutral points) may also result as indicated by an X.

<u>Case 1: Current in the same direction</u>

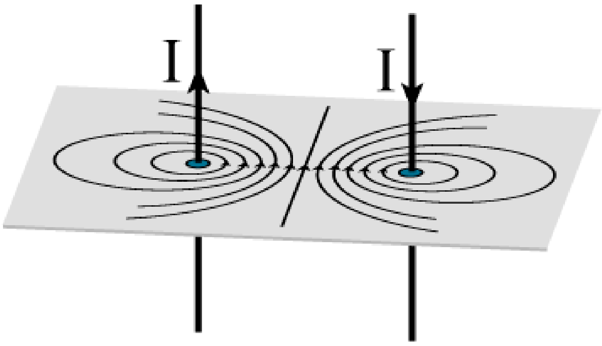


3-D Side View

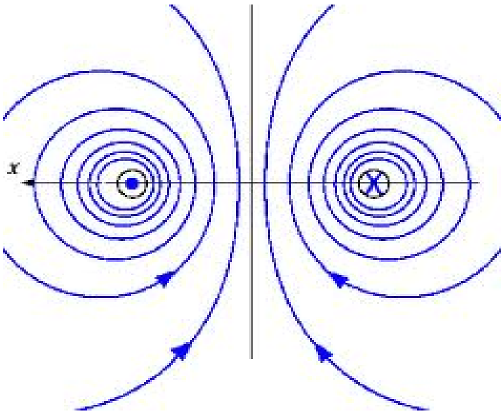


Top View

Case 2: Current in the opposite direction



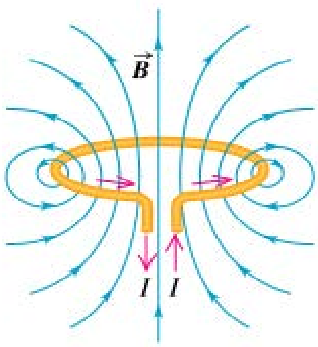
3-D Side View

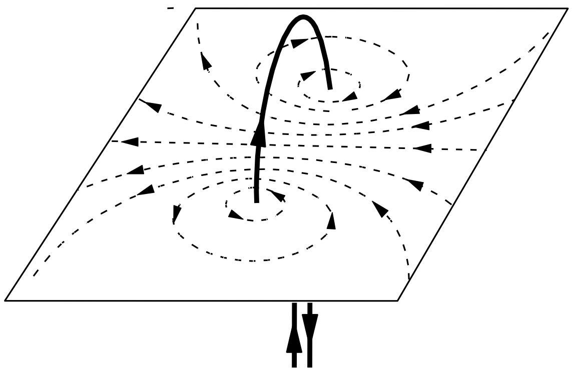


Top View

1.4 Magnetic Flux Pattern due to Flat Circular Coil

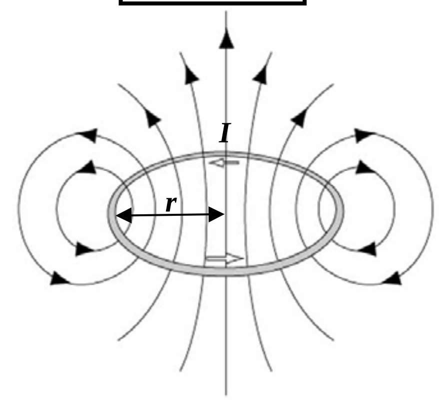
For a flat circular coil, the Right Hand Grip Rule can also be used to deduce the flux pattern. The thumb points along the direction of the current while the fingers point in the direction of the magnetic field around the coil.





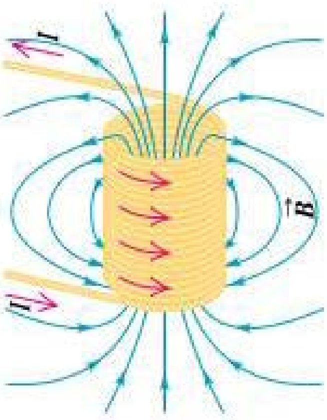
The magnetic flux density B at the center of a flat circular coil with N turns of radius r due to current I in placed in vacuum of permeability μ_0 is given by the expression

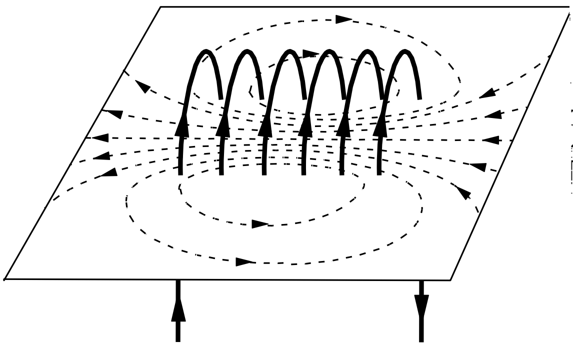
$$B = \frac{\mu_o N I}{2r}$$

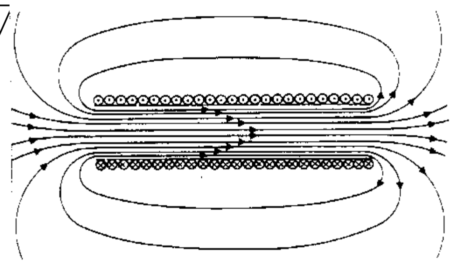


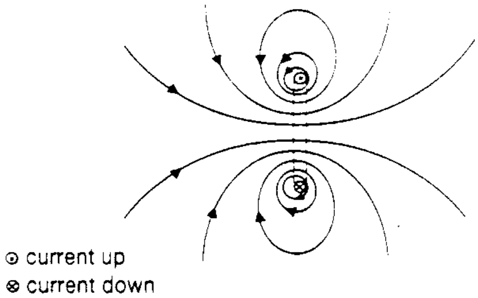
1.5 Magnetic Flux Pattern due to a Long Solenoid

A solenoid consists of many flat circular coils stacked together side by side. The resultant magnetic field can be obtained by the vector addition of magnetic field due to individual flat circular coils.







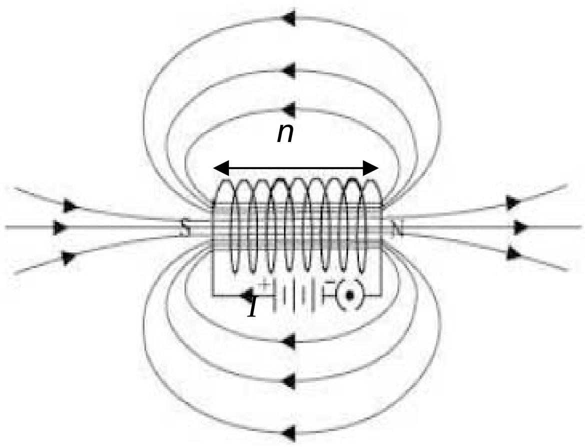


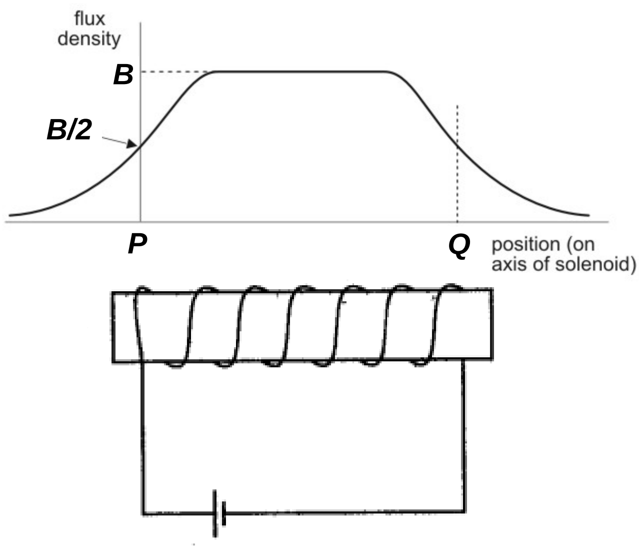
Flux pattern of a solenoid in different orientations

The magnetic flux density B inside a long solenoid with n number of turns per unit length along its axis due to current I in placed in vacuum of permeability μ_0 is given by the expression

$$B = \mu_o n I$$

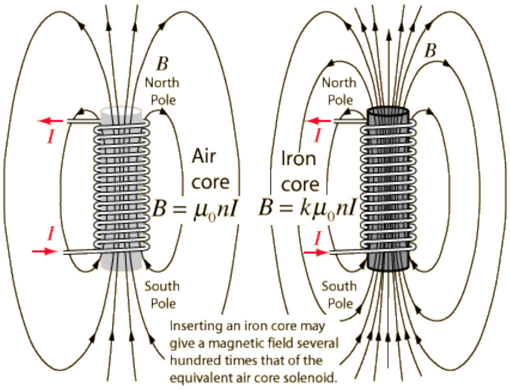
The magnetic field is uniform and the magnetic field lines are parallel along the axis of the solenoid





1.6 Effect of Ferrous Core

A ferrous (iron) core has the ability to align its dipoles in the direction of an external magnetic field. By inserting a ferrous core into a solenoid, the field lines are concentrated, thus strengthening the field. The ferrous core becomes magnetized and results in a much stronger magnetic field in the same direction.



Effect of a ferrous core

2 Electromagnetism

2.1 Force on a Current-carrying Conductor

When a current-carrying conductor is placed in an external magnetic field, a force will be exerted on it if the field is perpendicular to or has a component that is perpendicular to the current.

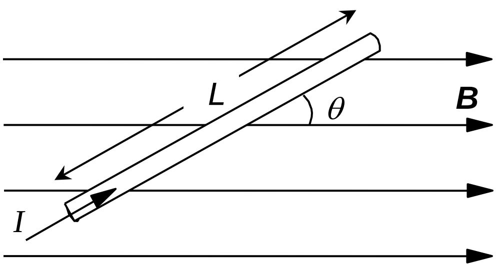
The magnitude of the force *F* on a conductor of length *L* carrying a current I placed at an angle θ to the magnetic field *B* is given by:

$$\begin{aligned} F_B &= BIL\sin\theta \\ &= (B\sin\theta)IL \\ &= B_\perp IL \\ &= B(I\sin\theta)L \\ &= BI_\perp L \end{aligned}$$

where θ is the angle between current and magnetic field

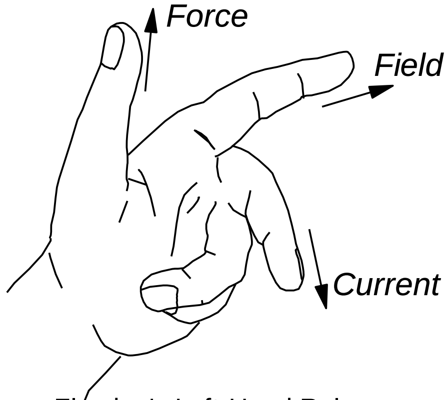
where $B\sin\theta$ or $B_\perp$ is the component of *B* that is perpendicular to I

where $I\sin\theta$ or $I_\perp$ is the component of I that is perpendicular to *B*



Uniform magnetic field in the plane of the paper and of flux density, *B*

The direction of the force *F* is given by Fleming's Left Hand Rule where the magnetic force experienced by the conductor is always perpendicular to both B and I.



Fleming's Left-Hand Rule

2.2 Magnetic Flux Density

Magnetic flux density is a measure of the strength of the magnetic field in a region of space. Its direction is at right angles to the force and current as indicated by Fleming's Left Hand Rule.

Symbol: B
Unit: Tesla, T

Definition of Magnetic Flux Density, B

Magnetic flux density is defined as the force acting per unit length per unit current on a conductor carrying current placed at right angles to the magnetic field.

$$B = \frac{F}{IL}, \theta = 90^o$$

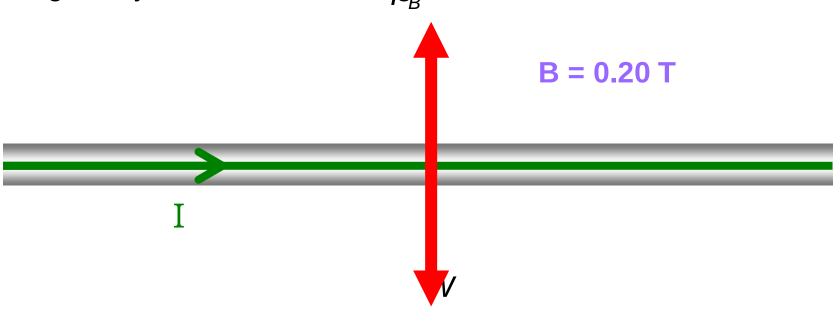
Definition of the unit Tesla, T

One tesla is defined as the magnetic flux density of a magnetic field in which a force per unit length of one newton per metre acts on a conductor which is carrying a current of one ampere in a direction perpendicular to the field.

Comparing units: $$1T = \frac{1N}{1A \times 1m}, \theta = 90^0$$

Example 2

A straight horizontal rod, of mass 50 g and length 0.50 m, is placed in a uniform horizontal magnetic field of flux density 0.20 T perpendicular to the rod. Determine the current in the rod if the force acting on it just balances its weight.

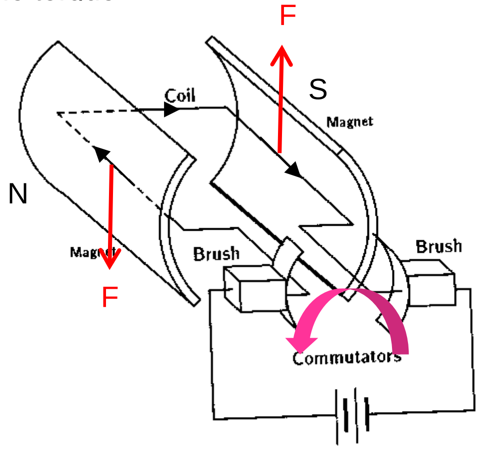


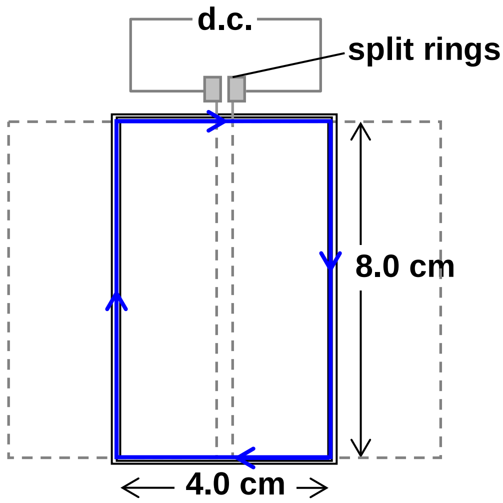
Taking upwards as positive

$$\sum F = ma$$
$$F_B - mg = ma$$
$$BIL\sin\theta - mg = ma$$
$$I = \frac{ma + mg}{BL\sin\theta}$$
$$= \frac{50\times10^{-3}\times0 + 50\times10^{-3}\times9.81}{0.20\times0.50\times\sin 90^0}$$
$$= 4.9\ \text{A}$$

Example 3

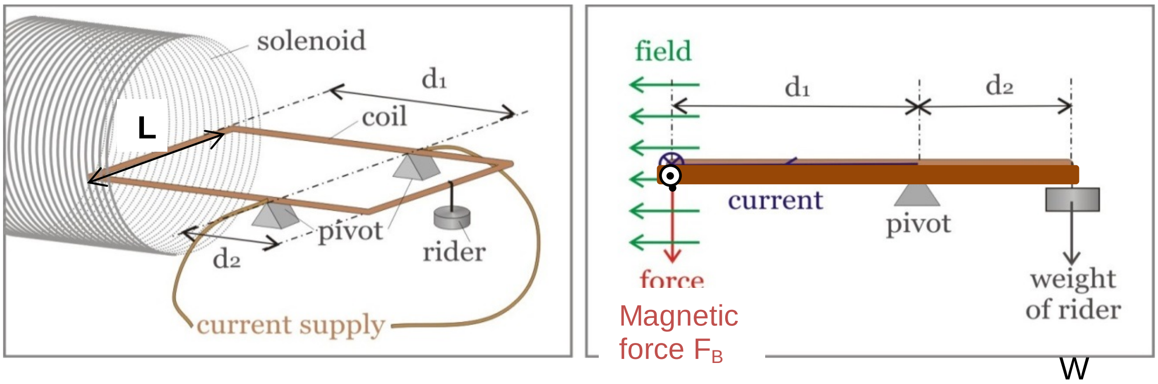
The diagram below shows a typical d.c. motor. The flux density due to the magnet is 25 mT. The coil has 50 turns of copper wire and carries a current of 200 mA. Calculate the torque produced by the magnetic force at this position. Indicate on the left diagram the direction of the torque.





$$\text{Force on 1 coil} = BIL\sin\theta$$
$$= 25\times10^{-3}\times200\times10^{-3}\times8.0\times10^{-2}\times\sin 90^{0}$$
$$= 4.0\times10^{-4}\text{ N}$$
$$\text{Force on 50 coils} = 50\times4.0\times10^{-4}$$
$$= 2.0\times10^{-2}\text{ N}$$
$$\text{Torque} = 2.0\times10^{-2}\times4.0\times10^{-2}$$
$$= 8.0\times10^{-4}\text{ N m Anti-clockwise}$$

2.2.1 Determination of Magnetic Flux Density using Current Balance



Part of a rectangular coil is placed inside a solenoid and the current is allowed to flow in the coil such that it experiences a downward magnetic force F_B.

A rider of known mass is hung at the opposite end of the coil.
The pivot is shifted until the coil is in rotational equilibrium.

Let the distances from the pivot be d_1 and d_2 respectively.
In rotational equilibrium, taking moments about the pivot,

$$F_B d_1 = mgd_2$$
$$BILd_1 = mgd_2$$
$$B = \frac{mgd_2}{ILd_1}$$

Example 4

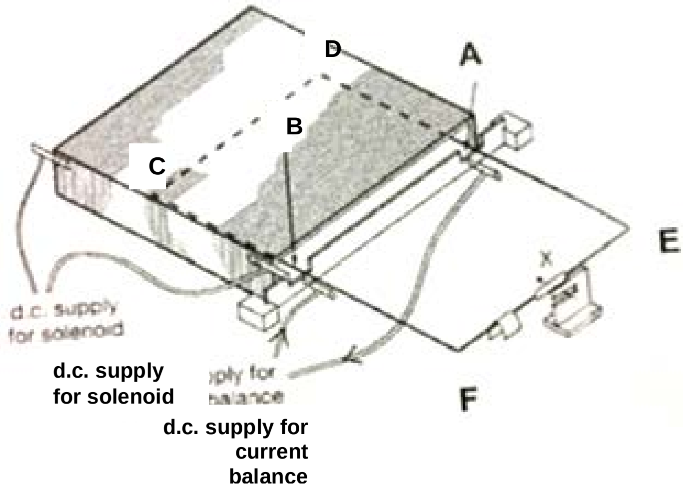
In the current balance below, the wire frame rests on two knife edges at A and B, which are exactly at the middle of the frame. Lengths AB and CD are 0.15 m while lengths CB and DA are 0.20 m. There is a current of 6.0 A which enters the left-hand side of the frame by means of the knife edge. A separate current is supplied to the solenoid to create a field which has a flux density of 3.5 mT.

(a) Explain why there is no magnetic force on AD and CB.

(b) Indicate the direction of the magnetic field in the diagram.

(c) Calculate the force acting on CD.

(d) Calculate the mass needed to be placed on the wire frame in order to restore equilibrium



(e) If the laboratory environment has a background magnetic field in the horizontal direction that may interfere with the setup, suggest and explain how the effect due to the background magnetic field may be reduced.

(a) Since direction of current is parallel to that of magnetic field, using $F = BILsin\theta$, there is no magnetic force on AD and CB.

(b) Current in wire frame is in direction BCDA. The magnetic field exerts a downward force on on CD which is balanced by the mass placed on FE. Hence, the field is pointing in the direction parallel to CB or DA.

(c) $F = BIL\sin\theta$

$= (3.5 \times 10^{-3})(6)(0.15)$

$= 3.15 \times 10^{-3}$ N

(d) By Principle of Moments, taking moments about the pivot,

$F_B \times L_{DA} = W \times L_{AE}$

$(3.15 \times 10^{-3}) = m\ (9.81)$

$m = 3.20 \times 10^{-4}$ kg

(e) Align the wire section CD such that it is parallel to the horizontal background magnetic field. This ensures that no force (and hence no torque) acts on section CD to disturb the equilibrium.

2.3 Forces between Current-Carrying Conductors

Two current-carrying conductors are placed near to one another. Since a magnetic field is set up when there is a current in a conductor, each conductor lies in the magnetic field created by the current in the other conductor. Each conductor hence experiences a magnetic force.

Steps to Determine the Direction of Force on a Current-carrying Conductor

❶ Find the field due to external sources at the location of the conductor

❷ Find the current in the conductor ❸ Apply Fleming's Left Hand Rule to find the force

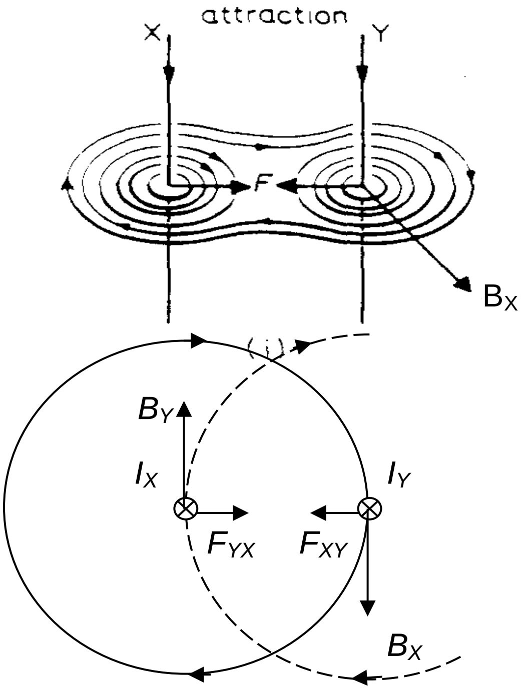


Figure (i): Forces between currents in same direction

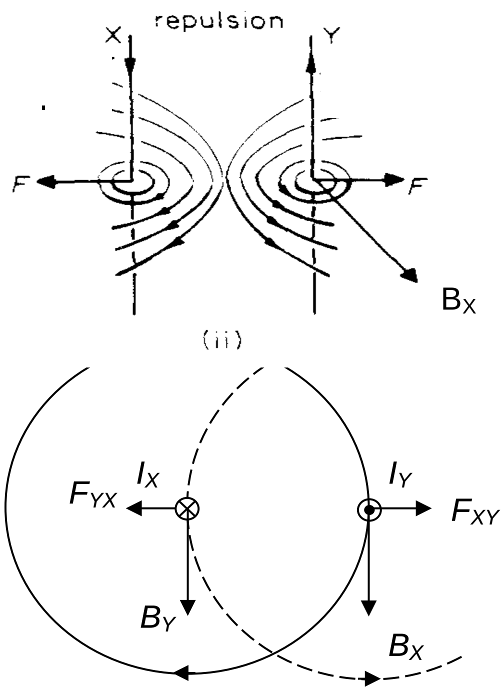


Figure (ii): Forces between currents in opposite direction

For conductor Y	B-Field due to X	Current due to Y	Force acting on Y
		Into page	
For conductor X	**B-Field due to Y**	**Current due to X**	**Force acting on X**
		Out of page	

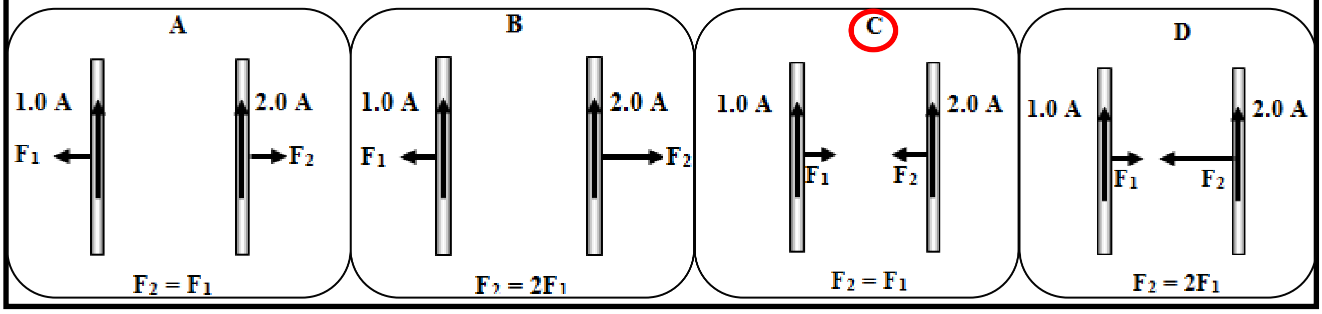
For conductor Y	B-Field due to X	Current due to Y	Force acting on Y
		Out of page	
For conductor X	**B-Field due to Y**	**Current due to X**	**Force acting on X**
		Into page	

When the currents in the two long straight conductors X and Y are in the <u>same</u> direction, there is a force of <u>attraction</u> between them which tends to pull the conductors towards each other [Fig. (i)]. When the currents flow in <u>opposite</u> directions, there is a <u>repulsive</u> force between them which tends to push the conductors away from one another [Fig. (ii)].

Example 5

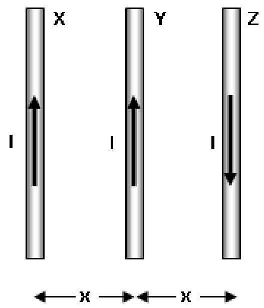
Two long, straight, parallel wires carry current of 1.0 A and 2.0 A respectively.

Which diagram shows the directions and relative magnitudes F_1 and F_2 of the forces per unit length on each of the wires?



Example 6

The diagram shows three parallel wires X, Y, and Z that carry currents of equal magnitude in the directions shown. The resultant force experienced by Y due to the current in X and Z is



A perpendicular to the plane of the paper

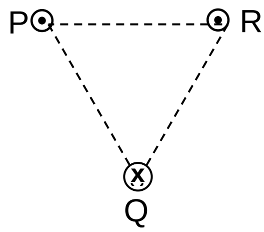
B to the left

C to the right

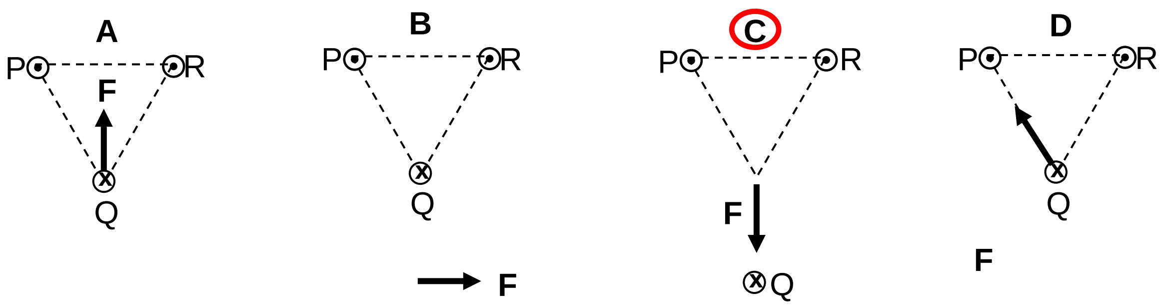
D zero

Example 7

Three long vertical wires pass through the corners of an equilateral triangle **PQR**. They carry equal currents into or out of the paper in the directions shown in the diagram.



Which direction shows the direction of the resultant force F on the wire Q?



2.4 Force on a Moving Charge in a Magnetic Field

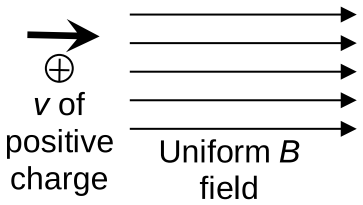
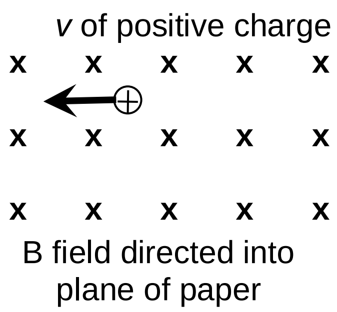
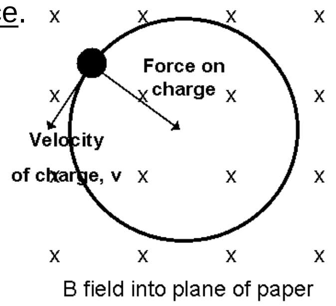
Previously, we have discussed how a current-carrying conductor experiences force in a magnetic field. Since current constitutes a flow of charges, moving charged particles also experience magnetic forces in a magnetic field.

The magnitude of the force F_B on a particle of charge q moving with speed v in a magnetic field of flux density B is given by

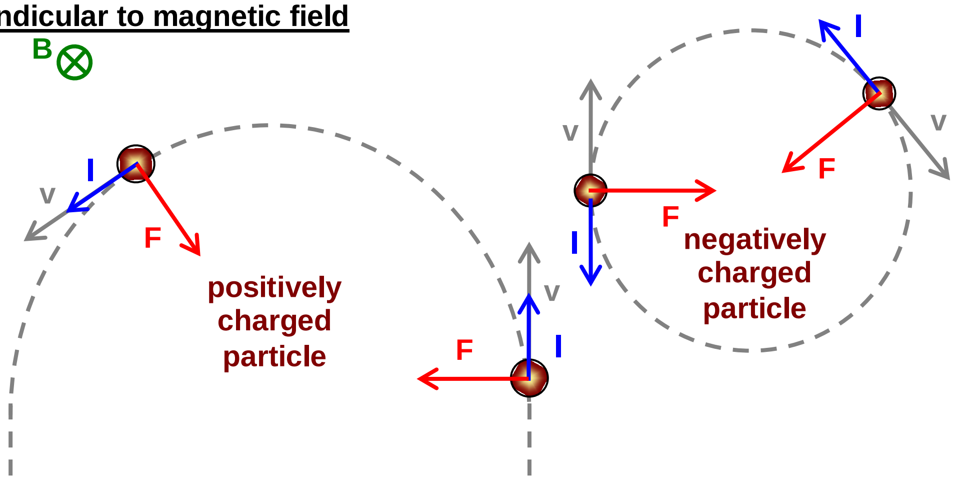
$$\boxed{F_B = Bqv\sin\theta}$$

where θ is angle between v and B

The direction of the force can be obtained by using Fleming's Left Hand Rule, where current is the direction of flow of positive charges.

Scenario	θ	Force due to B field	Path of Particle
 	0°	*Since v* <u>is in the direction of flux density *B*,</u> $F_B = B\,q\,v\sin\theta = 0$ No force experienced by charged particle	Charged particle continues to move <u>at constant speed in a straight line.</u>
 	90°	*Since v* <u>is perpendicular to *B*,</u> $F_B = B\,q\,v\sin\theta = B\,q\,v$	Path is a <u>circle</u>, with the force due to magnetic field on the charge acting as <u>centripetal force</u>.  Checklist for Circular Motion: □ Constant *F*? □ $F \perp v$?

Velocity perpendicular to magnetic field



When the velocity of the particle is perpendicular to the magnetic field, it gives rise to a force due to magnetic field. This force contributes to a centripetal force acting on the particle, causing it to move in uniform circular motion and Newton's Second Law can be applied.

$$\sum F = ma$$

$$F_B = \frac{mv^2}{r}$$

$$Bqv = \frac{mv^2}{r}$$

$$r = \frac{mv}{Bq}$$

From the equation, we can deduce that for particles entering the magnetic field with the same velocity, the radius is proportional to its mass-to-charge ratio (i.e. *m/q*).

Since magnetic force is always perpendicular to the velocity of the particle, there is no work done on the particle by the magnetic force. Thus the speed and kinetic energy of the particle remains constant as it moves in along the circular path in a magnetic field.

Example 8

An electron enters a magnetic field of flux density B_1 at right angles with speed 1.0 x 10^5 m s^{-1} and makes a circular path of radius 0.50 m. The flux density is changed to B_2 such that a proton with speed 7.5 x 10^4 m s^{-1} enters the field and makes a circular path identical to that of the electron. Determine the ratio of B_1/B_2 . Comment on the directions of B_1 and B_2 .

$$Bqv = \frac{mv^2}{r}$$

$$B = \frac{m\,v}{qr}$$

$$B_1/B_2 = \frac{\frac{m_e v_e}{er_e}}{\frac{m_p v_p}{er_p}}$$

$$= \frac{r_p m_e v_e}{r_e m_p v_p}$$

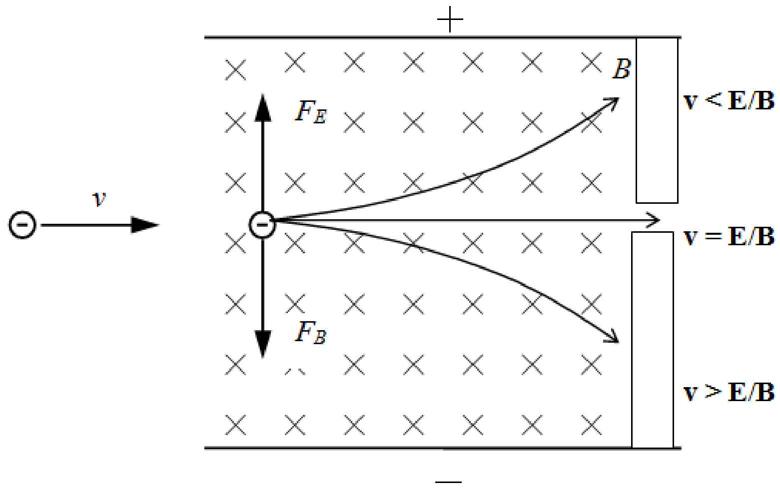
$$= \frac{0.50 \times 9.11 \times 10^{-31} \times 1.0 \times 10^5}{0.50 \times 1.67 \times 10^{-27} \times 7.5 \times 10^4}$$

$$= 7.27 \times 10^{-4}$$

B_1 and B_2 are in opposite direction.

2.5 Motion of Charged Particles in Crossed Fields

Consider an electron of velocity *v* projected into a crossed E- and B- fields as shown below.



Velocity	Relative Magnitude of F_E & F_B	Deflection
$v = E/B$	$F_E = F_B$	Undeflected
$v > E/B$	$F_E < F_B$	deflected ↓
$v < E/B$	$F_E > F_B$	deflected ↑

Upon entering the crossed fields, the electron experiences two forces, F_E and F_B due to the E- and B-fields respectively. $F_E = eE$ is directed upwards while $F_B = Bev$ is directed downwards.
By adjusting *B* and *E* such that $F_E = F_B$, the electron will be able to move through the crossed fields undeflected.

$$\sum F = ma$$
$$F_E - F_B = ma$$
$$= 0$$
$$qE - Bqv = 0$$
$$v = \frac{E}{B}$$

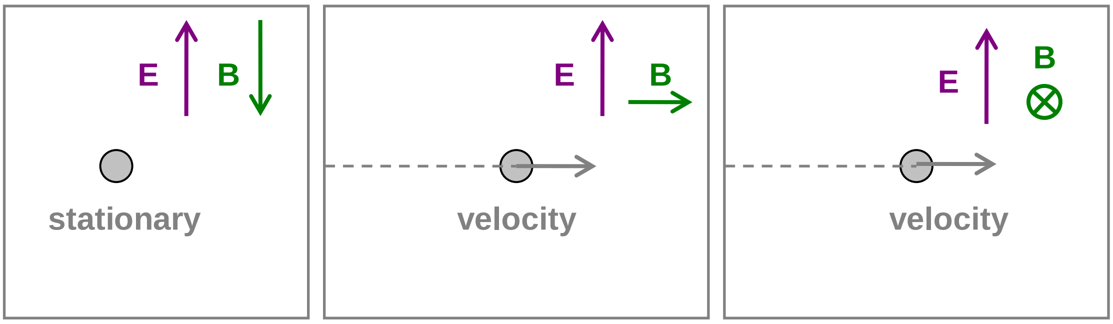
For a beam of charged particles having a <u>range of velocities</u>, only those moving with $v = E/B$ remain undeflected. Hence, only particles of this particular velocity will emerge at the slit while the rest are blocked. This is the working principle of the Velocity Selector. It allows us to achieve having all particles moving with the same speed, which is required in many experiments.

Refer to the table for the differences in the deflection of charged particles in a magnetic field and electric field.

Deflection in Magnetic Field	Deflection in Electric Field
Magnetic field can exert a magnetic force only on a moving charged particle. Stationary charge particles do not experience a force.	Electric field will always exert an electric force on a charged particle, whether it is stationary or not.
Magnetic force is perpendicular to the magnetic field and velocity of the charged particle.	Electric force acts along the direction of the electric field depending on the polarity of the charged particle.
Magnetic force is dependent on the speed and direction of motion of the charged particle.	Electric force is not dependent on the speed and direction of the charged particle.
Circular motion is obtained when a charged particle enters the uniform magnetic field perpendicularly.	Parabolic motion is obtained when a charged particle enters the uniform electric field perpendicularly.

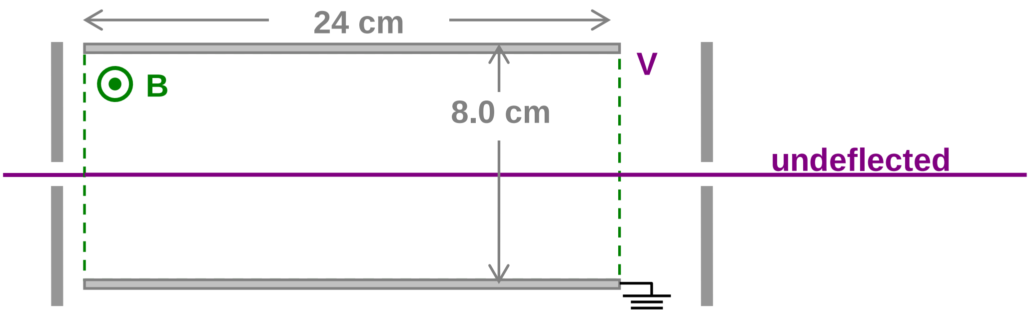
Example 9

Indicate with an arrow the direction of the resultant force on the proton in each of the cases shown below.



Example 10

Electrons of speed 2.0×10^5 m s^{-1} emerge from the crossed fields shown below undeflected. If the flux density of the field is 0.50 mT, calculate the potential V. Also deduce the speed of alpha particles that will emerge from the crossed fields undeflected.



$F_B = F_E$

$B\, q\, v = q\, E$

$E = B\, v = 0.50 \times 10^{-3} \times 2.0 \times 10^{5} = 1.0 \times 10^{2}$ N C^{-1}

$E = \Delta V / d = \Delta V / 0.080$

$\Delta V = 8.0$ V

The upper plate must be at a lower potential than the lower plate, thus $V = -8.0$ V

For undeflected particles

$v = E / B$ (independent of mass and charge)

Alpha particles (or any other particles) of the same speed (2.0×10^5 m s^{-1}) will not be deflected.

APPENDIX A: HALL PROBE

Source: Physics Review, September 2015, Vol. 25, Number 1, Main, P., Supersonic Skydive

Measuring electricity is really easy—we're all familiar with electrical equipment like voltmeter, ammeter and ohmmeter. But measuring magnetic flux density is a little bit harder. The Hall probe uses a clever bit of science discovered in 1879 by American physicist Edwin H. Hall (1855–1938) which was years ahead of its time—20 years before the discovery of the electron—and no one really knew what to do with it until decades later when semiconducting materials such as silicon became better understood. Let's take a closer look at how the Hall probe works!

When an electric current I flows through a conductor placed perpendicular to a magnetic field of flux density B, the magnetic field exerts a force on the moving charge carriers which tends to push them to one side of the conductor.

This buildup of charges at the either sides of the conductor produces an electric field. At steady current, the resulting electric force balances the magnetic force. The measurable potential difference between the two (transverse) sides of the conductor is the hall voltage V_H.

$F_B = Bev_d$

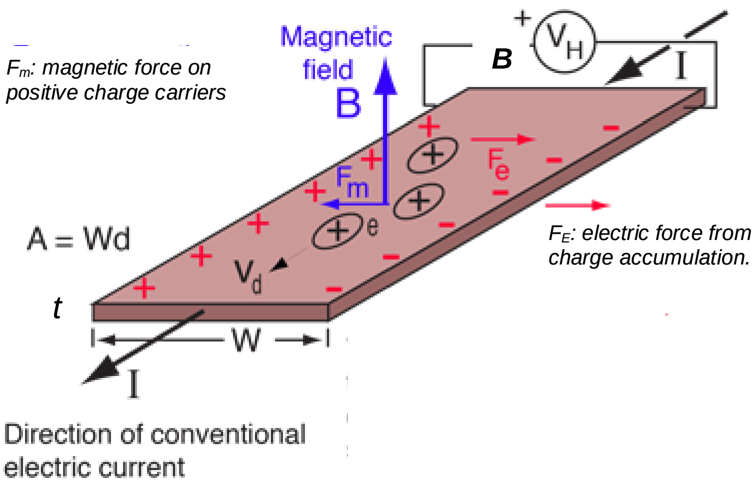
$F_E = Ee$

$F_B = F_E$

$Bev_d = Ee$

$E = V_H/w$

$Bev_d = (V_H/w)\ e$



$V_H = Bv_d\ w$

$I = nAv_de$

$v_d = I / nAe$

$V_H = B\ (I / nAe)\ w = B\ (I / ntwe)\ w = B\ (I / net)$

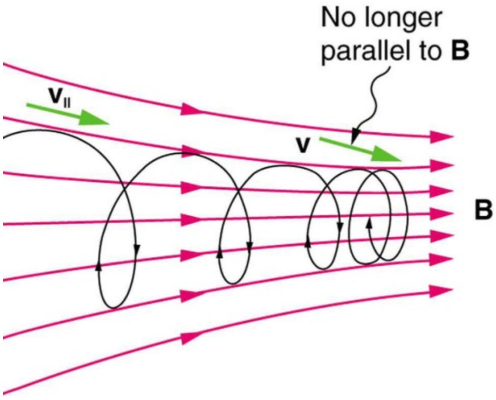
For the same sample of conductor at steady current, the hall voltage is proportional to the magnetic flux density, B. It is important to place the hall probe perpendicular to the magnetic field in order to measure the flux density accurately.

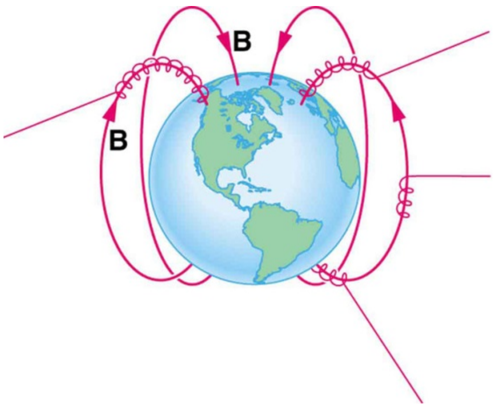
APPENDIX B: AURORA BOREALIS

The northern lights, or aurora borealis, offer an entrancing, dramatic, magical display that fascinates all who see it — but just what causes this dazzling natural phenomenon?

Solar winds stream away from the sun at speeds of about 1 million miles per hour. When they reach the earth, some 40 hours after leaving the sun, energetic charged particles within the solar winds enter the earth's atmosphere. These particles are channelled toward the poles by the magnetic force which causes them to spiral around the magnetic field lines of the earth.

In section 2.4, we learned about the circular motion of a charged particle when the magnetic field is aligned perpendicularly to the velocity of the particle. In the case of charged particles from the solar winds, the particles enter at an angle to the magnetic field. Resolving the velocity into components, The component of velocity perpendicular to the field causes the circular motion while the component of the velocity parallel to the field is unaffected, since the magnetic force is zero for motion parallel to the field. This produces helical motion rather than a circular motion.





These charged particles entering Earth’s magnetic field are energetic enough to ionize air molecules (mainly oxygen and nitrogen), so a considerable number of atoms and molecules are elevated to excited states. When they make the transition back to their ground states they emit light characteristic of the atoms and molecules – a taste of upcoming chapter on Quantum Physics. The color of the aurora depends on which atom is struck, and the altitude of the meeting.

You might be amazed to know that auroral displays can even make sounds! People have reported hearing claps and crackles during an aurora, and this has been verified by microphones placed by scientists. It is time to add them to your bucket list!

Colour of Aurora	Molecule that Charged Particles excite	Altitude
Green	**Oxygen**	< 150 miles
Red	**Oxygen**	> 150 miles
Blue	**Nitrogen**	< 60 miles
Purple/Violet	**Nitrogen**	**> 60 miles**