Anglo-Chinese School
(Independent)

FINAL EXAMINATION 2021

YEAR 3 INTEGRATED PROGRAMME

ADVANCED MATHEMATICS

PAPER 2

Tuesday **12 October 2021** **1 hour 30 minutes**

Additional Materials

Writing paper (7 sheets)

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A graphic display calculator may be used in this paper.
Answer all the questions on the answer sheets provided.
At the end of the examination, attach the answer sheets in a single bundle.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

INFORMATION FOR STUDENTS

The maximum mark for this paper is 80.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.

*Answer **all** the questions on the answer sheets provided.*

1. *[Maximum mark: 7]*

The polynomial, $P(x) = Ax^3 + 2x^2 + Bx - 2$, is exactly divisible by $(2x-1)$ and has a remainder of 42 when divided by $(x-2)$.

(a) Find the value of *A* and of *B*. [4 marks]

(b) Hence, show that $P(x) = 0$ has only one real solution. [3 marks]

2. *[Maximum mark: 7]*

(a) Express $\frac{2x-9}{2x^2+5x-3}$ as partial fractions. [4 marks]

(b) Hence, solve the equation, $\frac{14x-63}{2x^2+5x-3} = 2 + \frac{15}{x+3}$. [3 marks]

3. *[Maximum mark: 9]*

The function, f , is defined by $f : x \mapsto 2(x-p)^2 + q$, for the domain $p \le x \le 4$.

(a) Given that $f(0) = -6$ and $f(1) = -8$, find the value of *p* and of *q*. [5 marks]

(b) Explain why $f^{-1}(x)$ exists and hence, find an expression for $f^{-1}(x)$. [3 marks]

(c) State the domain of $f^{-1}(x)$. [1 mark]

4. *[Maximum mark: 9]*

The functions, *f* and *g*, are defined as $f : x \mapsto \frac{2x}{2-x}$, $x \neq 2$, and $g : x \mapsto 2x - 1$.

(a) Obtain an expression, in similar form, for $(gf)^{-1}$ and state the value(s) of *x* for which $(gf)^{-1}$ is undefined. [5 marks]

(b) Hence, solve $g^2 g^{-1}(x) = (gf)^{-1}$. [4 marks]

5. *[Maximum marks: 8]*

The graph of $y = f(x)$ is mapped onto the graph of $y_1 = 2f(x+2) - 2$ after undergoing a series of transformations.

(a) Describe these transformations fully. [3 marks]

It is further given that $f(x) = \frac{x}{3x+2}$.

(b) Write down the equation of $y_1 = 2f(x+2) - 2$ in terms of *x*. [2 marks]

(c) State the equations of the asymptotes of y_1. [2 marks]

(d) The point $B(-2, b)$ lies on the graph of y_1. State the value of *b*. [1 mark]

6. *[Maximum mark: 10]*

The function, $f(x) = 2x^3 + Ax^2 + 5x + B$, maps onto itself at $x = 1$ and $f(2) = 5$.

(a) Find the value of *A* and of *B*. [4 marks]

Another function, $g(x)$, is defined as $g(x) = f(x) + 3 - 4x$.

(b) (i) Factorise $g(x)$ completely. [4 marks]

(ii) Hence, solve $g(x^2) = 0$. [2 marks]

7. *[Maximum mark: 11]*

The functions, *g* and *h*, are defined as follows.

$g: x \mapsto \frac{3-x}{x+5},\ x \neq -5$ and $h: x \mapsto x+2$.

(a) Find

(i) $gh(3)+hg(3)$, [2 marks]

(ii) $h^2 g(8)$. [2 marks]

(b) Find the value of *k* if $h^{-1}g^{-1}(k) = -1$. [3 marks]

(c) Solve $g(x)-h(x)=0$. [4 marks]

8. *[Maximum mark: 8]*

(a) On the same axes, sketch the graph of $f(x) = 2x^3 - x + 1$, and the graph of $g(x) = \frac{5x}{x^2+1}$, for $-1 \leq x \leq 2$. On your diagram, indicate the axes intercepts and the co-ordinates of the turning points and the end points. [6 marks]

(b) Hence, find the range of values of *x* such that $f(x)-g(x) \leq 0$. [2 marks]

9. *[Maximum marks: 11]*

(a) Solve each of the following equations, for $180° \leq x \leq 360°$.

(i) $3\tan^2 x - 1 = 0$, [2 marks]

(ii) $\frac{-3}{\cos x - 1} - 8 = 0$. [3 marks]

(b) Solve each of the following equations, for $0 < x < 2\pi$, leaving your answers in terms of π.

(i) $\sqrt{2}\sin x = 1$, [2 marks]

(ii) $2\sin x + \tan x = 0$. [4 marks]

End of Paper 2

Answers

1. (a) $A=4,\ B=2$

2. (a) $\dfrac{2x-9}{2x^2+5x-3}=\dfrac{15}{7(x+3)}-\dfrac{16}{7(2x-1)}$

   (b) $x=-\dfrac{7}{2}$

3. (a) $p=1,\ q=-8$

   (b) $f^{-1}(x)=1+\sqrt{\dfrac{x+8}{2}}$

   (c) $-8\le x\le 10$

4. (a) $(gf)^{-1}=\dfrac{2(x+1)}{x+5},\ x\neq 2$ and $x\neq -5$

   (b) $x=-4.31, 0.812$

5. (a) T_1: Horizontal translation of 2 units along the negative x-axis.
   T_2: Stretch, factor 2, along the y-axis.
   T_3: Vertical translation of 2 units along the negative y-axis.

   (b) $y_1=\dfrac{-4(x+3)}{3x+8}$

   (c) $y=-\dfrac{4}{3}$ & $x=-\dfrac{8}{3}$

   (d) $b=-2$

6. (a) $A=-5,\ B=-1$

   (b) $g(x)=(x-2)(2x+1)(x-1),\ x=\pm\sqrt{2},\pm 1$

7. (a) $\dfrac{9}{5},\ \dfrac{47}{13}$

   (b) $k=\dfrac{1}{3}$

   (c) $x=-1,-7$

8. (a) $g(x)$: Intercept: $(0,0)$

End points: $(-1,-2.5),(2,2)$

Turning points: $(-1,2.5),(1,2.5)$

$f(x)$: Intercept: $(0,1)$

End points: $(-1,0),(2,15)$

Turning points: $(-0.408,1.27),(0.408,0.728)$

(b) $0.173 \le x \le 1.09$

9. (a) (i) $x = 210°, 330°$ (ii) $x = 308.7°$

(b) (i) $x = \frac{\pi}{4}, \frac{3\pi}{4}$ (ii) $x = \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$