Geylang Methodist School (Secondary)
Preliminary Examination 2024

Candidate Name	**WORKED SOLUTIONS & SUGGESTED ANSWER SCHEME**		
Class		**Index Number**	

MATHEMATICS

4052/01

Paper 1

4 Express
5 Normal (Academic)

Candidates answer on the Question Paper.

2 hours 15 minutes

Setter: Ms Nainee Ismail

Monday, 5 August 2024

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use
90

This document consists of **20** printed pages including the cover page.

[Turn Over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 Expand and simplify $(4x - y)(3x + 4y)$.

$(4x - y)(3x + 4y)$

$= 12x^2 - 3xy + 16xy - 4y^2$

$= 12x^2 + 13xy - 4y^2$

$\times$	$4x$	$-y$
$3x$	$12x^2$	$-3xy$
$4y$	$16xy$	$-4y^2$

Answer $12x^2 + 13xy - 4y^2$ [2]

---

2 **(a)** Find the lowest common multiple (LCM) of 108 and 140.

By Prime Factorisation,

2	108	140
2	54	70
3	27	35
3	9	35
3	3	35
5	1	7
7	1	1

LCM $= 2^2 \times 3^3 \times 5 \times 7$

$= 4 \times 27 \times 5 \times 7$

$= 3780$

Answer 3780 [1]

(b) Find the highest common factor (HCF) of 108 and 140.

By Prime Factorisation,

2	108	140
2	54	70
	27	35

HCF $= 2^2$

HCF $= 4$

Answer 4 [1]

---

3 Solve $4 - \frac{1}{3}x = 2$.

$4 - \frac{1}{3}x = 2$

$4 - 2 = \frac{1}{3}x$

$2 \times 3 = x$

$\therefore x = 6$

Answer $x =$ 6 [1]

4 10 23 34 40 25 35 17 44 23

(a) Find the median of the set of numbers.

Ascending Order:

10 17 23 23 $\boxed{\mathbf{25}}$ 34 35 40 44

Answer ....25.......... [1]

(b) Find the range of the set of numbers.

$\boxed{\mathbf{10}}$ 17 23 23 25 34 35 40 $\boxed{\mathbf{44}}$

Range = Highest – Lowest = 44 – 10 = 34

Answer ....34.......... [1]

---

5 **(a)** A hotel made breakfast milkshake for its guests.
A mixture of mango juice, milk and yogurt in the ratio 7 : 5 : 3 are mixed together.
2.5 litres of milk is used in the mixture.

(i) How much yogurt is used in the milkshake?

5 units $\rightarrow$ 2.5 litres

1 unit $\rightarrow$ $\frac{2.5}{5} = 0.5$ litres

3 units $\rightarrow$ $0.5 \times 3 = 1.5$ litres

Answer ....1.5.......... litres [1]

(ii) How much breakfast milkshake is made?

Total number of units	1 unit $\rightarrow$ 0.5 litres
= 7 + 5 + 3	15 units $\rightarrow$ 15×0.5
= 15 units	$\rightarrow$ 7.5 litres

Answer ....7.5.......... litres [1]

(b) Another milkshake is made using apple juice, peach juice and milk.
The ratio of apple juice : milk = 2 : 3.
The ratio of milk : peach juice = 5 : 4.

Find the ratio of apple juice : milk : peach juice.

Apple	:	**Milk**	:	**Peach**
2(5)	:	**3**(5)	:	
	:	**5**(3)	:	**4**(3)
10	:	15	:	12

Answer ....10.... : ....15.... : ....12.... [1]

6 Find the equation of the straight line passing through $(7, -7)$ and $(-4, 15)$.

Gradient, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{15-(-7)}{-4-7}$

$= \dfrac{15+7}{-11}$

$= \dfrac{22}{-11}$

Gradient, $m = -2$

y – intercept, c:

$y = mx + c$
$-7 = (-2)(7) + c$
$-7 = -14 + c$
$c = -7 + 14$
$c = 7$

Equation:

$y = mx + c$
$y = -2x + 7$

Answer $y = -2x + 7$ .... [3]

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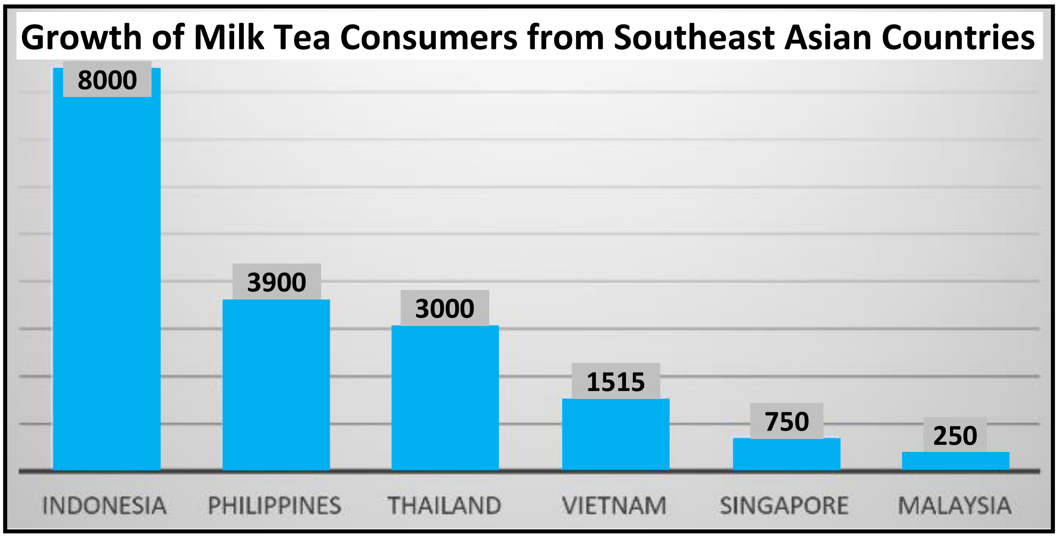
7 The graph shows the growth in milk tea consumers regionally.



Adapted from source: https://www.mdpi.com/

(a) State one misleading feature of the graph.

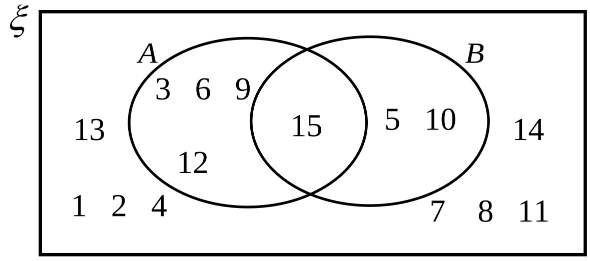
E*vidence*: Value does not specify if its actual value or percentage. No y-axis, not indicative of the length of bar and value cannot be verified for accuracy. Title states growth which indicates a basis of comparison which is not shown.
[Any comment with substantial and mathematically sound justification can be accepted] [1]

(b) Based on your answer in **part (a)**, explain how this feature affects the reader's interpretation of the graph.

Any **I***nterpretation* or **C***onclusion* with substantial and mathematically sound justification based on the **E***vidence* mentioned in **part (a)** can be accepted. [1]

[Turn Over

8 The Venn diagram shows the elements of $\xi = \{\text{integers } x: 1 \le x \le 15\}$ and two sets A and B.



(a) Use one of the symbols below to complete each statement.

$\varnothing \quad \subseteq \quad \in \quad \cap \quad \not\subset$

(i) { 15 } ……$\subseteq$…… $A \cap B$ [1]

(ii) 1 ……$\in$…… $(A \cup B)'$ [1]

(b) Suggest a description to define set A.

A = {integer x: x is a multiple of 3 **OR** x is divisible by 3}
………………………………………………………………………………… [1]

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9 The number of social media users globally grew from 4.72 billion in January 2023 to 5.04 billion in January 2024. The number increased by r % every month during that period. Find the value of r. (1 billion = 1×10^9)

Social media is a system which experiences ***exponential*** growth.
[**No** penalty for not using the info (1 billion = 1×10^9) given:

$$\text{Total amount } = P\left(1+\frac{r}{100}\right)^n$$

$$5.04\times10^9 = 4.72\times10^9\left(1+\frac{r}{100}\right)^{12}$$

$$\frac{5.04}{4.72} = \left(1+\frac{r}{100}\right)^{12}$$

$$\sqrt[12]{\frac{5.04}{4.72}} = 1+\frac{r}{100}$$

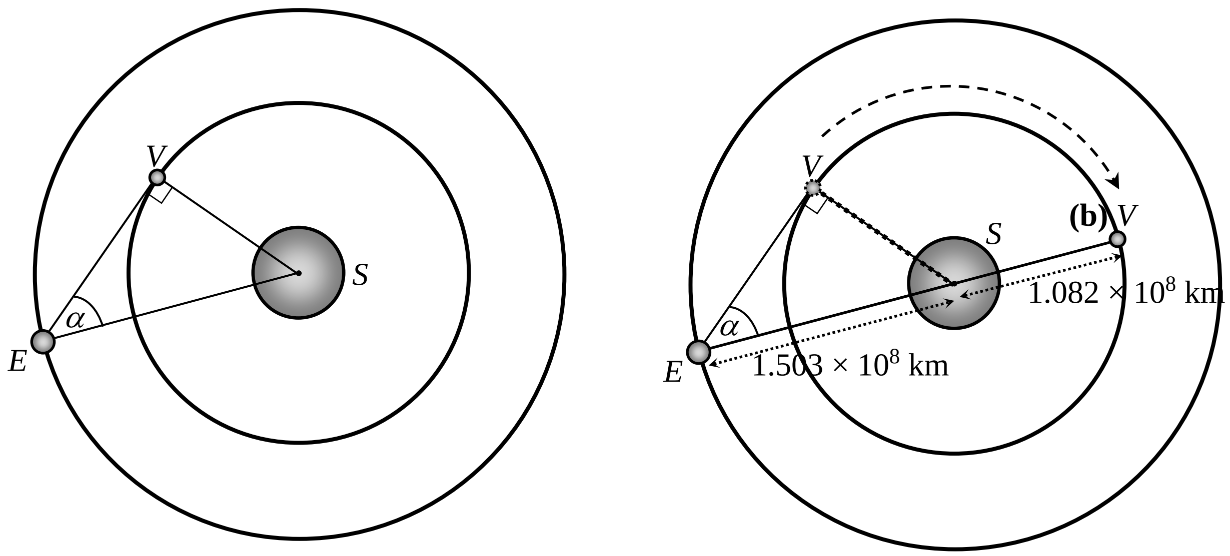
$$100\left(\sqrt[12]{\frac{5.04}{4.72}}-1\right) = r$$

r = 0.5481408453
r = 0.548

Answer r = ……0.548…… % [2]

10 The following diagram shows the positions of Earth, E, and Venus, V, and their orbit around the Sun, S. The radius of Venus' orbit is 1.082×10^8 km.

[Follow the degree of accuracy of the values specified in this question and leave your answers in standard form where necessary.]



(a) If the angle α is 46.054°, calculate the distance between Earth and the Sun.

Distance between Earth and the Sun = ES

$$\sin \alpha° = \frac{VS}{ES}$$

$$\sin 46.054° = \frac{1.082 \times 10^8}{ES}$$

$$ES = \frac{1.082 \times 10^8}{\sin 46.054°}$$

$ES = 1.502789924 \times 10^8$ km

$ES = 1.503 \times 10^8$ km

(follow the <u>standard form</u> and <u>degree of accuracy</u> given in question: <u>3dp</u>)

Answer 1.503×10^8 km [1]

(b) State the value of α when Earth is furthest from Venus and calculate this distance.

Earth furthest from Venus = $ES + VS$

When $\alpha = 0.000°$ (ES and VS are on the same straight line, collinear)

(follow degree of accuracy given in question: 3dp)

Distance $= ES + VS$

$= (1.502789924 \times 10^8) + (1.082 \times 10^8)$

Distance $= 2.584789924 \times 10^8$

$ES + VS = 2.585 \times 10^8$ km

(follow <u>degree of accuracy</u> given in question: <u>3dp</u>)

Answer $\alpha =$ 0.000°

2.585×10^8 km [2]

11 Simplify $\dfrac{x^3-4x}{2x^2-7x+6}$.

$$\frac{x^3-4x}{2x^2-7x+6}=\frac{x\left(x^2-4\right)}{(x-2)(2x-3)}$$
$$=\frac{x(x+2)(x-2)}{(x-2)(2x-3)}$$
$$=\frac{x(x+2)}{(2x-3)}$$

Factorise Numerator fully
Factorise Denominator fully
Simplify fraction correctly

Answer $\dfrac{x(x+2)}{(2x-3)}$ [3]

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12 **(a)** A box is said to contain blue marbles, red marbles and yellow marbles.
A marble is picked at random from the box.
The probability that the marble is blue is 0.27.
The probability that the marble is red is 0.73.

Deduce if there are any yellow marbles in the box, showing the calculations clearly.

E*vidence*:
P(yellow) = 1 – [P(blue) + P(red)]
= 1 – [0.27 + 0.73]
= 1 – 1
P(yellow) = 0

I*nterpretation*: P(yellow) = 0 means the chances is NIL and not possible to do so.
C*onclusion*: There are no yellow marbles in the box. [2]

(b) Another box contains 10 green marbles, 8 orange marbles and 3 purple marbles.
The probability of picking a purple marble is $\frac{1}{4}$ when x purple marbles are added in.
Find the total number of purple marbles in this box.

New $\mathrm{P}(\text{purple})$:

$$\frac{3+x}{10+8+3+x}=\frac{1}{4}$$
$$\frac{3+x}{21+x}=\frac{1}{4}$$
$$4(3+x)=1(21+x)$$

$12 + 4x = 21 + x$
$4x - x = 21 - 12$
$3x = 9$
$x = 3$

Total number of purple marbles
= 3 + 3
= 6 purple marbles

Answer 6 [2]

13 Factorise completely $8 - 2x - 24y + 6xy$.

$8 - 2x - 24y + 6xy$	
$= 2(4 - x - 12y + 3xy)$	Factorise common factor 2
$= 2[1(4 - x) - 3y(4 - x)]$	Factorise by grouping
$= 2(4 - x)(1 - 3y)$	Correctly simplified (equivalent)

Answer $2(4 - x)(1 - 3y)$ **or** $2(x - 4)(3y - 1)$ [3]

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14 A container of oil is 65% full. 20% of the oil is used for cooking.
15.6 litres of oil is left in the container.
If 14.8 litres of oil is then poured into the container, determine whether there will be a spill.
Show your working.

% of oil (in the container) used for cooking = (20% × 65%) = 13%
% of oil (in the container) left = 65% – 13% = 52%
Total capacity of oil still in the container:

52% → 15.6 litres (100% – 52% = 48% to be full)

$$1\% \to \frac{15.6}{52} \text{ litres}$$

$$48\% \to \frac{15.6}{52} \times 48$$

$$= 14.4 \text{ litres}$$

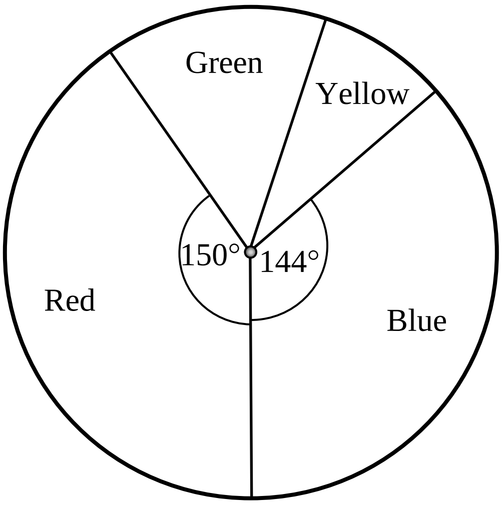
14.8 litres > 14.4 litres
Amount poured > Capacity (Space) left in container

Hence, there will be a spill.

[Accept any reasonable working with method marks allocated accordingly for each step.]

There will be a spill as the total amount of oil poured (14.8 litres) exceeds the empty capacity (space) in the container (14.4 litres). [4]

15 A survey is conducted on the number of students in Red, Green, Yellow and Blue House who participated in the Inter-House Games. The results are shown as a pie chart.



There are twice as many students that are from Green House as compared to Yellow House. Explain why this information may not be useful to find the number of students from the Red and Blue House who participated in the Inter-House Games.

E*vidence*: A pie chart displays the breakdown (by angles, percentage, fraction etc) of the survey of the different houses.
I*nterpretation*: However, the actual breakdown of numbers cannot be found.
C*onclusion*: This can only be found when the total number of students surveyed are given, which is a missing information here. [1]

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16 The Marina Coastal Expressway (MCE) tunnel is about 3.6 km long. The speed limit in expressway tunnels is 70 km/h. Find the shortest time possible for a car of length 4.8 m to pass through the MCE tunnel **completely**, within the speed regulations.
Give your answer in minutes and seconds (nearest whole number).

- Adding 4.8 m to Total Distance
- Correct Formula for Time and correct value in hours
- Correct Time for minutes and seconds

$$\text{Time taken} = \frac{\text{Total Distance}}{\text{Average Speed}}$$
$$= \frac{3.6\text{ km} + 4.8\text{ m}}{70\text{ km/h}}$$
$$= \frac{3.6\text{ km} + \frac{4.8}{1000}\text{ km}}{70\text{ km/h}}$$
$$= \frac{3.6048\text{ km}}{70\text{ km/h}}$$
$= 0.05149714286$ ($\times 60$ minutes $\times$ 60 seconds)
$= 185.3897143$ seconds
Time taken = 3 minutes 5.3897143 seconds
Time taken = 3 minutes 6 seconds (round up)
(pass through the MCE tunnel **completely**)

Answer ....3.......... minutes ....6.......... seconds [3]

17 $N = \dfrac{a^6}{b^3}$ where a and b are prime numbers.

(a) Explain why N is a perfect cube

$N = \dfrac{a^6}{b^3} = \dfrac{a^{2\times 3}}{b^{1\times 3}} = \left(\dfrac{a^2}{b}\right)^3$

$\Rightarrow N = \left(\dfrac{a^2}{b}\right)^3$; hence N is a perfect cube. Power is a multiple of 3. [1]

(b) Show that M is divisible by 6, given that $\dfrac{MN}{2} = (3ab)^2$ and M is a natural number.

Answer

$$\frac{MN}{2} = (3ab)^2$$
$$MN = 2(3^2)(ab)^2$$
$$M = \frac{18(ab)^2}{N}$$
$$M = 6\left(\frac{3a^2b^2}{N}\right)$$

Hence

$\Rightarrow M = 6\left(\dfrac{3a^2b^2}{N}\right)$

Attempt at
→ M as subject for formula
→ 6 extracted (multiple of 6 / divisible by 6)

[2]

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18 The expression $2 + px - x^2$ can be written in the form $q - (x-3)^2$.

(a) Find the value of p and the value of q.

$2 + px - x^2 = q - (x-3)^2$
$2 + px - x^2 = q - (x^2 - 6x + 9)$
$2 + px - x^2 = q - x^2 + 6x - 9$
$2 + px - x^2 = (q-9) + 6x - x^2$

Comparing coefficient of:
x : $p = 6$
x^0 : $2 = q - 9$
$q = 11$

Answer $p =$ 6

$q =$ 11 [2]

(b) Explain why when $x = 3$, the expression has its maximum value.

E*vidence*: $(x-3)^2$ in $q - (x-3)^2$ is of minimum value when $(x-3)^2 = 0$, which leads to $(x-3) = 0$ and $x = 3$. $[x^2 \geq 0]$
I*nterpretation*: Coefficient of x^2 is negative hence $-(x-3)^2 = 0$ is maximum when $x = 3$.
C*onclusion*: Hence when $x = 3$, $2 + px - x^2$ has its **max**imum value. [1]

19 **(a)** The force, F Newtons, between any two particles in the universe is inversely proportional to the square of the distance, r metres, between them.
It is known that $F = 100$ Newtons for a particular value r.
Find the value of F when this value of r is doubled.

$$F \ \alpha \ \frac{1}{r^2}$$
$$F = \frac{k}{r^2}$$
$$100 = \frac{k}{r^2}$$
$$k = 100r^2$$

$$F_2 = \frac{k}{(r_2)^2} \quad [r_2 = 2r]$$
$$F_2 = \frac{100r^2}{(2r)^2}$$
$$F_2 = \frac{100r^2}{4r^2}$$
$$F_2 = 25$$

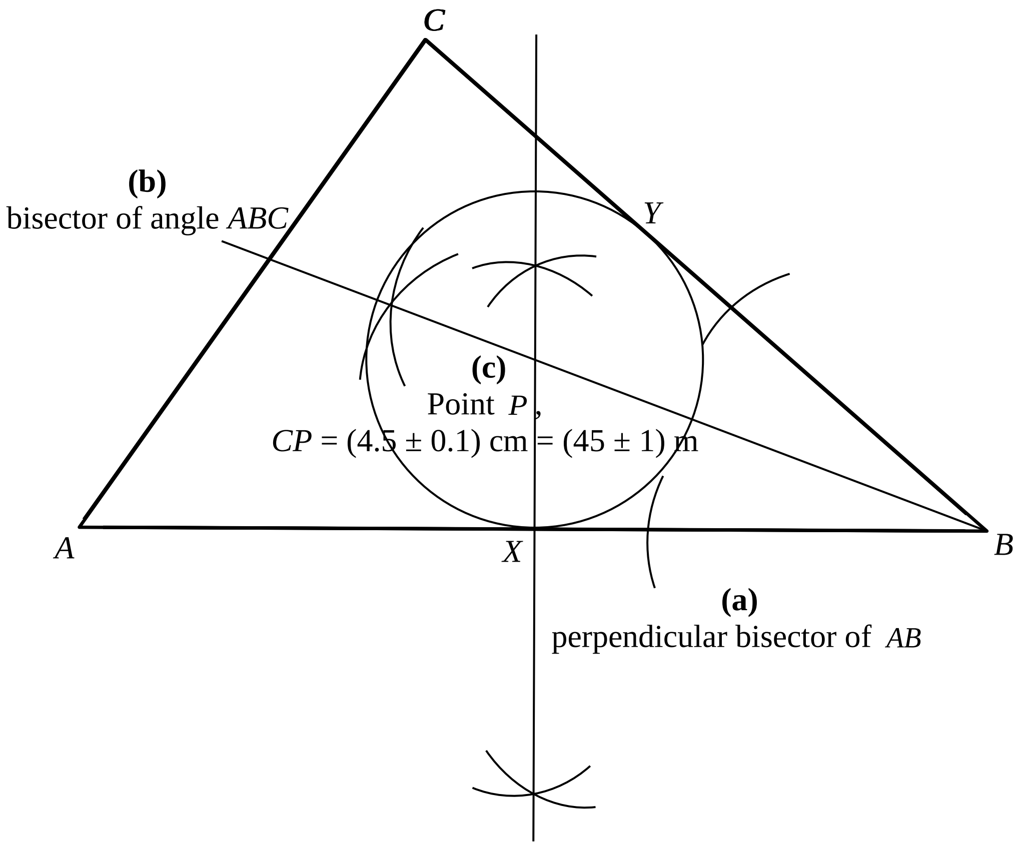
Answer ....25.......... Newtons [2]

(b) y is proportional to x^3.
Explain the effect on the value of x in order for y to be 2700% of its original value.

$$y_2 = 2700\% \text{ of } y \qquad (\Rightarrow 27y)$$
$$x_2 = \sqrt[3]{2700\%} \text{ of } y \qquad \left(\Rightarrow \sqrt[3]{27}\,x\right)$$
$$x_2 = \sqrt[3]{27}\,x$$
$$x_2 = 3x$$

....
$x_2 = 3x$ **OR** x is tripled of its original value **OR** x is 300% of its original value.
.... [1]

20 The diagram shows an eco-garden in a school in the shape of triangle *ABC*.
The diagram is drawn to a scale of 1 : 1000.



(a) Construct the perpendicular bisector of *AB*. [1]

(b) Construct the bisector of angle *ABC*. [1]

(c) A flagpole with the school flag is erected at a point *P* such that it is equidistant from *A* and *B* **and** equidistant from *AB* and *BC*.
Mark the point *P* on the diagram and measure the actual length of *CP*.

1 : 1000
1 cm : 1000 cm
(1 m = 100 cm)
1 cm : (1000 ÷ 10) m
1 cm : 10 m

$CP = (4.5 \pm 0.1)$ cm $= (45 \pm 1)$ m

Answer CP = ...(45 ± 1)...... m [1]

(d) A tiled path is to be constructed in the eco-garden, where each tile is equidistant from *P*. The tiled path also meets the line *AB* and *BC* at points *X* and *Y* respectively such that $BX = BY$. Describe the shape of the tiled path and find its actual length.

Tiled path is a circle with radius, $r = (2.2 \pm 0.1)$ cm $= (22 \pm 1)$ m

Length $= 2\pi r$
$= 2\pi(22)$ [$r = 22 \pm 1$]
Length $= 44\pi r = 138.230 \ldots$ m
Length $= 138$ m (3sf)
$131.9 \text{ m} \leq \text{Length} \leq 144.5 \text{ m}$

$BX = BY$
(Tangents from external point, *B*, are equal)

Answer Shape of tiled path: ......Circle......

Length of tiled path = ...138...... m [2]

[Turn Over

21 **(a)** $\dfrac{2^a \times \left(\sqrt[3]{64}\right)^c}{16^b} = 1.$

Find an expression for a in terms of b and c.

$$\frac{2^a \times \left(\sqrt[3]{64}\right)^c}{16^b} = 1$$

$$2^a \times 4^c = 16^b$$

$$\left(2^a\right) \times \left(2^2\right)^c = \left(2^4\right)^b$$

Simplify using Indices Law
Simplify to base 2

$$2^a \times 2^{2c} = 2^{4b}$$

$$2^{a+2c} = 2^{4b}$$

$\Rightarrow$ $a + 2c = 4b$

$\therefore$ $a = 4b - 2c$

Answer $a =$ ......$4b - 2c$...... [2]

(b) Simplify $\left(\dfrac{27y^{12}}{x^6}\right)^{-\frac{1}{3}}$.

$$\left(\frac{27y^{12}}{x^6}\right)^{-\frac{1}{3}} = \left(\frac{x^6}{27y^{12}}\right)^{\frac{1}{3}} = \left(\frac{x^6}{3^3 y^{12}}\right)^{\frac{1}{3}}$$

$$= \left(\frac{x^{6\times\frac{1}{3}}}{3^{3\times\frac{1}{3}} y^{12\times\frac{1}{3}}}\right) = \frac{x^2}{3y^4}$$

Answer ......$\dfrac{x^2}{3y^4}$...... [2]

22 The table shows the distribution of weight of 40 students in a class.

Weight (x kg)	Frequency
$40 < x \le 50$	2
$50 < x \le 60$	13
$60 < x \le 70$	20
$70 < x \le 80$	5

(a) Calculate an estimate for

(i) the mean weight of the students,

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{(2\times 45)+(13\times 55)+(20\times 65)+(5\times 75)}{40} = \frac{2480}{40}$$

Mean = 62 kg

Answer ......62...... kg [1]

(ii) the standard deviation of the weight.

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

$$= \sqrt{\frac{(2\times 45^2)+(13\times 55^2)+(20\times 65^2)+(5\times 75^2)}{40} - 62^2}$$

$$= 7.483314774$$

Standard Deviation = 7.48 kg

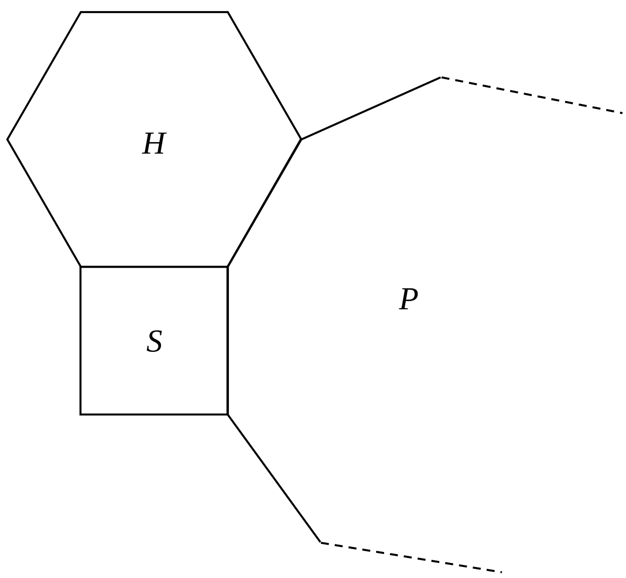
Answer ......7.48...... kg [1]

(b) Due to a fault in the weighing machine, the weight recorded was 2 kg lighter than the actual weight. Explain how this may have affected the mean and standard deviation and state its correct value.

Mean calculated is lower and correct value will increase by 2 kg (62 + 2 = 64 kg).

Standard deviation will remain the same at 7.48 kg. [2]

23 The diagram below shows a square S, a regular hexagon H and a part of a regular n – sided polygon P.



Find the number of sides, n, of the regular polygon P.

Each interior angle of regular hexagon, H

$= \dfrac{(6-2)\times 180°}{6}$

$= \dfrac{4\times 180°}{6}$

$= \dfrac{720°}{6}$

$= 120°$

Each interior angle of square, $S = 90°$

Each interior angle of regular polygon, P

$= 360° – 120° – 90°$

$= 150°$

[angles at a point]

Each exterior angle of regular polygon, P

$= 180° – 150°$

$= 30°$

Number of sides, n, of regular polygon P

$n = \dfrac{360°}{30°}$

$n = 12$ sides

[Accept working with sound properties of angles used]

Answer $n =$ ....12.......... [4]

24 **(a)** The first five terms of a sequence are 2, 7, 12, 17 and 22.

(i) T_n is the nth term of the sequence.
Find an expression, in terms of n for T_n.

$T_n = a + (n - 1)\,d$ $[a = 2 ,\ d = 7 - 2 = 5]$
$T_n = 2 + (n - 1)(5)$
$= 2 + 5n - 5$
$T_n = 5n - 3$

Answer $T_n =$ $5n - 3$ (or equivalent) [1]

(ii) The sum of the first n terms of this sequence is given by $an^2 + bn$.

When $n = 1$, $a + b = 2$.
Show that $4a + 2b = 9$.

Answer

Given: Sum $S_n = an^2 + bn$
When $n = 2$, $S_2 = a(2)^2 + b(2)$
$2 + 7 = 4a + 2b$
$4a + 2b = 9$ (Shown)

[1]

(iii) Solve the equations from **part (ii)**.

$a + b = 2,$
$4a + 2b = 9.$

$a + b = 2$ (1)
$4a + 2b = 9$ (2)
(1): $a = 2 - b$
(2): $4(2 - b) + 2b = 9$
$8 - 4b + 2b = 9$
$-2b = 9 - 8$
$-2b = 1$
$b = -\frac{1}{2}$

(1): $a = 2 - b$
$a = 2 - \left(-\frac{1}{2}\right)$
$a = 2 + \frac{1}{2}$
$a = 2\frac{1}{2}$

Answer $a =$ $2\frac{1}{2}$

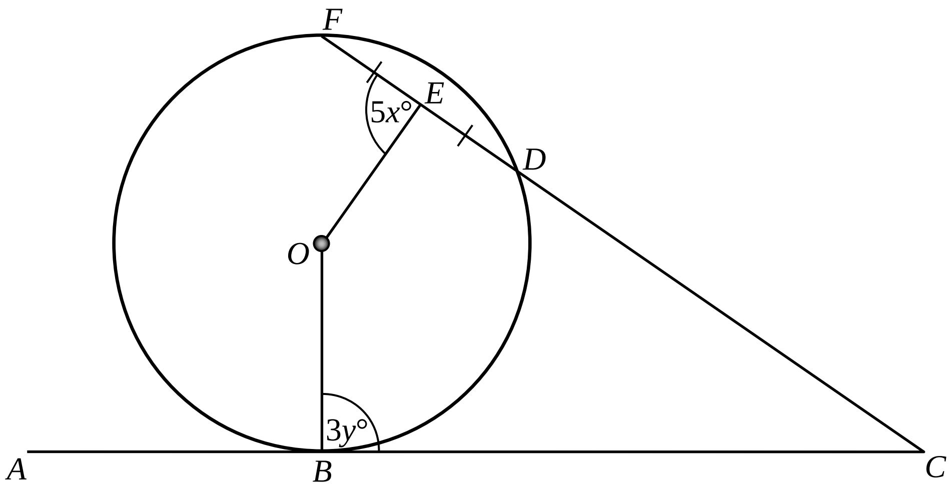
$b =$ $-\frac{1}{2}$ [2]

(b) The sum of the first n terms of a different sequence is given by $5n^2 + 3n$.
Find the 11th term of this sequence

Given: Sum $S_n = 5n^2 + 3n$
$T_{11} = S_{11} - S_{10}$
$= [5(11)^2 + 3(11)] - [5(10)^2 + 3(10)]$
$= 638 - 530$
$T_{11} = 108$

Answer 108 [2]

25 In the diagram, B, D and F lie on a circle, centre O.
AC is a tangent to the circle at B, $DE = FE$, angle $OEF = 5x°$ and angle $OBC = 3y°$.



(a) A statement was made that $BC = CE$.
Do you agree? Give reasons for your answer.

E*vidence*: AC is a tangent at B. In this case, ***CF* is NOT a tangent at *E***.
I*nterpretation*: For $BC = CE$, both AC and FC has to be *tangents* which are drawn from external point C (*tangents from external point*). E is not on the circumference.
C*onclusion*: **Disagree** that $BC = CE$ based on the evidence above and the interpretation. [1]

(b) Complete these statements.

Angle $OEF = 5x° = 90°$ (OE is a *perpendicular bisector of chord DF*)

$$5x = 90 \Rightarrow x = \frac{90}{5} = 18$$

(i) $x =$ 18 because OE is a *perpendicular bisector of chord DF*

(no marks awarded for correct value of x but incorrect reasons.) [1]

Angle $OBC = 3y° = 90°$ (AC is a *tangent perpendicular to radius OB*)

$$3y = 90 \Rightarrow y = \frac{90}{3} = 30$$

(ii) $y =$ 30 because AC is a *tangent perpendicular to radius OB*

(no marks awarded for correct value of y but incorrect reasons.) [1]

(c) Using the answers in **part (b)** and given that angle $BCD = (x + y)°$,
find the value of reflex angle BOE. Give a reason for each step of your answer.

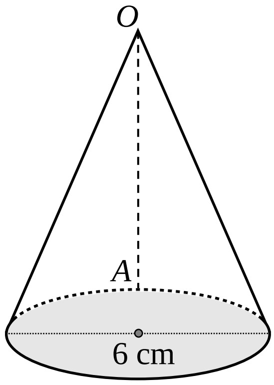
Angle $BCD = (x + y)° = (18 + 30)° = 48°$ [From **part (b)**]

Obtuse angle $BOE = [360 - 90 - 90 - 48]° = 132°$
[sum of angles in a quadrilateral, ⊥ bisector of chord, tangent perpendicular to radius]

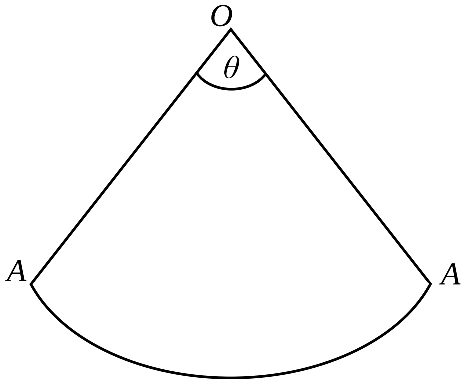
Reflex angle $BOE = (360 - 132)° = 228°$
[angles at a point]

Answer 228° [3]

26



A conical cup, shown above, has a diameter of 6 cm with a volume of 40π cm^3.
A cut is made along the dotted line, *OA*.
The cup is unfolded to form a sector subtended by angle θ, shown below.



(a) Show that θ is approximately 1.3792 radians (correct to 4 decimal places).

Answer

Radius of base of cup, $r = \frac{6}{2} = 3$ cm

Perpendicular height of cone, h

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$40\pi = \frac{1}{3}\pi(3)^2 h$$

$$40 = 3h$$

$$h = \frac{40}{3} = 13\frac{1}{3} \text{ cm}$$

$OA^2 = h^2 + r^2$

$$OA^2 = \left(\frac{40}{3}\right)^2 + 3^2 = \frac{1681}{9}$$

$$OA = \sqrt{\frac{1681}{9}} = \frac{41}{3} = 13\frac{2}{3} \text{ cm}$$

Circumference of base of circle (radius = r_1)
= Arc length of sector (radius = r_2)

$$2\pi r_1 = r_2\theta$$

$$2\pi(3) = (OA)\theta$$

$$6\pi = \left(\frac{41}{3}\right)\theta$$

$$\theta = 6\pi \div \frac{41}{3}$$

$$\theta = 1.379235799$$

θ = 1.3792 (4dp) (Shown)

<u>Alternative:</u>
Curved surface area of cone = Area of sector

$$\pi (r_1)^2 = \frac{1}{2}(r_2)^2\theta$$

[4]

(b) Hence, evaluate the area of the sector.

Area of sector $= \frac{1}{2}r^2\theta$ $\quad [r = r_2 = \frac{41}{3}]$

$$= \frac{1}{2}\left(\frac{41}{3}\right)^2(1.3792\ldots) \quad \text{(ecf)}$$

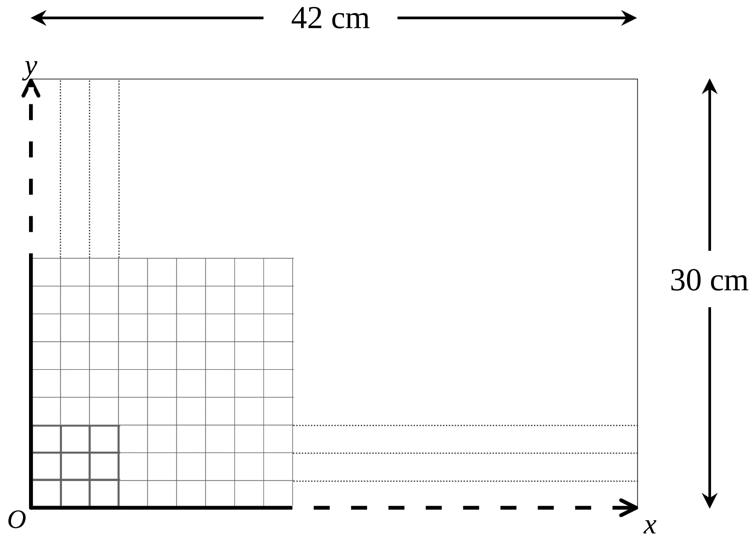
= 128.8052988

Area of sector = 129 cm^2 (3sf)

Answer ……………129……………… cm^2 [2]

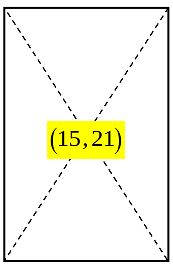
[Turn Over

27 A rectangular cardboard, *ABCD*, with dimensions 14 cm by 22 cm is to be drawn exactly at the centre of an A3-sized paper with dimensions 42 cm by 30 cm. For an accurate drawing, a grid with *x*-axis, *y*-axis and origin, *O*, at the bottom left-hand corner of the A3-sized paper, is drawn. A part of an incomplete grid (not drawn to scale) is shown below.
A scale of 1 cm to represent 1 unit is used.

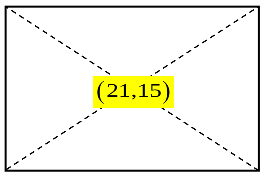


(a) State a possible coordinate for the centre of the A3-sized grid.

Two possible answers:



Orientation: **Portrait**
$(15, 21)$

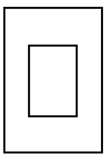


Orientation: **Landscape**
$(21, 15)$

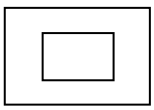
Answer (…………… , ……………) [1]

(b) State the possible coordinates for each of the points of the cardboard, *A*, *B*, *C* and *D*.

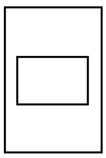
Four possible answers: Coordinates must be in order *ABCD*, *BCDA*, *CDAB*, *DABC* etc



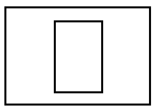
$(8,10)$
$(8,32)$
$(22,32)$
$(22,10)$



$(10,8)$
$(10,22)$
$(32,22)$
$(32,8)$



$(4,14)$
$(4,28)$
$(26,28)$
$(26,14)$



$(14,4)$
$(14,26)$
$(28,26)$
$(28,4)$

Answer *A* (…………… , ……………)

B (…………… , ……………)

C (…………… , ……………)

D (…………… , ……………) [2]

(c) The rectangular cardboard and the A3-sized paper are **not** similar.
The A3-sized paper is to be kept as it is as the grid is already drawn as a reference.
Suggest how the cardboard can be cut to make it similar to the A3-sized paper, such that there is minimal wastage and cutting.

Best Option: Cut the **<u>longer side</u>** of cardboard by **<u>2.4 cm</u>**. Ratio of length = $\frac{14}{30} = \frac{22 - \mathbf{2.4}}{42}\left(= \frac{19.6}{42}\right) = \frac{\mathbf{7}}{\mathbf{15}}$
……………………………………………………………………………………………

Next Option: Cut the **<u>longer side</u>** of cardboard by **<u>12 cm</u>**. Ratio of length = $\frac{14}{42} = \frac{22 - \mathbf{12}}{30}\left(= \frac{10}{30}\right) = \frac{\mathbf{1}}{\mathbf{3}}$
…………………………………………………………………………………………… [2]

(Minimal wastage and cutting required so BEST OPTION is the correct answer.)

End of Paper