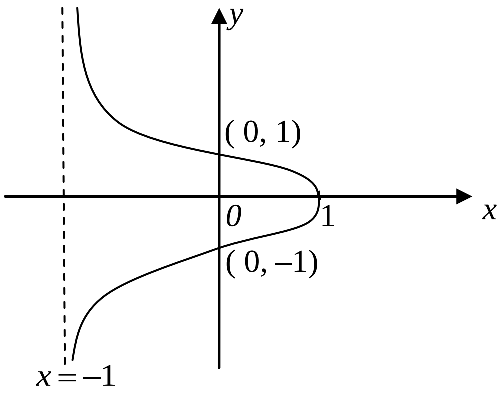
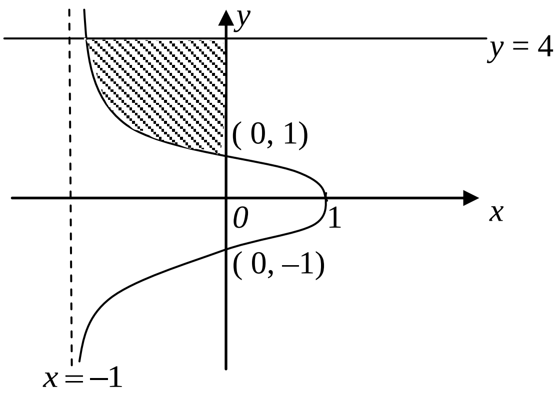
Qn	Suggested Solution
1	$\mathrm{f}(x) = ax^3 + bx^2 + cx + d$ where $a, b, c, d \in \mathbb{R}$. Given $\mathrm{f}(0) = 1$, $d = 1$. $\mathrm{f}'(x) = 3ax^2 + 2bx + c$ From the graph of $y = \mathrm{f}'(x)$, $\mathrm{f}'(-2) = 0 \Rightarrow 12a - 4b + c = 0 - (1)$ $\mathrm{f}'(7) = 0 \Rightarrow 147a + 14b + c = 0 - (2)$ $\mathrm{f}'(2.5) = -9 \Rightarrow 18.75a + 5b + c = -9 - (3)$ OR $\mathrm{f}''(2.5) = 0 \Rightarrow 15a + 2b = 0 - (4)$ Using GC to solve (1), (2) & (3): $a = \frac{4}{27}, b = -\frac{10}{9}, c = -\frac{56}{9}$ $\therefore \mathrm{f}(x) = \frac{4x^3}{27} - \frac{10x^2}{9} - \frac{56x}{9} + 1$
	For $\mathrm{f}(x)$ is concave downwards $\Leftrightarrow \mathrm{f}''(x) < 0$, $x < 2.5$.
2(i)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 6 \\ 3 \\ 0 \end{pmatrix}$ $\overrightarrow{AP} = \begin{pmatrix} 1+2\lambda \\ -2+\lambda \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2\lambda+2 \\ \lambda+1 \\ 0 \end{pmatrix} = \frac{(\lambda+1)}{3}\begin{pmatrix} 6 \\ 3 \\ 0 \end{pmatrix}$ Since $\overrightarrow{AP} = \frac{\lambda+1}{3}\overrightarrow{AB}$ for $\lambda \neq -1$ and *A* is a common point, this shows that *A*, *B* and *P* are collinear.

2(ii)	Given that area of triangle *OAP* is $162\sqrt{5}$, $\frac{1}{2}\left\lvert\overrightarrow{OA}\times\overrightarrow{AP}\right\rvert=162\sqrt{5}$ $\frac{1}{2}\left\lvert\begin{pmatrix}-1\\-3\\2\end{pmatrix}\times\frac{\lambda+1}{3}\overrightarrow{AB}\right\rvert=162\sqrt{5}$ $\frac{1}{2}\left\lvert\begin{pmatrix}-1\\-3\\2\end{pmatrix}\times\frac{\lambda+1}{3}\begin{pmatrix}6\\3\\0\end{pmatrix}\right\rvert=162\sqrt{5}$ $\lvert\lambda+1\rvert\left\lvert\begin{pmatrix}-1\\-3\\2\end{pmatrix}\times\begin{pmatrix}2\\1\\0\end{pmatrix}\right\rvert=324\sqrt{5}$ $\lvert\lambda+1\rvert\left\lvert\begin{pmatrix}-2\\4\\5\end{pmatrix}\right\rvert=324\sqrt{5}$ $\lvert\lambda+1\rvert\sqrt{(-2)^2+(4)^2+(5)^2}=324\sqrt{5}$ $\lvert\lambda+1\rvert\sqrt{45}=324\sqrt{5}$ $\lvert\lambda+1\rvert=108$ $\lambda+1=108$ or $\lambda+1=-108$ $\lambda=107$ or $\lambda=-109$ Since <u>*P* is on *BA* produced</u>, $\overrightarrow{AP}=k\overrightarrow{AB}$ for a negative value of *k*. Hence $\lambda=-109$.
3 1^{st} part	$\mathrm{f}(x)=\mathrm{e}^{x^2}\left(\sqrt{1+2x}\right)$ $=\left(1+x^2+\cdots\right)\left(1+\frac{1}{2}(2x)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}(2x)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(2x)^3+\cdots\right)$ $=\left(1+x^2+\cdots\right)\left(1+x-\frac{1}{2}x^2+\frac{1}{2}x^3+\cdots\right)$ $=\left(1+x-\frac{1}{2}x^2+\frac{1}{2}x^3+\cdots\right)+\left(x^2+x^3+\cdots\right)$ $=1+x+\frac{1}{2}x^2+\frac{3}{2}x^3+\cdots$
3(a)	When $x=\frac{1}{3}$,

	$\mathrm{e}^{\left(\frac{1}{3}\right)^2}\sqrt{1+2\left(\frac{1}{3}\right)} \approx 1+\frac{1}{3}+\frac{1}{2}\left(\frac{1}{3}\right)^2+\frac{3}{2}\left(\frac{1}{3}\right)^3$ $\mathrm{e}^{\frac{1}{9}}\sqrt{\frac{5}{3}} \approx 1+\frac{1}{3}+\frac{1}{2}\left(\frac{1}{3}\right)^2+\frac{3}{2}\left(\frac{1}{3}\right)^3=\frac{13}{9}$ $\sqrt{135}\mathrm{e}^{\frac{1}{9}}=9\sqrt{\frac{5}{3}}\mathrm{e}^{\frac{1}{9}} \approx 9\times\frac{13}{9}=13$
3(b)	$\mathrm{f}'(x)=1+x+\frac{9}{2}x^2+\cdots$ Using $\mathrm{f}'(x)=1+x+\frac{9}{2}x^2+\cdots$, $\mathrm{f}'(x)=2x\mathrm{e}^{x^2}\sqrt{1+2x}+\frac{\mathrm{e}^{x^2}}{\sqrt{1+2x}}$ $\mathrm{f}'(x)=2x\,\mathrm{f}(x)+\frac{\mathrm{e}^{x^2}}{\sqrt{1+2x}}$ $\frac{\mathrm{e}^{x^2}}{\sqrt{1+2x}}=\mathrm{f}'(x)-2x\,\mathrm{f}(x)$ $=\frac{\mathrm{d}}{\mathrm{d}x}\left(1+x+\frac{1}{2}x^2+\frac{3}{2}x^3+\cdots\right)$ $-2x\left(1+x+\frac{1}{2}x^2+\frac{3}{2}x^3+\cdots\right)$ $=\left(1+x+\frac{9}{2}x^2+\cdots\right)-\left(2x+2x^2+\cdots\right)$ $=1-x+\frac{5}{2}x^2+\cdots$ **Alternative method:** $\frac{\mathrm{e}^{x^2}}{\sqrt{1+2x}}=\mathrm{e}^{x^2}\left(\sqrt{1+2x}\right)(1+2x)^{-1}$ $=\left(1+x+\frac{1}{2}x^2+\frac{3}{2}x^3+\cdots\right)\left(1-2x+4x^2+\cdots\right)$ $=1-2x+4x^2+x-2x^2+\frac{1}{2}x^2$ $=1-x+\frac{5}{2}x^2+\cdots$

4(i)	$x=ut^2$ $\frac{\mathrm{d}x}{\mathrm{d}t}=2tu+t^2\frac{\mathrm{d}u}{\mathrm{d}t}$ $t^2\left(2tu+t^2\frac{\mathrm{d}u}{\mathrm{d}t}\right)-2t\left(ut^2\right)+\left(ut^2\right)^2=0$ $t^4\frac{\mathrm{d}u}{\mathrm{d}t}=-u^2t^4$ $\frac{\mathrm{d}u}{\mathrm{d}t}=-u^2\ \text{(shown)}$
4(ii)	$\int\frac{1}{u^2}\ \mathrm{d}u=\int-1\mathrm{d}t$ $-\frac{1}{u}=-t+C'$ $\frac{t^2}{x}=t-C'$ $x=\frac{t^2}{t+C}$ where $C=-C'$ Since there was 0.2 milligrams of bacteria after 15 minutes, then $0.2=\frac{(0.25)^2}{0.25+C}\Rightarrow 0.05+0.2C=0.0625$ $\Rightarrow C=\frac{1}{16}$ $\therefore x=\frac{16t^2}{16t+1}$
	When t = 4, $\therefore x=\frac{16(4)^2}{16(4)+1}=\frac{256}{65}$ or 3.94
4(iii)	As $t\to\infty,\ \frac{16t^2}{16t+1}\to\infty.$ The particular solution of the DE suggests that the **<u>amount of bacteria in the Petri dish will grow indefinitely as time passes</u>**. Hence the model is not a realistic one.

5(i)	
5(ii)	$x = \cos 2t, \qquad y = \tan t$ $\frac{dx}{dt} = -2\sin 2t, \quad \frac{dy}{dt} = \sec^2 t$ $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{\sec^2 t}{-2\sin 2t}$ When $t = \frac{\pi}{3}$, $P(-0.5, \sqrt{3})$ and $\frac{dy}{dx} = \frac{\sec^2 \frac{\pi}{3}}{-2\sin\frac{2\pi}{3}} = -\frac{4}{\sqrt{3}}$ So, $\frac{\sqrt{3}-0}{-0.5-b} = \frac{\sqrt{3}}{4}$ $b = -4.5$
5(iii)	 $x = \cos 2t = 0 \Rightarrow t = \frac{\pi}{4} \Rightarrow y = \tan\frac{\pi}{4} = 1$

Area

$= \left|\int_1^4 x\mathrm{d}y\right|$ or $-\int_1^4 x\mathrm{d}y$

$= -\int_{\frac{\pi}{4}}^{\tan^{-1}4} \cos 2t \sec^2 t\mathrm{d}t$

$= -\int_{\frac{\pi}{4}}^{\tan^{-1}4} \left(2\cos^2 t - 1\right)\sec^2 t\mathrm{d}t$

$= -\int_{\frac{\pi}{4}}^{\tan^{-1}4} 2 - \sec^2 t\mathrm{d}t$

$= -\left[2t - \tan t\right]_{\frac{\pi}{4}}^{\tan^{-1}4}$

$= -(2\tan^{-1} 4 - 4) + (\frac{\pi}{2} - 1)$

$= 3 + \frac{\pi}{2} - 2\tan^{-1} 4$

6

$x = 1$ --- (1)

$2x + y + az = 5$ --- (2)

Substitute (1) into (2):

$2 + y + az = 5$

$y = 3 - az$

Let $z = \lambda$,

Hence, $\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 - a\lambda \\ \lambda \end{pmatrix}$, where λ is a real number. (Shown)

6(a)

The angle between l and p_3 is $60°$. This implies

$$\sin 60° = \frac{\left|\begin{pmatrix} 0 \\ -a \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}\right|}{\left(\sqrt{a^2+1}\right)\left(\sqrt{1^2+2^2+1^2}\right)}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\left|-2a+1\right|}{\left(\sqrt{a^2+1}\right)\left(\sqrt{6}\right)}$$

$$\Rightarrow 4\left(4a^2 - 4a + 1\right) = 18\left(a^2+1\right)$$

$$\Rightarrow 16a^2 - 16a + 4 = 18a^2 + 18$$

$$\Rightarrow 2a^2 + 16a + 14 = 0$$

$$\Rightarrow a^2 + 8a + 7 = 0$$

$$\Rightarrow (a+7)(a+1) = 0$$

$$\Rightarrow a = -7 \quad \text{or} \quad a = -1$$

6(b)	Since $\overrightarrow{ON}$ is parallel to the normal vector of p_3, $\overrightarrow{ON}$ $=\pm\frac{\sqrt{6}}{3}\left(\frac{1}{\sqrt{1^2+2^2+1^2}}\begin{pmatrix}1\\2\\1\end{pmatrix}\right)$ $=\pm\frac{\sqrt{6}}{3}\left(\frac{1}{\sqrt{6}}\begin{pmatrix}1\\2\\1\end{pmatrix}\right)$ $=\begin{pmatrix}1/3\\2/3\\1/3\end{pmatrix}$ or $\begin{pmatrix}-1/3\\-2/3\\-1/3\end{pmatrix}$ Alternative Method: Let N be the foot of perpendicular from the origin to p_3. Since $\overrightarrow{ON}$ is parallel to the normal vector of p_3, and $\left\|\overrightarrow{ON}\right\|=\frac{\sqrt{6}}{3}$ $\sqrt{\alpha^2+4\alpha^2+\alpha^2}=\frac{\sqrt{6}}{3}$ $\sqrt{6\alpha^2}=\frac{\sqrt{6}}{3}$, $6\alpha^2=\frac{6}{9}$ $\alpha=\pm\frac{1}{3}$ $\overrightarrow{ON}=\begin{pmatrix}1/3\\2/3\\1/3\end{pmatrix}$ or $\begin{pmatrix}-1/3\\-2/3\\-1/3\end{pmatrix}$
6(c)	Given that p_1 , p_2 and p_3 do not have common point, then line l must be parallel to p_3 . Hence $\begin{pmatrix}0\\-a\\1\end{pmatrix}\cdot\begin{pmatrix}1\\2\\1\end{pmatrix}=0$

	$-2a+1=0$ $a=\frac{1}{2}$ Also, a point (1, 3, 0) in l must not lie in p_3. Hence $\begin{pmatrix}1\\3\\0\end{pmatrix}\cdot\begin{pmatrix}1\\2\\1\end{pmatrix}\neq b$ $b\neq 7$
7 (a)(i)	$r=\frac{a+d}{a+3d}=\frac{a+3d}{a+8d}$ $\Rightarrow (a+d)(a+8d)=(a+3d)^2$ $\Rightarrow a^2+9ad+8d^2=a^2+6ad+9d^2$ $\Rightarrow d^2-3ad=0$ $\Rightarrow d(d-3a)=0$ $\Rightarrow d=0$ (rej.) or $d=3a$ (shown) Alternatively, let b be the first term of the geometric series. Then $d=\frac{b-br}{5}=\frac{br-br^2}{2}$ $\Rightarrow 2b-2br=5br-5br^2$ $\Rightarrow 5r^2-7r+2=0$ $\Rightarrow (5r-2)(r-1)=0$ $\Rightarrow r=\frac{2}{5}$ or $r=1$ (rej because otherwise $d=0$) Hence $d=\frac{b-b\left(\frac{2}{5}\right)}{5}=-\frac{3}{25}b=-\frac{3}{25}(a+8d)$ $25d=-3a-24d$ $d=3a$ (shown)
7 (a) (ii)	$r=\frac{a+d}{a+3d}=\frac{a+3a}{a+9a}=\frac{4a}{10a}=\frac{2}{5}$. Since $\left\lvert\frac{2}{5}\right\rvert<1,$ the geometric series is convergent.

	Sum to infinity $= \dfrac{a+8d}{1-r}$ $= \dfrac{a+24a}{1-\frac{2}{5}}$ $= \dfrac{5}{3}(25a)$ $= \dfrac{125}{3}a$
7(b) (i)	The distance the mountaineer climbs for each hour follows an arithmetic progression with first term 300 metres and common difference (– 10) metres. Total distance travelled after n hours $\le x$ $\frac{n}{2}\left[2(300)+(n-1)(-10)\right] \le x$ $\frac{n}{2}(600-10n+10) \le x$ $\frac{n}{2}(610-10n) \le x$ $n(305-5n) \le x$ $-5n^2+305n \le x$ (shown) $p=-5,\ q=305$
7(b) (ii)	If x = 2500, then $-5n^2+305n \le 2500$ $-5n^2+305n-2500 \le 0$ $n \le 9.757$ or $n \ge 51.24$ Hence $n=9$.
8	$\mathrm{p}(-3)=0$ $\Rightarrow (-3)^3+m(-3)^2-7(-3)+15=0$ $\Rightarrow -27+9m+21+15=0$ $\therefore m=-1$
8(i)	$z^3-z^2-7z+15=0$ $(z+3)(z^2-4z+5)=0$ $z=-3$ or $z=\dfrac{-(-4)\pm\sqrt{16-4(1)(5)}}{2(1)}$ $=\dfrac{4\pm\sqrt{-4}}{2}$ $=2\pm \mathrm{i}$ $\therefore z_1=2-\mathrm{i},\ \ z_2=2+\mathrm{i},\ \ z_3=-3$

8(ii)	Since z_1 and z_2 are complex conjugates, $\left(z_1^{\ n}\right)^* = \left(z_1^{\ *}\right)^n = z_2^{\ n}$. $z_1^{\ n}$ and $z_2^{\ n}$ are complex conjugates as well. Thus $z_1^{\ n} - z_2^{\ n} = z_1^{\ n} - \left(z_1^{\ n}\right)^*$ $\qquad\qquad = (2\mathrm{i})\,\mathrm{Im}\left(z_1^{\ n}\right)$ which is purely imaginary. **Alternative method:** $z_1^{\ n} - z_2^{\ n} = (2-\mathrm{i})^n - (2+\mathrm{i})^n$ $\qquad = \left(\sqrt{5}\mathrm{e}^{-\mathrm{i}\left(\tan^{-1}0.5\right)}\right)^n - \left(\sqrt{5}\mathrm{e}^{\mathrm{i}\left(\tan^{-1}0.5\right)}\right)^n$ $\qquad = 5^{\frac{n}{2}}\left(\mathrm{e}^{-\mathrm{i}\left(\tan^{-1}0.5\right)n} - \mathrm{e}^{\mathrm{i}\left(\tan^{-1}0.5\right)n}\right)$ $\qquad = 5^{\frac{n}{2}}\begin{pmatrix}\cos\left(n\tan^{-1}0.5\right) - \mathrm{i}\sin\left(n\tan^{-1}0.5\right) \\ -\cos\left(n\tan^{-1}0.5\right) - \mathrm{i}\sin\left(n\tan^{-1}0.5\right)\end{pmatrix}$ $\qquad = -(2)5^{\frac{n}{2}}\mathrm{i}\sin\left(n\tan^{-1}0.5\right)$ Since $\mathrm{Re}\left(z_1^{\ n} - z_2^{\ n}\right) = 0$, $z_1^{\ n} - z_2^{\ n}$ is purely imaginary.
8(iii)	Since $\lvert z_1\rvert = \sqrt{5}, \lvert z_3\rvert = 3, \lvert z_1\rvert \neq \lvert z_3\rvert$. The locus of complex numbers satisfying the equation $\lvert w\rvert = a$, for some positive constant *a*, will not pass through all the points representing the complex numbers z_1, z_2 and z_3.
9(i)	Let P_n be the statement $u_n = \dfrac{1}{n(n+1)}$, where $n \in \mathbb{Z}^+$. When $n = 1$, $LHS = u_1 = \dfrac{1}{2}$ (Given) $RHS = \dfrac{1}{(1)(1+1)} = \dfrac{1}{2}$ Hence, P_1 is true. Suppose/assume P_k is true for **some** $k \in \mathbb{Z}^+$, i.e. $u_k = \dfrac{1}{k(k+1)}$.

	Need to show that P_{k+1} is also true, i.e. $u_{k+1}=\frac{1}{(k+1)(k+2)}$ $u_{k+1}=u_k-\frac{2}{k(k+1)(k+2)}$ $=\frac{1}{k(k+1)}-\frac{2}{k(k+1)(k+2)}$ $=\frac{(k+2)-2}{k(k+1)(k+2)}$ $=\frac{k}{k(k+1)(k+2)}$ $=\frac{1}{(k+1)(k+2)}$ Hence P_{k+1} is true if P_k is true. Since P_1 is true and P_k is true implies that P_{k+1} is true, by mathematical induction, $u_n=\frac{1}{n(n+1)}$ for all positive integers *n*.
9(ii)	$\sum_{r=1}^{N}\frac{1}{r(r+1)(r+2)}=\frac{1}{2}\sum_{r=1}^{N}\left(u_r-u_{r+1}\right)$ $=\frac{1}{2}(u_1-u_2$ $+u_2-u_3$ $+u_3-u_4$ $\vdots$ $+u_N-u_{N+1})$ $=\frac{1}{2}\left(u_1-u_{N+1}\right)$ $=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{(N+1)(N+2)}\right)$
9(iii)	$\frac{1}{3^3}+\frac{1}{4^3}+\frac{1}{5^3}\ldots=\lim_{N\to\infty}\sum_{r=1}^{N}\frac{1}{\left(r+2\right)^3}$ For $r\in\mathbf{Z}^+$, $(r+2)^3>r(r+1)(r+2)$ OR $\frac{1}{(r+2)^3}<\frac{1}{r(r+1)(r+2)}$ OR $\sum_{r=1}^{N}\frac{1}{(r+2)^3}<\sum_{r=1}^{N}\frac{1}{r(r+1)(r+2)}=\frac{1}{4}-\frac{1}{2(N+1)(N+2)}<\frac{1}{4}$ Hence $\lim_{N\to\infty}\sum_{r=1}^{N}\frac{1}{\left(r+2\right)^3}<\lim_{n\to\infty}\frac{1}{4}=\frac{1}{4}$

9(iv)	Let $r-2=j$. Then $r-1=j+1$ and $r=j+2$. $\sum_{r=10}^{N}\frac{1}{r(r-1)(r-2)}=\sum_{j+2=10}^{j+2=N}\frac{1}{j(j+1)(j+2)}$ $=\sum_{j=8}^{j=N-2}\frac{1}{j(j+1)(j+2)}$ $=\sum_{j=1}^{j=N-2}\frac{1}{j(j+1)(j+2)}-\sum_{j=1}^{j=7}\frac{1}{j(j+1)(j+2)}$ $=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{N(N-1)}\right)-\frac{1}{2}\left(\frac{1}{2}-\frac{1}{(8)(9)}\right)$ $=\frac{1}{144}-\frac{1}{2N(N-1)}$
10 (a)(i)	$y=\frac{2x^2-a^2x+4a^2}{x^2-a^2}$ By long division, $y=2+\frac{6a^2-a^2x}{x^2-a^2}$ Vertical asymptotes: $x=-a$ or $x=a$ Horizontal asymptote: $y=2$
10 (a) (ii)	From $y=2+\frac{6a^2-a^2x}{x^2-a^2}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=0+\frac{\left(-a^2\right)\left(x^2-a^2\right)-\left(6a^2-a^2x\right)\left(2x\right)}{\left(x^2-a^2\right)^2}$ $=\frac{\left(a^2\right)\left(-x^2+a^2-12x+2x^2\right)}{\left(x^2-a^2\right)^2}$ $=\frac{\left(a^2\right)\left(2x^2-12x+a^2\right)}{\left(x^2-a^2\right)^2}$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=0$$

$$\Rightarrow \frac{\left(a^2\right)\left(x^2-12x+a^2\right)}{\left(x^2-a^2\right)^2}=0$$

$$\Rightarrow x^2-12x+a^2=0$$

Given C has two turning points,

$$\therefore b^2-4ac>0$$

$$\Rightarrow (-12)^2-4(1)\left(a^2\right)>0$$

$$144-4a^2>0$$

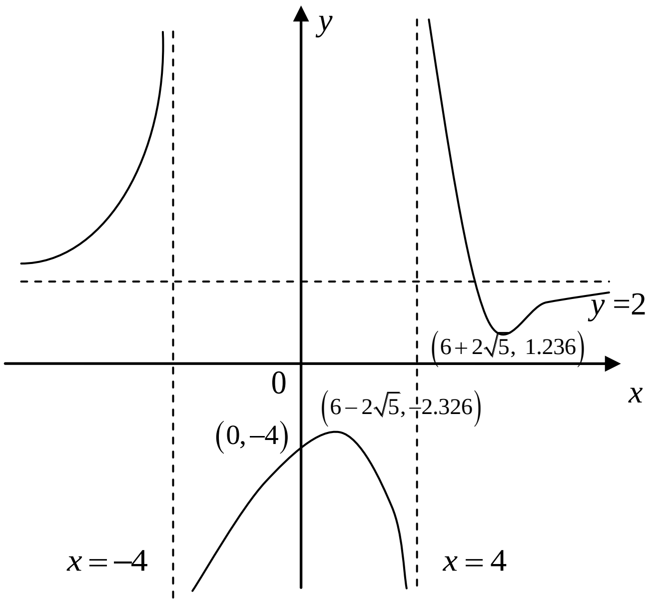
$$a^2-36<0$$

$$(a-6)(a+6)<0$$

$$-6<a<6$$

Since a is a positive constant, $0<a<6$. (Shown)

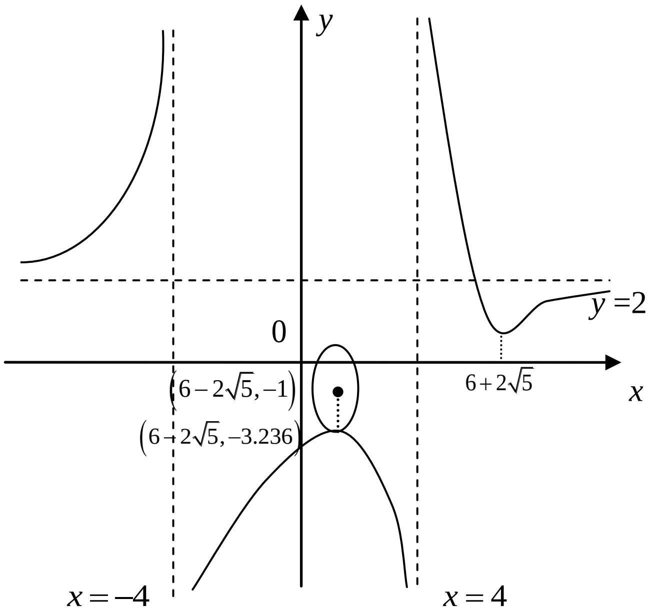
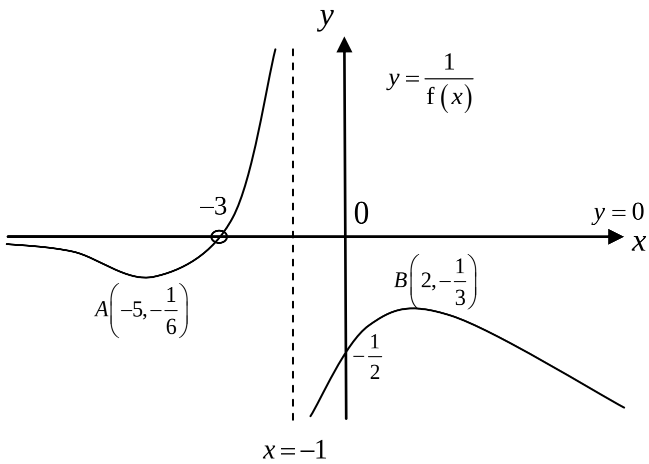
10



$$h^2\left(x-6+2\sqrt{5}\right)^2+(y+1)^2=h^2$$

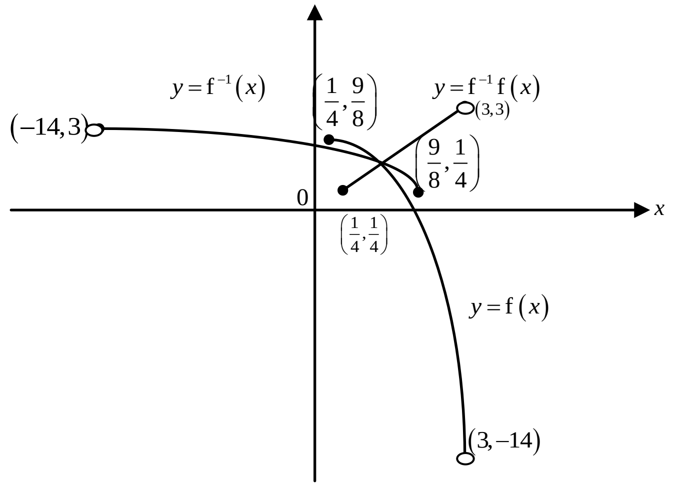
$$\Rightarrow \left(x-6+2\sqrt{5}\right)^2+\frac{(y+1)^2}{h^2}=1$$

It is an ellipse centred at $\left(6-2\sqrt{5},-1\right)$.

	Maximum turning point of C occurs at $\left(6-2\sqrt{5},-3.236\right)$.  For the ellipse to intersect the curve C more than once, $h>2.236.$. Since h is a positive integer, $h\geq 3$.
10 (b)	

y

11(i)



$\mathrm{f}^2(x) = x$

$\mathrm{f}(x) = \mathrm{f}^{-1}(x)$

$\mathrm{f}(x) = x$

$-2x^2 + x + 1 = x$

$2x^2 = 1$

$x = \dfrac{\sqrt{2}}{2}$ or $-\dfrac{\sqrt{2}}{2}$

Since $x \geq \dfrac{1}{4}$, $x = \dfrac{\sqrt{2}}{2}$.

11 (ii)

$R_g = (-\ln 2, \ln(1-2k)) \subseteq (-14, 9/8] = D_{f^{-1}}$

For least k, $\ln(1-2k) = 9/8$

$k = (1 - e^{9/8})/2$ or -1.03

11 (iii)

$\mathrm{f}(x) \leq \mathrm{h}(x)$

$x - 2x^2 + 1 \leq 18|x-2| - 18$

Consider $x \geq 2$
$x - 2x^2 + 1 \leq 18(x-2) - 18$
$2x^2 + 17x - 55 \geq 0$
$(2x-5)(x+11) \geq 0$
$x \leq -11$ or $x \geq 2.5$
Then $x \geq 2.5$

	Consider $x \le 2$ $x - 2x^2 + 1 \le 18(-x + 2) - 18$ $2x^2 - 19x + 17 \ge 0$ $(x - 8.5)(x - 1) \ge 0$ $x \le 1$ or $x \ge 8.5$ Then $x \le 1$ $\frac{1}{4} \le x \le 1$ or $2.5 \le x < 3$