H2 Topic 2

Kinematics

Circus stuntmen being shot out of cannons are human projectiles. In the absence of air resistance, they move in parabolic paths until they land in a strategically placed net.

Taken from College Physics, Serway/Faughn

Content

- Rectilinear motion
- Non-linear motion

Learning Outcomes

Candidates should be able to:

(a) Define displacement, speed, velocity and acceleration.

(b) Use graphical methods to represent distance travelled, displacement, speed, velocity and acceleration.

(c) Find displacement from the area under a velocity-time graph.

(d) Use the slope of a displacement-time graph to find the velocity.

(e) Use the slope of a velocity-time graph to find the acceleration.

(f) Derive, from the definitions of velocity and acceleration, equations which represent uniformly accelerated motion in a straight line.

(g) Solve problems using equations which represent uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance.

(h) Describe qualitatively the motion of bodies falling in a uniform gravitational field with air resistance.

(i) Describe and explain motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular direction.

References:

1 Advanced Level Physics by Loo Kwok Wai
2 College Physics by Young and Geller
3 Physics for Scientist and Engineers (5th Edition) by Serway

Physics Tip!

To understand Physics, you must be able to understand it

(i) **Qualitatively –** Define, Explain, State, Describe, Discuss, Suggest…
(ii) **Quantitatively –** Calculate, Measure, Determine, Show, Estimate, Suggest…
(iii) **Graphically –** Sketch, Determine, Estimate, Deduce…
(iv) **Pictorially –** Sketch (applied to diagrams)

In plain English, you must be able to explain Physics in words, equations & numbers, graphs and diagrams.

Data

speed of light in free space,	c	$= 3.00 \times 10^{8}$ m s^{-1}
permeability of free space,	μ_o	$= 4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space,	ε_o	$= 8.85 \times 10^{-12}$ F m^{-1}
		$(1/(36\pi)) \times 10^{-9}$ F m^{-1}
elementary charge,	e	$= 1.60 \times 10^{-19}$ C
the Planck constant,	h	$= 6.63 \times 10^{-34}$ J s
unified atomic mass constant,	u	$= 1.66 \times 10^{-27}$ kg
rest mass of electron,	m_e	$= 9.11 \times 10^{-31}$ kg
rest mass of proton,	m_p	$= 1.67 \times 10^{-27}$ kg
molar gas constant	R	$= 8.31$ J K^{-1} mol^{-1}
the Avogadro constant,	N_A	$= 6.02 \times 10^{23}$ mol^{-1}
the Boltzmann constant,	k	$= 1.38 \times 10^{-23}$ J K^{-1}
gravitational constant,	G	$= 6.67 \times 10^{-11}$N m^{2} kg^{2}
acceleration of free fall,	g	$= 9.81$ m s^{-2}

Formulae

uniformly accelerated motion,	s	$= ut + \frac{1}{2}at^2$
	v^2	$= u^2 + 2as$
work done on/by a gas,	W	$= p\Delta V$
hydrostatic pressure,	p	$= \rho gh$
gravitational potential,	ϕ	$= -\frac{GM}{r}$
displacement of particle in s.h.m.	x	$= x_o \sin \omega t$
velocity of particle in s.h.m.,	v	$= v_o \cos \omega t$
		$= \pm\omega\sqrt{x_o^2 - x^2}$
mean kinetic energy of a molecule of an ideal gas	E	$= \frac{3}{2}kT$
resistors in series,	R	$= R_1 + R_2 + \ldots$
resistors in parallel,	$\frac{1}{R}$	$= \frac{1}{R_1} + \frac{1}{R_2} + \ldots$
electric potential,	V	$= \frac{Q}{4\pi\varepsilon_o r}$
alternating current/voltage,	x	$= x_o \sin \omega t$
transmission coefficient,	T	$\propto \exp(-2kd)$
	where k	$= \sqrt{\frac{8\pi^2 m(U-E)}{h^2}}$
radioactive decay,	x	$= x_o \exp(-\lambda t)$
decay constant,	λ	$= \frac{0.693}{t_{\frac{1}{2}}}$

MATHEMATICAL REQUIREMENTS

Arithmetic

Candidates should be able to:

(a) recognise and use expressions in decimal and standard form (scientific) notation.

(b) use appropriate calculating aids (electronic calculator or tables) for addition, subtraction, multiplication and division. Find arithmetic means, powers (including reciprocals and square roots), sines, cosines, tangents (and the inverse functions), exponentials and logarithms (lg and ln).

(c) take account of accuracy in numerical work and handle calculations so that significant figures are neither lost unnecessarily nor carried beyond what is justified.

(d) make approximate evaluations of numerical expressions (e.g. $\pi^2 = 10$) and use such approximations to check the magnitude of machine calculations.

Algebra

Candidates should be able to:

(a) change the subject of an equation. Most relevant equations involve only the simpler operations but may include positive and negative indices and square roots.

(b) solve simple algebraic equations. Most relevant equations are linear but some may involve inverse and inverse square relationships. Linear simultaneous equations and the use of the formula to obtain the solutions of quadratic equations are included.

(c) substitute physical quantities into physical equations using consistent units and check the dimensional consistency of such equations.

(d) formulate simple algebraic equations as mathematical models of physical situations, and identify inadequacies of such models.

(e) recognise and use the logarithmic forms of expressions like ab, a/b, x^n, e^{kx}; understand the use of logarithms in relation to quantities with values that range over several orders of magnitude.

(f) manipulate and solve equations involving logarithmic and exponential functions.

(g) express small changes or errors as percentages and *vice versa.*

(h) comprehend and use the symbols $<$, $>$, $\leqslant$, $\geqslant$, $\ll$, $\gg$, $\approx$, $/$, $\propto$, $\langle x \rangle$ ($= \overline{x}$), Σ, Δx, δx, $\sqrt{}$.

Geometry and trigonometry

Candidates should be able to:

(a) calculate areas of right-angled and isosceles triangles, circumference and area of circles, areas and volumes of rectangular blocks, cylinders and spheres.

(b) use Pythagoras' theorem, similarity of triangles, the angle sum of a triangle.

(c) use sines, cosines and tangents (especially for 0°, 30°, 45°, 60°, 90°). Use the trigonometric relationships for triangles:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C};\ a^2 = b^2 + c^2 - 2bc\cos A$$

(d) use $\sin\theta \approx \tan\theta \approx \theta$ and $\cos\theta \approx 1$ for small θ; $\sin^2\theta + \cos^2\theta = 1$.

(e) understand the relationship between degrees and radians (defined as arc/radius), translate from one to the other and use the appropriate system in context.

Vectors

Candidates should be able to:

(a) find the resultant of two coplanar vectors, recognising situations where vector addition is appropriate.

(b) obtain expressions for components of a vector in perpendicular directions, recognising situations where vector resolution is appropriate.

Graphs

Candidates should be able to:

(a) translate information between graphical, numerical, algebraic and verbal forms.

(b) select appropriate variables and scales for graph plotting.

(c) for linear graphs, determine the slope, intercept and intersection.

(d) choose, by inspection, a straight line which will serve as the line of best fit through a set of data points presented graphically.

(e) recall standard linear form $y = mx + c$ and rearrange relationships into linear form where appropriate.

(f) sketch and recognise the forms of plots of common simple expressions like $1/x$, x^2, $1/x^2$, $\sin x$, $\cos x$, e^{-x}.

(g) use logarithmic plots to test exponential and power law variations.

(h) understand, draw and use the slope of a tangent to a curve as a means to obtain the gradient, and use notation in the form dy/dx for a rate of change.

(i) understand and use the area below a curve where the area has physical significance.

Any calculator used must be on the Singapore Examinations and Assessment Board list of approved calculators.

ANNEX A

LIST OF APPROVED SCIENTIFIC CALCULATORS

Not to worry about the approved date. It is up to the vendor to apply to SEAB for approval

The following models of Scientific Calculators are suitable for:

- **PSLE Mathematics and Foundation Mathematics Examinations**
- **GCE N(T), N(A), O and A-Level Examinations**

S/N	Calculator Brand	Calculator Model	For use until Year
1	AURORA	SC 550	2013
2	FIAMO	SC 6	2013
3		SC 20	2013
4	CANON	F-715S	2013
5	CASIO	FX 82AU	2013
6		FX 82MS	2013
7		FX 85MS	2013
8		FX 95MS	2013
9		FX 95 SG Plus	2013
10		FX 350MS	2013
11		FX 820MS	2013
12		FX 992S	2013
13		FX 96 SG Plus	2017
14	HEWLETT PACKARD	HP 8S	2013
15		HP 9S	2013
16		HP10S	2013
17	HOSEKI	H-1030	2013
18		H-1031	2013
19	HUBBLE COMPUTING	SC-10B	2015
20	REDSPOT COMPUTING	SC-10B	2015
21		SC-10C	2015
22	SHARP	EL 509VM	2013
23		EL 509 W	2013
24		EL 509WM	2013
25		EL 509WS	2013
26		EL 546VA	2013
27		ELW531S	2014
28		ELW531M	2016
29		EL 533X	2017
30	TEXAS INSTRUMENTS	TI 30XIIB	2013
31		TI 30XIIS	2013
32		TI 34 II	2013

LIST OF APPROVED GRAPHING CALCULATORS

The following models of Graphing Calculators are approved for use ONLY in subjects examined at H1, H2 and H3 Levels of the A-Level curriculum.
Note: All graphing calculators must be reset prior to any examination.

S/N	Calculator Brand	Calculator Model	For use until Year
1	CASIO	CFX-9850GC PLUS	2013
2		FX-9860G	2013
3		FX-9860G Slim	2013
4	TEXAS INSTRUMENTS	TI-83 PLUS	2013
5		TI-84 PLUS	2015
6		TI-84 PLUS Silver Edition	2015
7		TI-84 PLUS Pocket SE	2016

GLOSSARY OF TERMS USED IN PHYSICS PAPERS

It is hoped that the glossary will prove helpful to candidates as a guide, although it is not exhaustive. The glossary has been deliberately kept brief not only with respect to the number of terms included but also to the descriptions of their meanings. Candidates should appreciate that the meaning of a term must depend in part on its context. They should also note that the number of marks allocated for any part of a question is a guide to the depth of treatment required for the answer.

1. *Define (the term(s) …)* is intended literally. Only a formal statement or equivalent paraphrase, such as the defining equation with symbols identified, being required.

2. *What is meant by …* normally implies that a definition should be given, together with some relevant comment on the significance or context of the term(s) concerned, especially where two or more terms are included in the question. The amount of supplementary comment intended should be interpreted in the light of the indicated mark value.

3. *Explain* may imply reasoning or some reference to theory, depending on the context.

4. *State* implies a concise answer with little or no supporting argument, e.g. a numerical answer that can be obtained 'by inspection'.

5. *List* requires a number of points with no elaboration. Where a given number of points is specified, this should not be exceeded.

6. *Describe* requires candidates to state in words (using diagrams where appropriate) the main points of the topic. It is often used with reference either to particular phenomena or to particular experiments. In the former instance, the term usually implies that the answer should include reference to (visual) observations associated with the phenomena. The amount of description intended should be interpreted in the light of the indicated mark value.

7. *Discuss* requires candidates to give a critical account of the points involved in the topic.

8. *Deduce/Predict* implies that candidates are not expected to produce the required answer by recall but by making a logical connection between other pieces of information. Such information may be wholly given in the question or may depend on answers extracted in an earlier part of the question.

9. *Suggest* is used in two main contexts. It may either imply that there is no unique answer or that candidates are expected to apply their general knowledge to a 'novel' situation, one that formally may not be 'in the syllabus'.

10. *Calculate* is used when a numerical answer is required. In general, working should be shown.

11. *Measure* implies that the quantity concerned can be directly obtained from a suitable measuring instrument, e.g. length, using a rule, or angle, using a protractor.

12. *Determine* often implies that the quantity concerned cannot be measured directly but is obtained by calculation, substituting measured or known values of other quantities into a standard formula, e.g. the Young modulus, relative molecular mass.

13. *Show* is used when an algebraic deduction has to be made to prove a given equation. It is important that the terms being used by candidates are stated explicitly.

14. *Estimate* implies a reasoned order of magnitude statement or calculation of the quantity concerned. Candidates should make such simplifying assumptions as may be necessary about points of principle and about the values of quantities not otherwise included in the question.

15. *Sketch,* when applied to graph work, implies that the shape and/or position of the curve need only be qualitatively correct. However, candidates should be aware that, depending on the context, some quantitative aspects may be looked for, e.g. passing through the origin, having an intercept, asymptote or discontinuity at a particular value. On a sketch graph it is essential that candidates clearly indicate what is being plotted on each axis.

16. *Sketch,* when applied to diagrams, implies that a simple, freehand drawing is acceptable: nevertheless, care should be taken over proportions and the clear exposition of important details.

17. *Compare* requires candidates to provide both similarities and differences between things or concepts.

TEXTBOOKS

Teachers may find reference to the following books helpful.

Practice in Physics (3rd Edition), by Akrill et al, published by Hodder & Stoughton, ISBN 0-340-75813-9

New Understanding Physics for Advanced Level (4th Edition), by J. Breithaupt, published by Nelson Thornes, ISBN 0-748-74314-6

Advanced Physics (4th Edition), by T. Duncan, published by John Murray, ISBN 0-719-57669-5

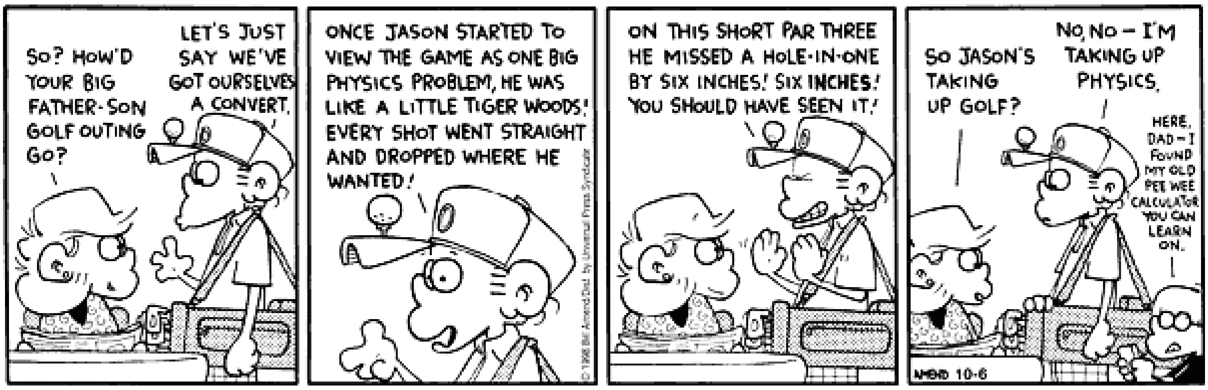
Advanced Physics (2nd Edition), by K. Gibbs, published by Cambridge University Press, ISBN 0-521-56701-7

Bath Advanced Science: Physics (2nd Edition), by R. Hutchings, published by Nelson Thornes, ISBN 0-174-38731-8

Physics for Scientists and Engineers with Modern Physics (5th Edition), by R. Serway, published by Saunders, ISBN 0-030-20974-9

Fundamental of Physics (Extended – 6th Edition), by R. Resnick, D. Halliday & J. Walker, published by Wiley, ISBN 0-471-22863-X

Physics: Principles with Applications (5th Edition), by D.C. Giancoli, published by Prentice Hall, ISBN 0-13611-971-9



Kinematics is a branch of Newtonian (or classical) mechanics concerned with describing the motion of objects without considering factors that cause the motions.

In this topic we will concentrate on two types of motion,

- Motion along a straight line (Rectilinear motion)
- Motion on a plane (Non-linear motion)

The following vector and scalar quantities are usually used to describe motion:

- Distance (scalar)
- Displacement (vector)
- Speed (scalar)
- Velocity (vector)
- Acceleration (vector)

2.1 Rectilinear Motion

Learning Outcomes
(a) Define displacement, speed, velocity and acceleration.

2.1.1 Distance and Displacement

Distance, d	The total length of path covered by a moving object irrespective of the direction of motion.

Distance is a <u>scalar</u> quantity.

Displacement, s	The linear distance of the position of the moving object from a given reference point.

Displacement is a <u>vector</u> quantity.

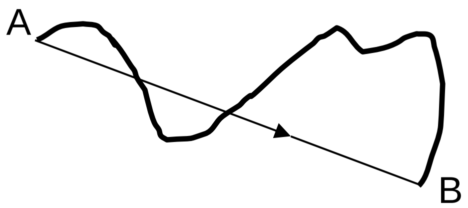


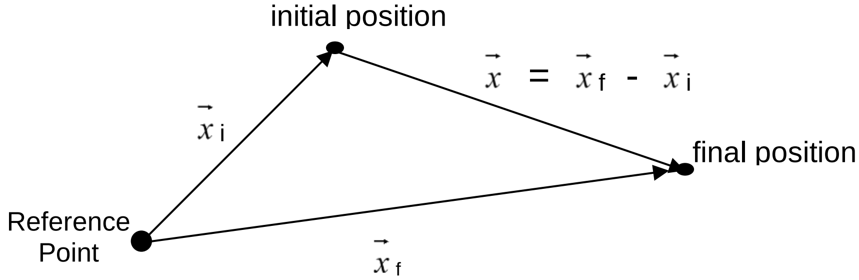
Figure 1

Considering Figure 1 above, the **distance** travelled by the particle to move from A to B is the actual length of the irregular path (in bold). The length of the straight line that joins A to B is the magnitude of the **displacement** of the particle from A while the arrow provides its direction.

Displacement of an object
= (displacement of final position from a fixed reference point) – (displacement of initial position from the same reference point)

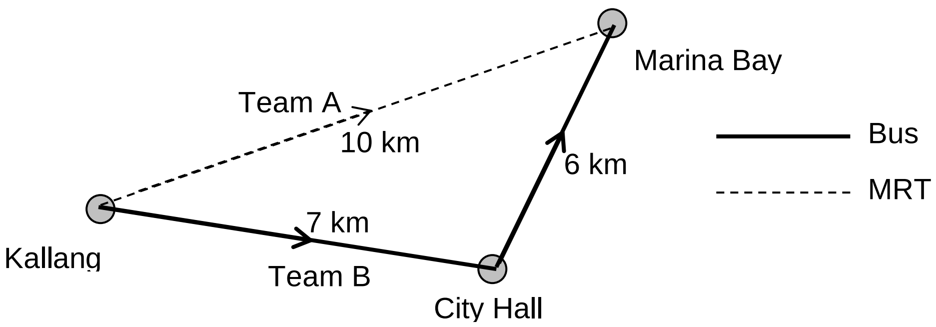
$$\vec{x} = \vec{x}_f - \vec{x}_i$$

Graphically:



Example 1

Two teams had to travel from Kallang to Marina Bay during an Amazing Race. Team A took the MRT directly from Mountbatten to Marina, while team B took a bus that bypassed City Hall first.



What is the distance travelled and the *magnitude* of the displacement for both teams?

Answers:

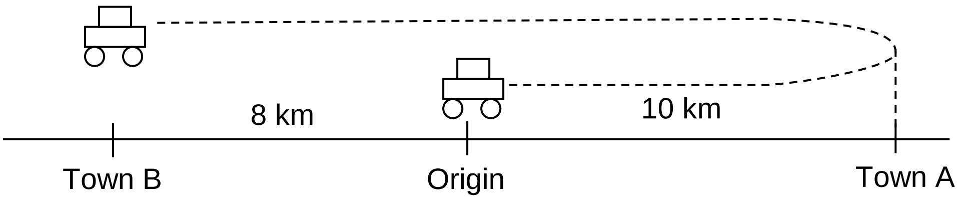
Team A Distance = **10 km** Team A Displacement = **10 km**

Team B Distance = **13 km** Team B Displacement = **10 km**

Example 2

Mr Tan travels 10 km east to Town A before turning back and travelling 18 km west to Town B. Determine the total distance travelled by Mr Tan to (i) town A, d_A (ii) town B, d_B, (iii) the displacements, s_A and s_B when Mr Tan is at town A and B respectively.

Solution



To differentiate between the east (right) and west (left) directions, take the direction towards the east to be positive:

(i) Total distance travelled to town A from origin, d_A = **10 km**
(ii) Total distance travelled to town B from origin , d_B = **10 + 18 = 28 km**

(iii) The displacement of Mr Tan at town A from origin is, s_A = **+ 10 km**
The displacement of Mr Tan at town B from origin, s_B = **– 8 km**

2.1.2 Speed and Velocity

Speed is the distance travelled per unit time.

Average speed, <u>	Average speed is the ratio of the total distance travelled to the time interval during which a particle travels over that distance. $$\langle u\rangle = \frac{d}{t}$$

Speed is a <u>scalar</u> quantity.

Velocity is defined as the rate of change of displacement with time. It acts in the direction of change of displacement.

Average velocity, <v>	Average velocity is the ratio of displacement to the time interval during which the displacement occurs. $$\langle v\rangle = \frac{s}{t}$$

Velocity is a <u>vector</u> quantity.

Example 3

Using information from Example 2, and given that Mr Tan took 10 minutes to drive to Town A and another 20 minutes to Town B, what is his average speed and velocity for the entire journey?

Solution

Average speed <u> $= \dfrac{\text{Total Distance}}{\text{Time Taken}} = \dfrac{28000}{30\times 60} = 15.55 = 16 \text{ m s}^{-1}$

Average velocity $= \dfrac{\text{Total Displacement}}{\text{Time Taken}} = \dfrac{-8000}{30\times 60} = -4.444 = -4.4 \text{ m s}^{-1}$

2.1.3 Instantaneous Speed and Velocity

Instantaneous speed of a particle is equal to the magnitude of its instantaneous velocity.

Instantaneous velocity is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally short so

Mathematically,

Instantaneous velocity, $\text{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t}$ or $\text{v} = \dfrac{ds}{dt}$

***Note*:** The mathematical equations you see here are standard forms used in derivative calculus. The mathematical expression $\lim\limits_{x \to 0} f(x)$ simply means the value of function $f(x)$ when x approaches zero. The expression Δs refers to a change in the quantity s. Hence instantaneous velocity, v can be expressed as $\text{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t}$. i.e. the ratio of a very small change in displacement (δs) to a small change in time (δt). Or simply $\text{v} = \dfrac{ds}{dt}$

2.1.4 Acceleration

Acceleration is the rate of change of velocity *with* time. It acts in the direction of the change of velocity. An object is said to be accelerating if its velocity is changing, either in magnitude or direction.

Average Acceleration, <a>	It is the ratio of the change in its velocity divided by the time interval over which this change occurs. $\langle a\rangle = \frac{\Delta v}{\Delta t}$
Instantaneous Acceleration, a	Defined as the limit of average acceleration as the time interval Δt goes to zero, that is, $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

Acceleration is a <u>vector</u> quantity.

Points to Ponder

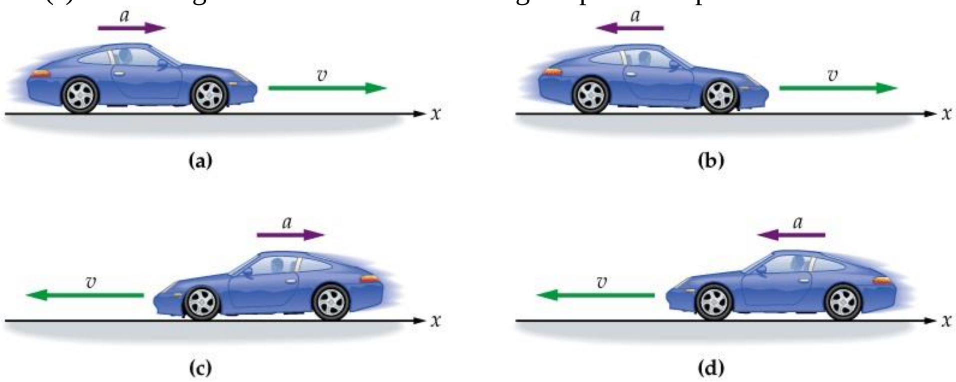
- A car moving in the eastward direction with a speed of 50 km h^{-1} makes a gentle turn towards the northward direction while maintaining its speed. Is the car accelerating? Why?

  Since the direction of the car is changing, its velocity is changing as well. Hence the car must be accelerating

- If a car is travelling to the right, can its acceleration be to the left? Explain.

  Yes. One example is when a car is braking.

- Which car(s) in the diagram below is/are increasing in speed? Explain.



  In diagrams (a) and (d). In both cases, the acceleration and velocity vectors are in the same direction.
  In general, when v and a are in the same direction, the object is moving faster.
  When v and a are in the opposite direction, the object is slowing down.

Learning Outcomes

(b) Use graphical methods to represent distance travelled, displacement, speed, velocity and acceleration.
(c) Find displacement from the area under a velocity-time graph.
(d) Use the slope of a displacement-time graph to find the velocity.
(e) Use the slope of a velocity-time graph to find the acceleration.

2.2 Graphical Methods for Linear Motion

2.2.1 Finding velocity from Displacement-Time Graphs

We can represent the displacement of any object from a reference point using a displacement – time graph. The displacement-time graph expresses how the displacement of a particle or body changes with time. Since $v = \frac{ds}{dt}$, the velocity at any instant can be found from the gradient (slope) of the displacement-time graph.

The illustration below distinguishes between finding average velocity and finding the instantaneous velocity from a displacement-time graph.

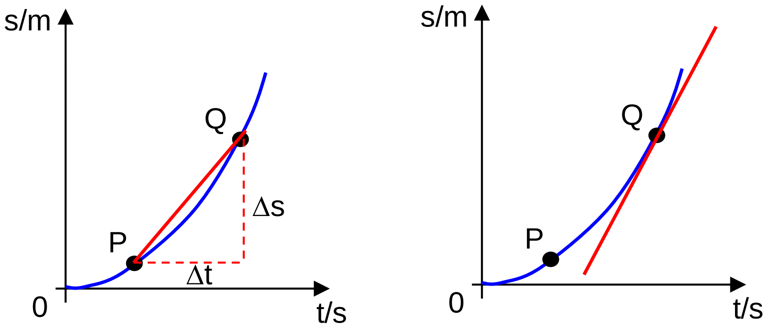
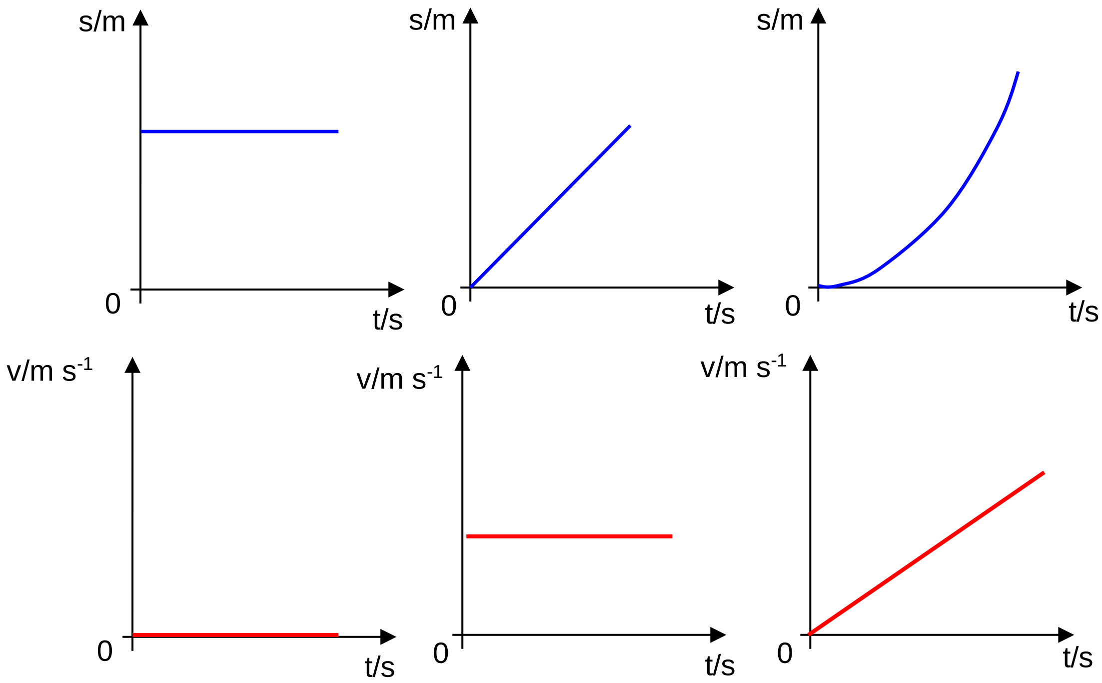


Fig 2(a) Fig 2(b)

The ratio $\frac{\Delta s}{\Delta t}$ in Fig 2(a) gives the magnitude of the average velocity for the time interval Δt.
The instantaneous velocity at Q is found by finding the gradient of the tangent (slope) at point Q as shown in Fig 2(b).

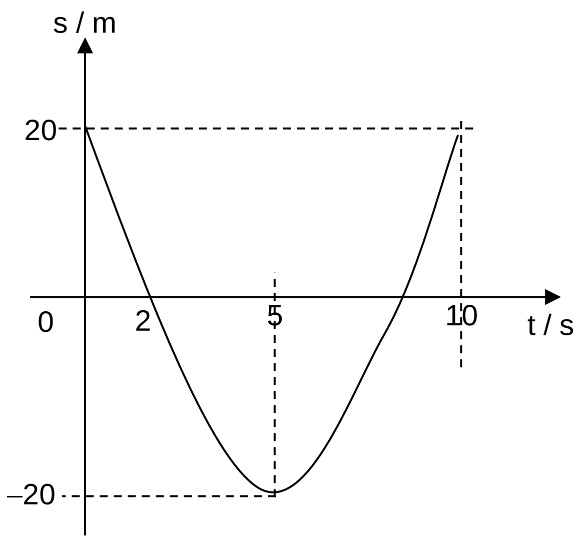
Example 4

(a) Sketch the velocity-time graph for the displacement-time graph given. Note that the velocity-time graph expresses how the instantaneous velocity of a particle or body changes with time.



(b) Determine the following information from a displacement – time graph shown below. You may assume that the graph from t = 0 s to t = 2 s is a straight line.

Displacement at t = 5 s	**– 20 m**
Average velocity from t = 0 s to t = 10 s	**0 m s^{-1}**
Instantaneous speed at t = 1 s	**10 m s^{-1}**
Instantaneous velocity at t = 1 s	**– 10 m s^{-1}**



Point to Ponder

Is it possible to find the total distance travelled from the displacement – time graph? Why?

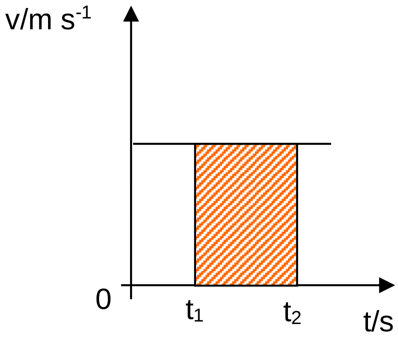
2.2.2 Finding displacement from Velocity-Time Graphs

We can also represent the motion of an object using a velocity – time graph.

Since $v = \dfrac{ds}{dt}$

$$\Rightarrow \vec{s} = \int_0^t \vec{v}\, dt$$

$\therefore$ The displacement can be calculated by finding the area under the velocity-time graph.



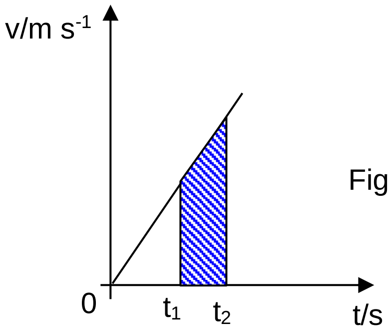


Fig 3

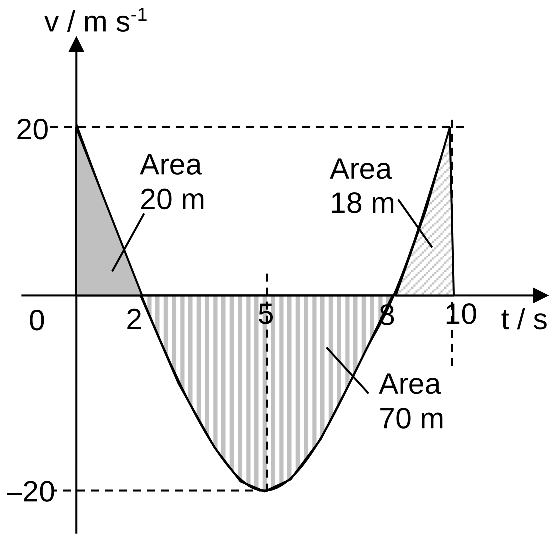
The shaded area in Fig 3 represents the displacement of the object from time t_1 to t_2.

Can you now sketch the displacement-time graph given the velocity-time graph? (Consider the graphs which you have sketched earlier in Example 4).

Example 5

Determine the following information from a velocity – time graph shown below.

Displacement at t = 10 s	**–32 m**
Average Velocity from t = 0 s to t = 10 s	**–3.2 m s^{-1}**
Instantaneous Velocity at t = 10 seconds	**20 m s^{-1}**
Average Acceleration from t = 0 s to t = 5 s	**-8.0 m s^{-2}**
Instantaneous Acceleration at t = 5 s	**0 m s^{-2}**

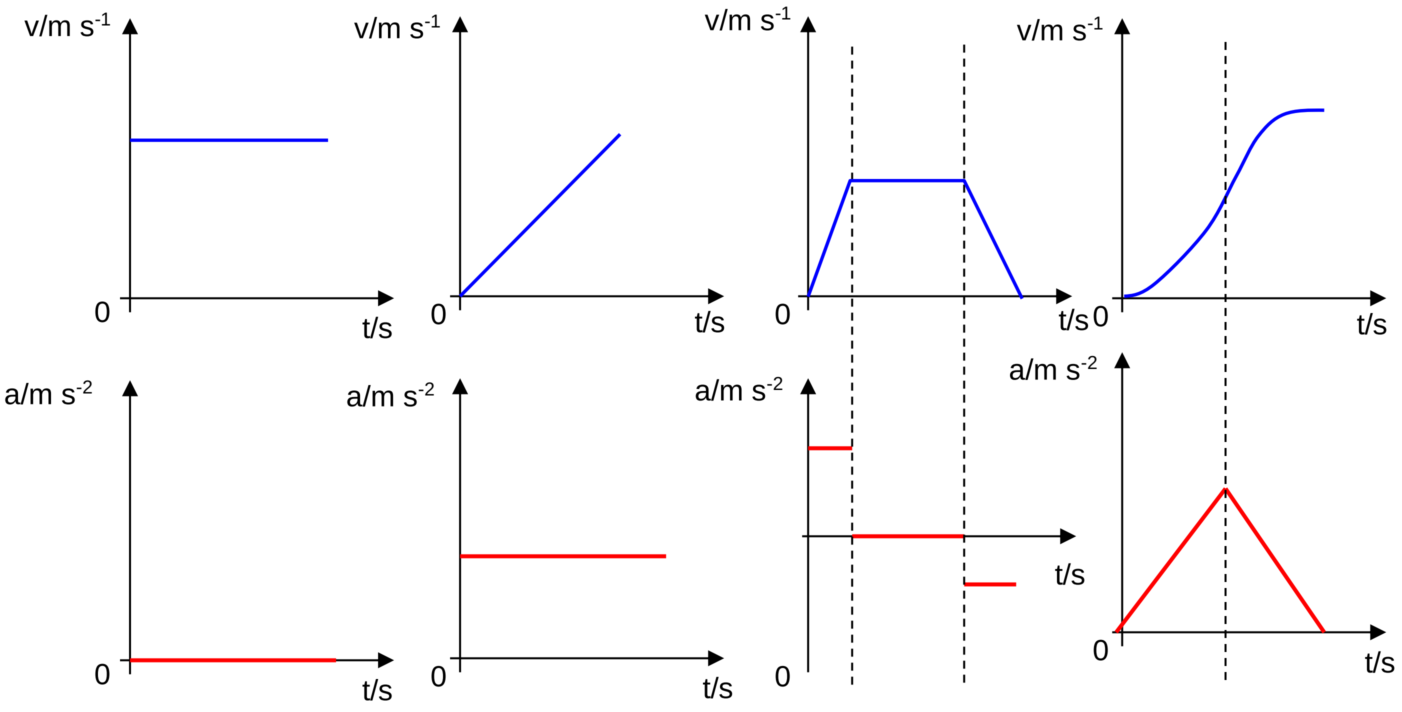


2.2.3 Finding Acceleration from Velocity-Time Graphs

Since $a = \frac{dv}{dt}$, the acceleration at any instant can be found from the <u>gradient (slope) of the velocity-time graph</u>.

Example 6

Sketch the acceleration-time graph for the velocity-time graph given.



Def

2.3 Equations of Motion

Learning Outcomes
(f) Derive, from the definitions of velocity and acceleration, equations which represent uniformly accelerated motion in a straight line.

It would be very tedious if we had to plot a velocity – time graph or displacement – time graph each time we need to solve a kinematics problem. Thankfully, we can solve kinematics questions with equations of motions derived from the graphs above. However, these equations of motions apply only to <u>uniformly accelerated</u> motions <u>in a straight line</u>.

2.3.1 Derivation of Equations of Motion

Consider a particle moving with ***uniform (constant) acceleration***, *a*, in a straight line.

u : initial velocity $\quad$ v : final velocity
a : acceleration $\quad$ t : time taken
s : displacement along a straight line

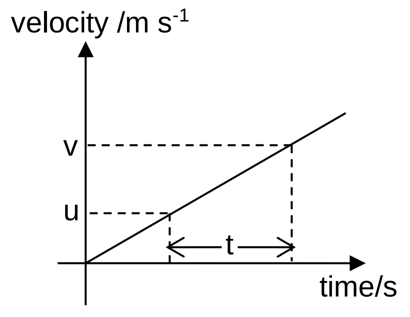
From the velocity – time graph, the instantaneous acceleration at time *t* is the gradient of the graph at the same time.

Hence,

$$a = \frac{v-u}{t}$$

or

$$\boxed{v = u + at} \quad \text{(Eqn 1)}$$



Example of uniformly accelerated motion

We can also write down the average velocity <v> as

$$<v> = \frac{1}{2}(u+v)$$

Since displacement is the area under the graph, we get

$$s = \frac{(u+v)}{2} \times t$$

Substituting $v = u + at$, $s = \frac{(u+u+at)}{2} \times t$

Simplifying, we get $\boxed{s = ut + \frac{1}{2}at^2}$ (Eqn 2)

Example 7

Show that the 3rd equation of motion is $\boxed{v^2 = u^2 + 2as}$ by substituting equation 1 found earlier into equation 2.

From eqn 1, $t = \frac{v-u}{a}$,

Substitute it into Equation 2 above, we get

$$s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

$$s = \frac{vu-u^2}{a} + \frac{1}{2}\left(\frac{v^2-2uv+u^2}{a}\right)$$

$$s = \frac{1}{2a}\left(v^2-u^2\right)$$

$\therefore v^2 = u^2 + 2as$ $\quad$ proven

2.3.2 Sign Conventions

The equations of motion are in fact vector equations, and vectors have magnitude and directions. For all rectilinear vector problems, we can indicate the directions using either the positive or negative sign.

For example, if we take the direction towards our right as positive, then the direction towards our left will be negative. Whatever direction that we define as positive would not affect our final solution, but it is important to state and keep track of your directions correctly.

Example 8

Junjie threw a ball vertically upwards with a velocity of 15.0 m s^{-1}, find the time taken to reach maximum height. Take g = 9.81 m s^{-2}

Solution

Taking **upwards** as positive

u = 15.0 m s^{-1}
v = 0.0 m s^{-1}
a = – 9.81 m s^{-2}

using $v = u + at$,

$$t = \frac{v-u}{a}$$
$$= \frac{0-15}{-9.81}$$
$$= 1.53s$$

Let **downwards** be positive

u = – 15.0 m s^{-1}
v = 0.0 m s^{-1}
a = 9.81 m s^{-2}

using $v = u + at$,

$$t = \frac{v-u}{a}$$
$$= \frac{0-(-15)}{9.81}$$
$$= 1.53s$$

All directions be positive

u =15.0m s^{-1}
v = 0.0 m
a = 9.81 m s^{-2}

using $v = u + at$,

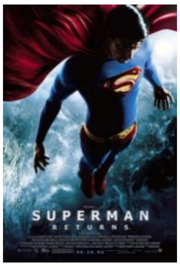
$$t = \frac{v-u}{a}$$
$$= \frac{0-15}{9.81}$$
$$= -1.53s$$

WRONG!

As can be seen, it does not matter if you choose the upward direction or the downward direction as positive initially. So long as you stick with the convention chosen throughout your problem, you would arrive at the same result.

Example 9

Superman wanted to leap from the ground to the top of the Taipei 101 building, 500 m above the ground.



(a) What is the minimum initial velocity required for him to reach the top of the building?
(b) What is his initial velocity if he reached the top of the building in 1.0 s?

Solution

(a) For minimum initial velocity, the maximum height of his jump will be just equal that of the building. When he reaches the top of the building, his velocity will be 0 m s^{-1}.

Taking upwards as positive

u = ?, v = 0 m s^{-1}, a = –9.81 m s^{-2}, s = 500 m, t = ?

Using $v^2 = u^2 + 2as$

$$0^2 = u^2 + 2(-9.81)(500)$$

$$u^2 = 9810$$

u = 99.0 m s^{-1} (to 3 sf)

(b) Taking upwards as positive

u = ?, v = ?, t = 1.0 s, s = 500 m, a = -9.81 m s^{-2}

Using $s = ut + \frac{1}{2}at^2$, we get

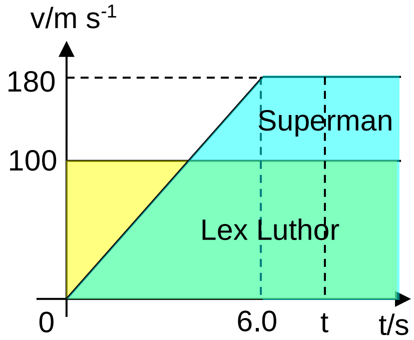
$$500 = u + 0.5(-9.81)$$

u = 505 m s^{-1}
= 510 m s^{-1} (to 2 sf)

Example 10

Lois Lane was held captive by Lex Luthor in an airplane travelling at a constant velocity of 100 m s^{-1}. The plane passed superman, who was initially stationary. Superman then chased after the plane with an acceleration of 30 m s^{-2} until he reached the top speed of 180 m s^{-1}. How far would superman have flown when he caught up with Lois Lane?

Solution 1 – Graphical Method



For superman to catch up with the plane, the area under both graphs must equal.

100 × t = 0.5 × 6.0 × 180 + 180 × (t-6.0)

80t = 540

t = 6.75 s

Therefore total distance travelled by superman is

~~100 × 6.75 = 675 m~~

Solution 2 – Equations of Motion

Note that acceleration is 30 m s^{-2} for 1^{st} 6.0 seconds and zero subsequently. Kinematics equations only applies for constant acceleration motion therefore care must be taken to solve this problem in 2 segments:

For the first 6.0 seconds,

Considering the airplane, total displacement of airplane is 100 × 6.0 = 600 m

Considering Superman, given u = 0 m s^{-1}, a = 30 m s^{-2}, v = 180m s^{-1}, s=?

Using $v^2 = u^2 + 2as$,

$$180^2 = 0 + 2(30)s$$

$$\therefore s = 540\,\text{m}$$

After the first 6.0 seconds, Superman flies 80 m s^{-1} faster than the airplane and is 60 m away.

Therefore, time taken for Superman to catch up the plane from that point onwards is simply

$$t = 60/80 = 0.75\text{s}$$

Hence total displacement from starting point is 6.75×100 = 675 m

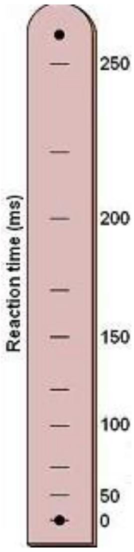
Learning Outcomes

(g) Solve problems using equations which represent uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance.

Steps to solve kinematics problems using the equations of motion:

1. Define the positive direction.
2. Write down all data given with the correct sign.
3. Write down all the 3 equations of motions. Identify unknowns in each equation.
4. Find the connection between the unknown and the data given.
5. Decide which equation(s) to use, and in what order.
6. Start solving.

Example 11



(a) The figure below shows a simple device for measuring your reaction time. A friend holds the strip vertically, with thumb and forefinger at the dot on the top. You then position your thumb and forefinger at the other dot on the bottom, being careful not to touch the strip. Your friend releases the strip and you try to pinch it as soon as possible after you see it begin to fall. The mark at the place where you pinch the strip gives your reaction time.

(i) How far from the bottom dot should you place the 50 ms? [1.23 cm]

Taking downwards as +ve,
u = 0 m s^{-1}, t = 50 ms, a = +*g* = +9.81 m s^{-2}, s=?

Choose an appropriate equation of motion

Use $s = ut + \frac{1}{2}at^2$

$s = 0 + ½\ 9.81\ (0.050)^2$
$= +0.0123$ m

(ii) Explain clearly how many times higher should the marks for 100, 150, 200 and 250 ms be? [4, 9, 16 and 25 times]

Since $s = ½\, g\, t^2$
therefore s is directly proportional to the square of time t,
hence the marks for 100, 150, 200, and 250 ms are at distances that are 2^2, 3^2, 4^2, and 5^2 greater than the 50 ms mark.

Example 12

A boy throws a ball vertically upwards and catches it 3.0 s later at the same location Neglecting air resistance,

(a) (i) find the speed with which the ball leaves his hand,

Taking upwards as +ve
$v = - u, t = 3.0\ s, a = - 9.81\ m\ s^{-2}$

Using $v = u + at$
$-u = u + (-9.81)(3)$
$u = 14.7\ m\ s^{-1}$

Hence speed = $14.7\ m\ s^{-1}$

(ii) find the maximum height to which it rises.

At maximum height, the ball will be instantaneously at rest.

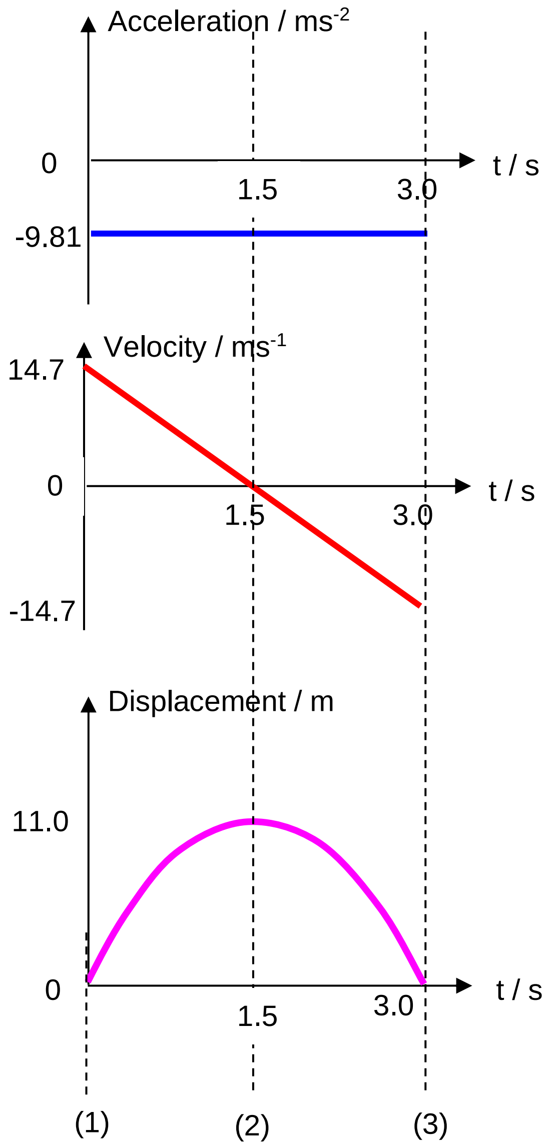
Taking upwards as +ve
$u = +14.7\ m\ s^{-1};\ v = 0\ m\ s^{-1},\ t = 1.5\ s,\ a = -9.81\ m\ s^{-2}$

Using $s = ut + \frac{1}{2}at^2$
$s = 14.7\ (1.5) - \frac{1}{2}\ 9.81\ (1.5)^2$
$= 11.0\ m$

Hence, max height is 11.0 m

(b) Sketch from 0 to 3 seconds,
(i) an acceleration-time graph,
(ii) a velocity-time graph, and
(iii) a displacement-time graph.

Indicate clearly on your graphs, the values of the *times* at which
(1) the ball leaves his hand,
(2) it comes to its maximum height,
(3) it reaches his hand again.



2.4 Motion under Gravity (Free Fall)

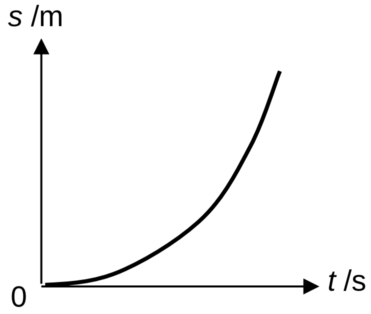
In kinematics, we often deal with motion that is under the influence of gravity. If a ball is dropped from rest, the displacement, velocity and acceleration are in the same direction (downwards). Hence we can apply kinematics equations to derive information of the motion required.

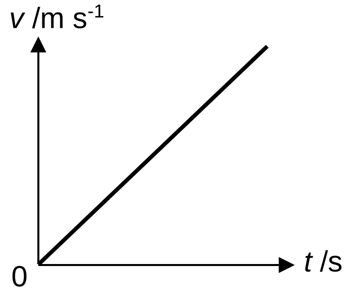
2.4.1 Air resistance is negligible

For this case, the only force acting on the ball is gravity. After time t, the velocity of the ball released from rest would be given by:

$$v = u + at$$
$$= gt \qquad \text{(since } u = 0\text{)}$$

Thus the displacement and velocity changes with time as shown in the graphs below





Learning Outcomes

(h) Describe qualitatively the motion of bodies falling in a uniform gravitational field with air resistance.

2.4.2 Motion due to Air resistance

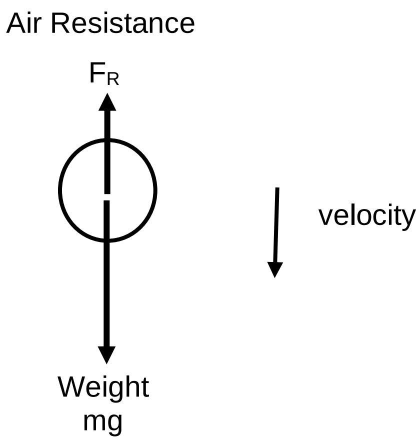
In practice, if the air resistance is taken into consideration, the velocity of the falling object will not increase with time indefinitely. In general, the air resistance opposing motion tends to increase approximately with the square of the speed at high velocities and increase proportionately with speed at low velocities.

Hence, the larger the velocity, the greater the larger the retarding force (air resistance). The actual acceleration is the combined free-fall acceleration and the retardation due to air-resistance.

Taking downward as positive

$$\text{Net force} = ma = mg - F_R$$

$$\therefore a = g - \frac{F_R}{m}$$

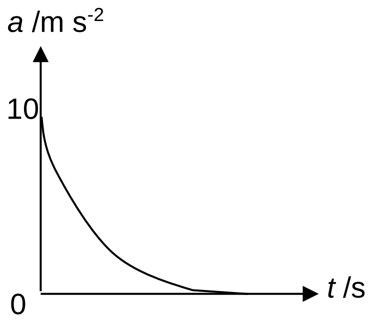


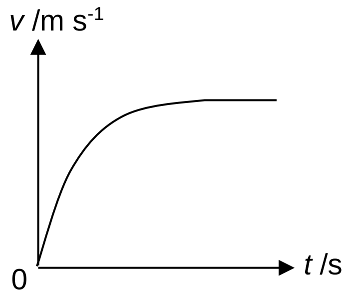
For bodies falling from great heights (aeroplane, clouds), the air resistance can be very large, so much that the retardation is equal to *g*. As such, the net force is 0 N and we say that the body has reached terminal velocity, v_T. Bodies falling in a fluid medium behave in the same way.

Example 13

Sketch the *v*-*t* and *a*-*t* graphs of a body falling from a great height when air resistance is not negligible.

Solution





Note:

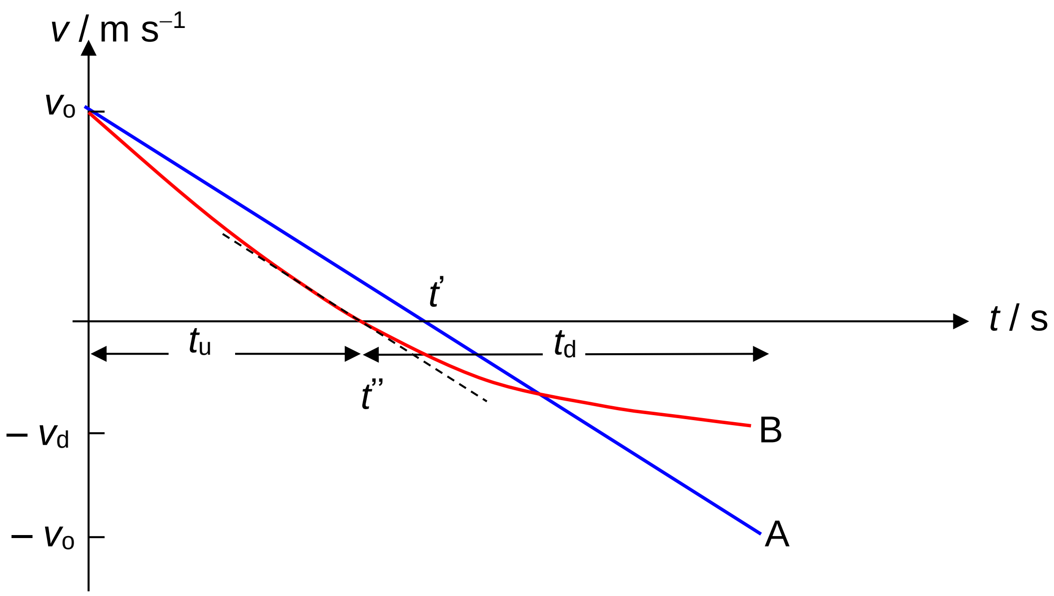
In general *g* is not a constant. It decreases with height above the Earth's surface. However, for height << Earth radius (e.g. few hundred metres), g varies negligibly with height and can be taken as a constant (= 9.81 m s^{-2})

"Released" or "dropped" implies that initial velocity *u* equal to 0 m s^{-1}

"Thrown or "fired" implies that initial velocity *u* is not equal to 0 m s^{-1}

Example 14

A ball is thrown vertically upwards with an initial velocity v_o. Sketch its velocity-time graph showing the time until the ball reaches the thrower's hand, in the case when
(a) air resistance is neglected, labelled A
(b) air resistance is considered, labelled B
Explain qualitatively the shapes of both graphs.



The slope of the graph without any air resistance is a straight line with a constant negative slope because

the acceleration due to gravity is constant and is directed downwards.

When the ball is on its upward journey, the slope of graph B is steeper than that for graph A because

the retarding effect of air resistance acting in the same direction as the ball's weight is such that the net deceleration of the ball due to the weight of the ball and air resistance is greater than that due to gravity alone.

When the ball is on its downward journey, the slope of graph B is gentler than that for graph A because
the retarding effect of air resistance is such that the net acceleration of the ball downwards due to the weight of the ball and the opposing effect of air resistance is less than that due to gravity alone.

When the ball is at the maximum height, the slope of graph B is equal to that for graph A because

at the maximum height, the ball is instantaneously at rest, hence air resistance is zero, and the acceleration of the ball is due to gravity alone.

For the upward journey, the area under graph B is less than for graph A because

some energy is used to overcome air resistance, and so the gain in gravitational potential energy maximum height reached is less than its initial kinetic energy. Hence the maximum height reached is less than the case for that without air resistance.

When the ball is on its upward journey, the time taken to reach maximum height is less for graph B (t'') than for graph A (t') because

with air resistance, the ball experiences a greater retarding force and hence deceleration on its upward journey, and so is decelerated to rest in a shorter time.

Considering graph B alone, the time t_u taken for the ball to reach the top when going up less than the time t_d for the ball to reach the thrower's hand when coming down because

on the upward journey, there is a greater deceleration as the air resistance helps to decelerate the ball, whereas on the downward journey, there is a less acceleration as the air resistance opposes the downward acceleration of the ball.

Consider graph B alone. The velocity with which the ball leaves the thrower's hand is v_o. The velocity of the ball just before it reaches the thrower's hand is $-v_d$. The magnitude of v_d is less than that for v_o because

some energy is used to overcome air resistance, and so the kinetic energy that the ball returns to the thrower's hand is less than before.

Example 15

A body is thrown vertically upwards. Which of the following provides the correct explanation when the times of flight for the upward motion t_u and the downward motion t_d (to return to the same level) are compared if air resistance is not ignored?

A $t_d > t_u$, because the body moves faster on its downward flight as the downward acceleration is larger.

B. $t_d > t_u$, because at a given speed the net acceleration when the body is moving downwards is smaller than the net deceleration when it is moving upwards.

C $t_d < t_u$, because at a given speed the net acceleration when the body is moving downwards is greater than the net deceleration when it is moving upwards.

D $t_d = t_u$, because the net acceleration when the body is moving downwards is equal to the net deceleration when it is moving upwards.

2.5 Non-linear Motion

Learning Outcomes

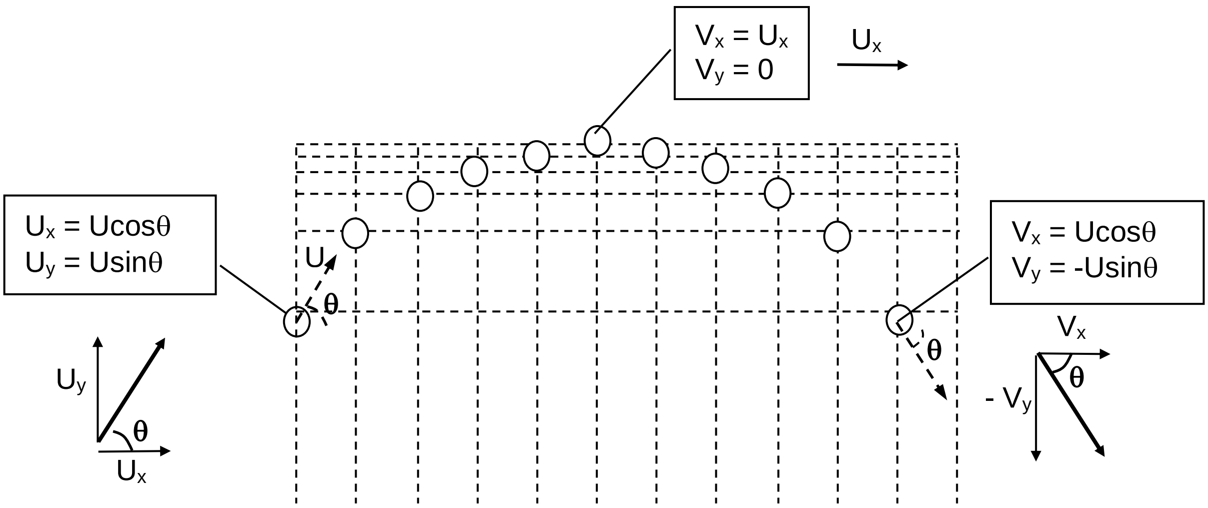
(i) Describe and explain motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular direction.

So far, we had only considered objects along straight-line paths. Now let us look at objects moving in a plane, that is, two-dimensional motion. Projectile motion deals with the motion of a particle or body when it undergoes motion in which there is a uniform velocity in one direction and a uniform acceleration in a perpendicular direction.

Examples in which projectile motion can occur include the following:
a. A body moving in a perpendicular to uniform gravitational field.
b. A charged particle moving perpendicular to uniform electric field.

When an object is projected into the air at an angle θ with respect to the ground with speed *v*, the body will move in a parabolic path. An example of projectile motion is an Olympic athlete throwing a javelin

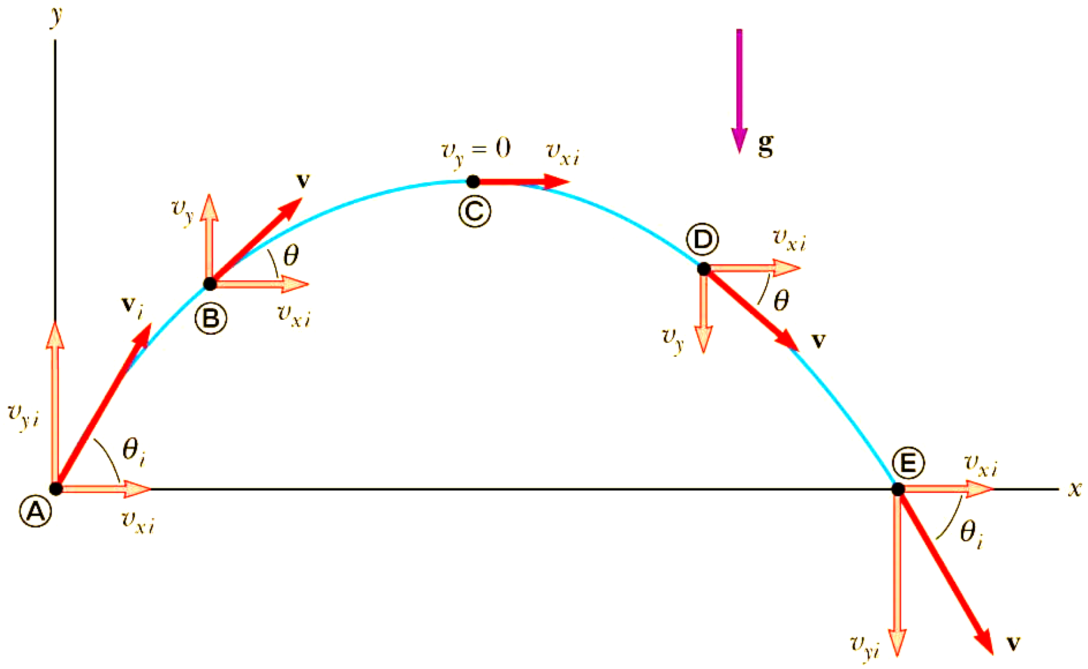
Though the motion is two-dimensional, it can be analyzed by considering the straight line motion in the horizontal direction and vertical direction independently.



Horizontal motion

The body is moving with a constant velocity. There is no resultant force in the horizontal direction and therefore the horizontal acceleration is **zero**.

Horizontal Acceleration	$a_x = 0$	
Horizontal Velocity	$v_x = u_x + a_x t$	$= u_x$
Horizontal Displacement	$s_x = u_x t + \frac{1}{2} a_x t^2$	$= u_x t$



Note the **length of the vectors** representing the horizontal and vertical components of the velocity at each stage of the motion. The length of the horizontal component is constant.

Vertical motion

The body moves under the influence of gravity with a **constant** acceleration (g) hence the vertical velocity **changes** with time. Taking downwards (gravity) as positive, the kinematics equation for motion applies to this case

Vertical Acceleration	$a_y = g$
Vertical Velocity	$v_y = u_y + gt$
Vertical Displacement	$s_y = u_y t + \frac{1}{2}gt^2$
Time independent Eqn	$v_y^2 = u_y^2 + 2gs_y$

Note: A parabola is a shape found commonly in mathematics. In fact, if you plot quadratic equations ($ax^2 + bx + c$) in a graph, you get a parabola.

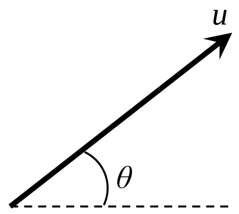
Velocity of projectile at any time is given by the vector sum of v_x and v_y at that time.

In magnitude, $v = \sqrt{v_x^{\,2} + v_y^{\,2}}$

In direction, $\theta = \tan^{-1} \dfrac{v_y}{v_x}$

Notes: In kinematics and subsequent Physics topics, we would have to resolve vectors in the perpendicular directions to solve problems. Hence it is important to revise your vectors.

2.5.1 Maximum Height



At the maximum height, $v_y = 0$

Using $v_y^{\,2} = u_y^{\,2} + 2\, a_y\, y$

$= (u \sin\theta)^2 - 2\, g\, H$

$\therefore$ maximum height reached , $H = \dfrac{u^2 \sin^2\theta}{2g}$

2.5.2 Time to reach Maximum Height

At the maximum height, $v_y = 0$

Using $v_y = u_y + a_y\, t$

$= u \sin\theta - g\, t_{up}$

$t_{up} = \dfrac{u \sin\theta}{g}$

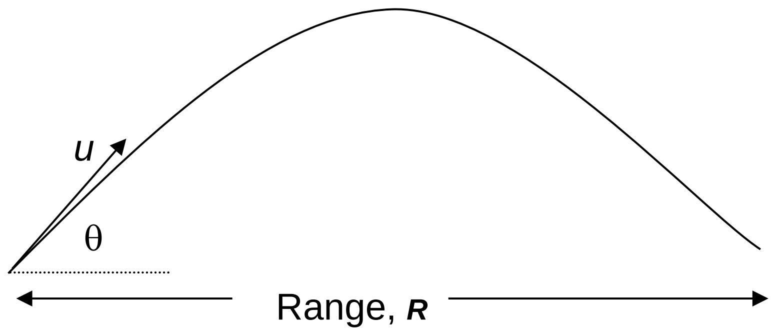
2.5.3 Total Time of Flight, T

$t_{up} = t_{down}$

$\therefore T = 2\, t_{up}$

$T = \dfrac{2u \sin\theta}{g}$

2.5.4 Alternative derivation for Total Time of Flight, T



Considering the whole motion, when the projectile has reached the ground, $y = 0$

Taking the upward direction as positive, $a_y = -g$, $u_y = u\sin\theta$

Using $s_y = u_y t + ½ a_y t^2$

$0 = (u\sin\theta)t + ½(-g)t^2$

For $y = 0$, $t\,(u\sin\theta - ½ g t) = 0$

$$\Rightarrow \quad t = 0 \quad \text{or} \quad t = \frac{2u\sin\theta}{g} = T$$

2.5.5 Range, R

The maximum horizontal distance covered by the projectile is known as the horizontal range, R.

$$R = v_x T = (u\cos\theta)\frac{2u\sin\theta}{g}$$

$$R = \frac{u^2\sin 2\theta}{g}$$

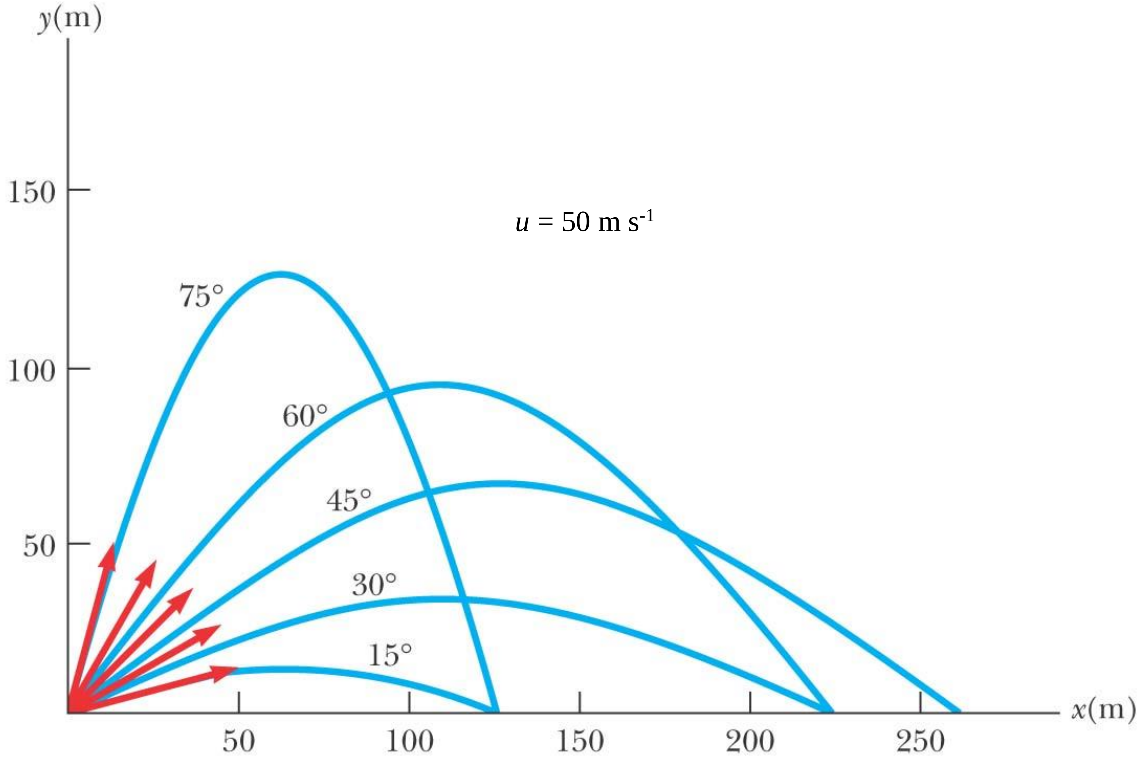
For a given speed of projection, u, the horizontal range is maximum when

$$\sin 2\theta = 1 \Rightarrow \theta = 45°$$

thus maximum range, $R_{max} = \dfrac{u^2}{g}$

<u>IMPORTANT!</u>
EQUATIONS FOR RANGE, TIME OF FLIGHT, MAXIMUM HEIGHT ETC SECTION 2.5.1 to 2.5.5 CANNOT BE USED W/O PROOF FOR NUMERICAL PROBLEMS.
Except for MCQ where working is not required

The figure below shows a projectile launched with an initial speed of 50 m s^{-1} at various angle of projection. Note that the complementary values of angle of projection result in the same value of R.



Problem Solving Hints for Projectile Problems in Gravitational Field

1. Resolve the initial velocity into the x and y components.

2. Solve the constant velocity horizontal motion and constant accelerated motion vertically. Note the x and y motion share the same t.

Example 16

An object is projected at an angle to the horizontal in a gravitational field and it follows a parabolic path, PQRST. These points are the positions of the objects after successive equal time intervals, T being the highest point reached.

The displacements PQ, QR, RS and ST

A are equal

B decrease at a constant rate

C. have equal horizontal components

D increase at a constant rate

Example 17

From the top of a building we throw three identical rocks. One is thrown vertically upwards, one horizontally sideways, and one dropped vertically downwards. The rocks thrown vertically upwards and horizontally have the same initial speed. Which rock takes the shortest time to reach the ground? [Ignore air resistance]

A The one thrown vertically upwards

B The one thrown horizontally sideways.

C. The one dropped vertically downwards.

D Both the rocks thrown horizontally sideways and dropped vertically downwards

Example 18

A fighter plane was flying horizontally at 90 m s^{-1} at a height of 800 m when it released a bomb.

(a) State the initial horizontal and vertical velocities of the bomb.

Since the bomb is on the plane and hence it will initially travel with the velocity of the plane,

$u_x = 90$ m s^{-1} ***in the direction of flight of the plane.***
$u_y = 0$ m s^{-1}

(b) Determine the time needed for the bomb to reach the ground.

Remember that for a projectile problem, we can always solve the vertical and horizontal motions separately. They are independent.

Taking downwards as +ve,
$u_x =$ m s^{-1}, $u_y =$ m s^{-1}, $a_x = 0$ m s^{-2}, $a_y = +g = + 9.81$ m s^{-2}, $s_y = +800$ m

Using $s_y = u_y t + ½ a_y t^2$

$800 = 0 + ½ (9.81) t^2$

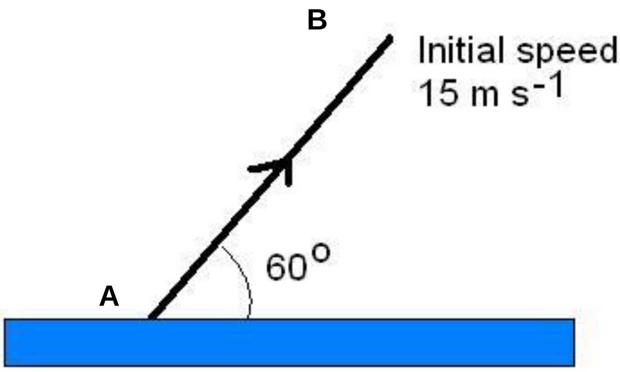
$= 12.8$ s

(c) Determine the horizontal distance the bomb travelled before it reached the ground, assuming air resistance is negligible.

Taking downwards as +ve,
$u_x = 90$ m s^{-1}, $u_y = 0$ m s^{-1}, $a_x = 0$ m s^{-2}, $a_y = +g = + 9.81$ m s^{-2}, $s_y = +800$ m, $t = 12.8$ s
Using $s_x = u_x t + ½ a_x t^2$
$s_x = u_x t + ½ (0) t^2$
$= 90 (12.8)$
$= 1150$ m

Example 19

A ball is thrown with an initial velocity of 15 m s^{-1} at an angle of 60° to the horizontal, as shown in the diagram below.



(a) Explain why AB represents the velocity of the ball and not just its speed.

AB represents the velocity as the length of the arrow gives an indication of the magnitude of the velocity and the direction provides its initial direction of projection.

(b) Calculate, for this ball, the initial values of

(i) the vertical component of the velocity,

$u_y = 15 \sin 60$
$= 13 \text{ m s}^{-1}$

(ii) the horizontal component of the velocity.

$u_x = 15 \cos 60$
$= 7.5 \text{ m s}^{-1}$

(c) Assuming that air resistance can be neglected, use your answers in (b) to determine

(i) the maximum height to which the ball rises,

Taking upwards as +ve,
$u_y = +13 \text{ m s}^{-1}$, ***at max. height:*** $v_y = 0 \text{ m s}^{-1}$, $a_y = -9.81 \text{ m s}^{-2}$

Using $v^2 = u^2 + 2as$
$0 = 13^2 + 2(-9.81)(s_y)$
$s_y = 8.61 \text{ m}$

(ii) the time of flight, that is, the time interval between the ball being thrown and returning to ground level,

Since the ball returns to ground level, $s_y = 0$

Using $s_y = u_y t + \frac{1}{2} a_y t^2$
$0 = 13t + \frac{1}{2}(-9.81)(t)$
$t(13 - 4.905) = 0$
hence $t = 0$ ***or*** $t = 2.65 \text{ s}$

(d) Use your answers to (b) to sketch the path of the ball, assuming air resistance is negligible. Label this path N.

(e) (i) On your sketch in (d), draw the path of the ball, assuming that air resistance cannot be neglected. Label this path A.

(ii) Suggest an explanation for any differences between the two paths N and A.

In the horizontal direction, the ball experiences a net deceleration, therefore the horizontal speed decreases resulting in a shorter range.

As the ball rises, it has to do work against air resistance, hence mechanical energy is lost and the maximum height is reduced (as per Conservation of Energy)

As the ball rises, it experiences a net deceleration greater than g because air resistance is acting in the same direction as the gravitational force. Hence, its vertical speed decreases to zero in a shorter time. As a result, it reaches a max height sooner, explaining why the max height is displaced to the left compared to path N.

Path A is asymmetrical because horizontal speed is not uniform and in the vertical direction, the vertical acceleration experienced by the ball is not constant.

Kinematics Summary

Scalar: **Distance** is length of path taken by the object.

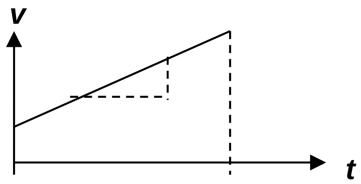
Speed is defined as distance travelled per unit time.

Vector **Displacement** of an object measures the change in position of the object along a straight line and is defined as the distance and direction of a position of a point as measured from a reference position.

Velocity is the *rate of change of displacement with time*. It acts in the direction of change of displacement. It is a vector quantity.

Acceleration is the *rate of change of velocity with time*. It acts in the direction of the change of velocity.

Graph



Gradient = acceleration
Area = Displacement

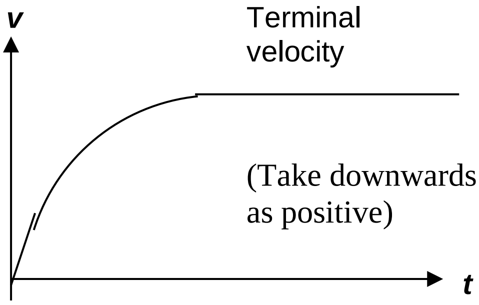
Equations of Motion for a body traveling in a straight line with ***constant acceleration.***

$\boldsymbol{v} = u + at$
$s = ut + ½ at^2$
$v^2 = u^2 + 2 a s$
$s = ½ (u+v)t$

> Note: Need to know how to derive these eqns from definitions of velocity and acceleration

Note: Usually, the direction of the initial velocity u is taken as the positive direction. Vector (v, s or a) which has an opposite direction is given a negative sign.

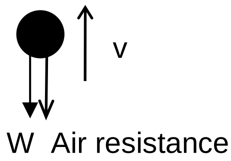
Object falling in a Uniform Gravitational Field with Air Resistance



As the object falls, its velocity increases and the air resistance acting on the object also increases. This opposes the downward motion and causes the acceleration to decrease. At a point when the weight is offset by the air resistance, the net force and hence the acceleration is zero, the object moves with a constant terminal velocity.

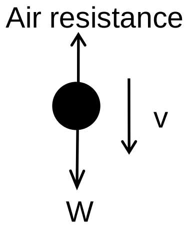
Motion of a Body Moving Vertically in a Uniform Gravitational Field with air resistance

Consider a body thrown vertically upwards with an initial velocity u in a uniform gravitational field with air resistance

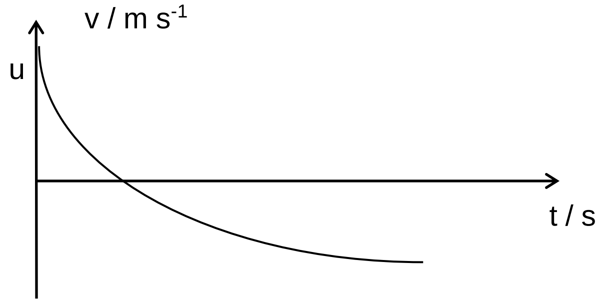


As a body rises, both air resistance and weight act downward, causing a larger resultant force. Its initial upward deceleration is greater and it takes a shorter time to reach the greatest height.

As a body falls, it accelerates downwards, and air resistance opposes its weight, causing a smaller resultant force. The magnitude of the acceleration is smaller and it takes a longer time to reach the original position.



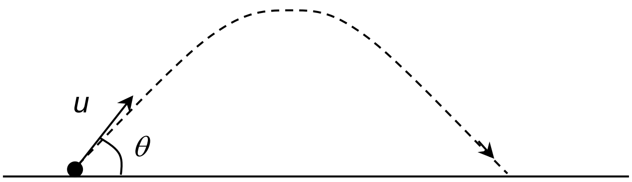
The gradient of the graph on the upward journey is steeper than that of the downwards journey. When its velocity is 0 m s^{-1}, the gradient of the graph is 9.81 ms^{-2}.



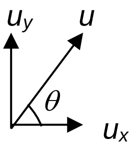
(Take upwards as positive)

Projectile Motion

A mass is projected with speed *u* at an angle θ in a space with *no* air resistance.



$u_x = u \cos\theta$
$u_y = u \sin\theta$
$u = \sqrt{u_x^2 + u_y^2}$



For a projectile motion, the horizontal and vertical components of the motion are **independent** of each other.

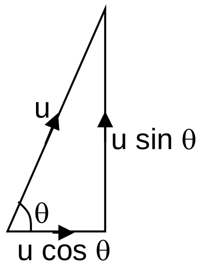
horizontal: $F_x = 0$, $\mathbf{a_x = 0}$, $\mathbf{v_x}$ **is constant**,
$\mathbf{u_x = u \cos\theta}$ $v_x = u_x$, $s_x = u_x t$.

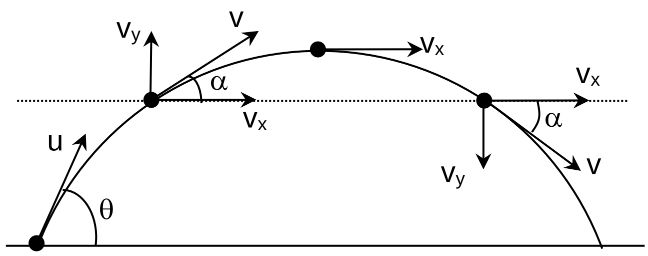
vertical: $F_y = W$, $\mathbf{a_y}$ **is constant** **(9.81 m s^{-2} downwards)**
$\mathbf{u_y = u \sin\theta}$

$v_y = u_y + a_y t$
$s_y = u_y t + \frac{1}{2} a_y t^2$
$v_y^2 = u_y^2 + 2 a_y s_y$

$\mathbf{v_y = 0}$ **(at highest point)**

Note: Time is the only quantity that is the same for both the horizontal and vertical motions.





Special cases:

<u>Initial vertical velocity</u>

u

Vertical
$\boldsymbol{u_y = u, a_y = 9.81 ms^{-2}}$ **(down)**

Horizontal
$\boldsymbol{u_x = 0, a_x = 0}$,

Apply

$v = u + at$
$s = ut + \frac{1}{2} a t^2$
$v^2 = u^2 + 2as$

in 2 directions

<u>Initial horizontal velocity</u>

u

Vertical
$\boldsymbol{u_y = 0}$
$\boldsymbol{a_y = 9.81 ms^{-2}}$ **(down)**

Horizontal
$\boldsymbol{u_x = u, a_x = 0}$,

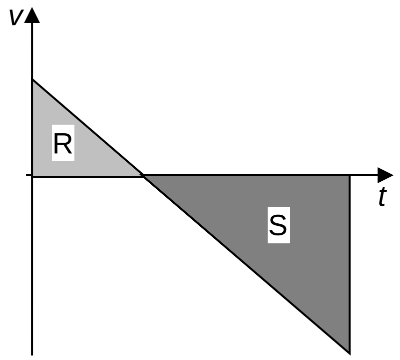
Kinematics Tutorial

Self-Attempt Questions

Rectilinear Motion: Concept of distance, displacement, speed, velocity and acceleration

1. A stone is thrown upwards from the top of a cliff. After reaching its maximum height, it falls past the cliff-top and into the sea.

The graph shows how the vertical velocity *v* of the stone varies with time *t* after being thrown upwards. R and S are the magnitudes of the areas of the two triangles.



(a) What is the height of the cliff-top above the sea?

(b) What is the magnitude of the maximum height reached by the stone as measured from the cliff-top?

(c) What is the total distance travelled by the stone from the instant of release until it hits the sea

A R **B** S **C** R+S **D** S – R

Ans: *D, A, C, Modified from N98/I/3*

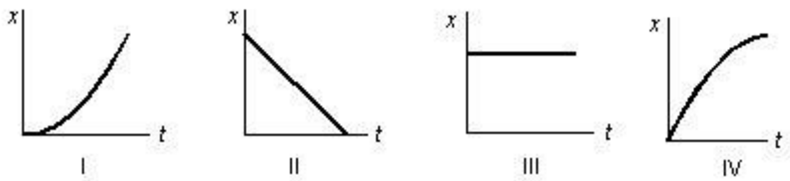
2. The average speed of a moving object during a given interval of time is always:

A the magnitude of its average velocity over the interval
B the distance covered during the time interval divided by the time interval
C one-half its speed at the end of the interval
D one-half its acceleration multiplied by the time interval.

Ans: *B*

Rectilinear Motion: Graphical Analysis

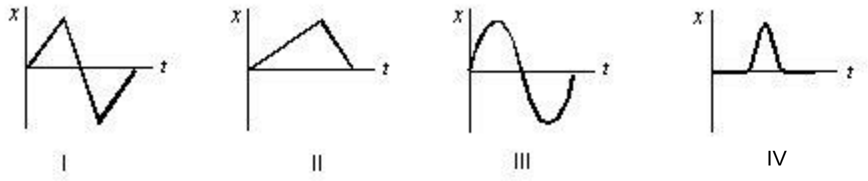
3. Which of the following four displacement versus time graphs represents the motion of an object moving with a constant speed?



A I B II C III D IV

Ans: *B*

4. A car accelerates from rest on a straight road. A short time later, the car decelerates to a stop and then returns to its original position in a similar manner, by speeding up and then slowing to a stop. Which of the following graphs best describes the motion?



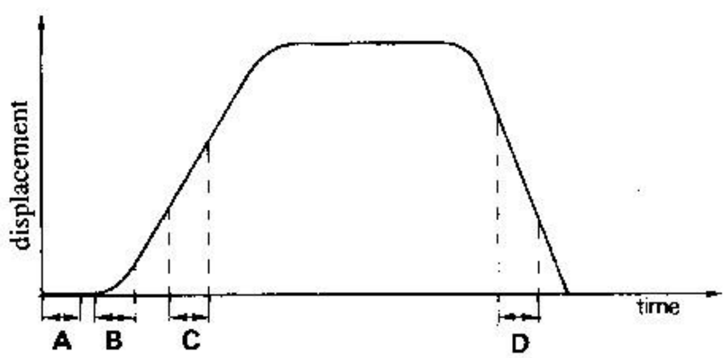
A I B II C III D IV

Ans: *D*

Discussion Questions

5. (a) Define displacement, speed, velocity and acceleration.
(b) Are the following situations possible? Use suitable real-life examples to illustrate your answers.
(i) Can a body have zero velocity and be accelerating?
(ii) Can a body have zero acceleration but a non-zero velocity?
(iii) Can a body be moving in one direction but accelerating in the opposite direction?
(iv) Can a body be accelerating when its velocity is constant?
(c) Using your definition of displacement, explain how it is possible for a car to travel a certain distance and yet have zero displacement.
(d) Explain why it is technically incorrect to define speed as distance travelled per second. Include in your answer the correct statement defining speed.

6 The graph represents how displacement varies with time for a vehicle moving along a straight line.

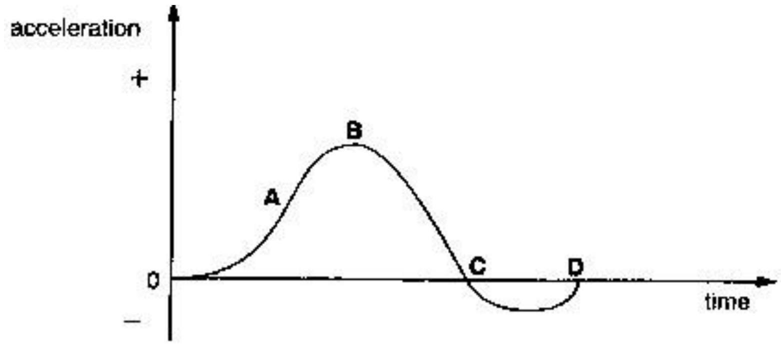


During which time interval does the acceleration of the vehicle have its greatest value?

[N02/I/3]

7 A car is travelling along a straight road. The graph shows the variation with time of its acceleration during part of the journey.

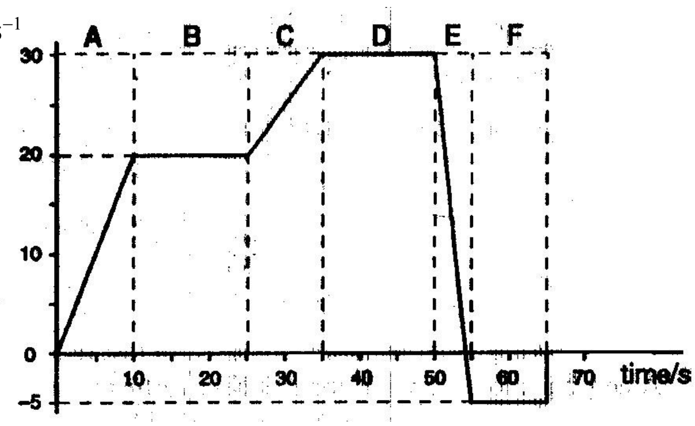
At which point on the graph does the car have its greatest velocity?



[N96/I/4]

8 The figure below shows a velocity-time graph for a journey lasting 65 s.

Velocity / m s^{-1}

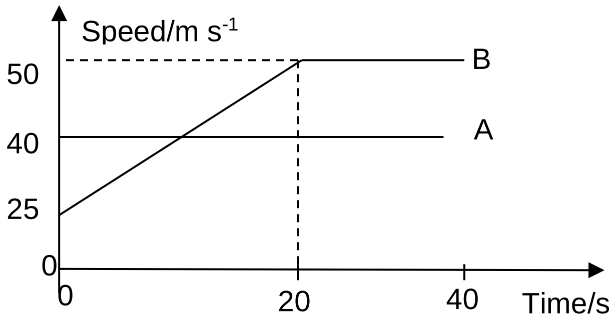


(a) Using information from the graph, obtain
   (i) the acceleration in section A and E separately.
   (ii) the distance travelled in section B and C separately.

(b) Describe qualitatively in words what happens in section E and F of the journey.
(c) Sketch a corresponding distance-time graph. Detailed calculations of the distance travelled are not required.

[N97/II/1 part]

9 The graph below shows the speeds of two cars A and B which are travelling in the same direction over a period of time of 40 s. Car A, travelling at a constant speed of 40 ms^{-1} overtakes car B at time t=0 s. In order to catch up with car A, car B immediately accelerates uniformly for 20 s to reach a constant speed of 50 m s^{-1}.



(a) How far does car A travel during the first 20 s?

(b) Calculate the acceleration of car B in the first 20 s.

(c) How far does car B travel in this time?

(d) What additional time will it take for car B to catch up with car A?

(e) How far will each car have then travelled since t = 0?

(f) What is the maximum distance between the cars before car B catches up with car A?

[N93/II/2]

<u>Self-Attempt Questions</u>

Rectilinear Motion: Equations of Motion

10. A car, initially at rest, travels 20 m in 4.0 s along a straight line with constant acceleration. The acceleration of the car is:

A 0.40 m s^{-2}
B 1.3 m s^{-2}
C 2.5 m s^{-2}
D 4.9 m s^{-2}

Ans: *C*

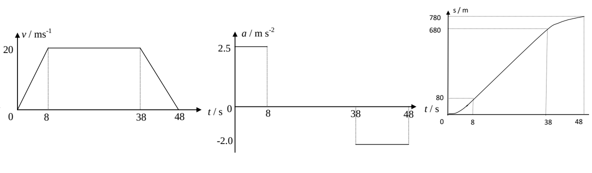
11 (a) An MRT train accelerates uniformly from rest for 8.0 s until its velocity reaches 20 m s^{-1}. Find
(i) its initial acceleration,
(ii) its distance travelled from rest.

(b) The MRT train then travels for another 30 s at this constant velocity of 20 m s^{-1}. Calculate the additional distance travelled for this section of its journey.

(c) The MRT train then decelerates uniformly to come to rest in a further 10 s. Find
(i) its deceleration over this section of its journey,
(ii) the additional distance travelled for this section of its journey.

(d) For this motion, sketch
(i) a velocity-time graph,
(ii) an acceleration-time graph,
(iii) a displacement-time graph.

Ans: *2.5 m s^{-2}, 80 m, 600 m, –2.0 m s^{-2}, 100 m,*

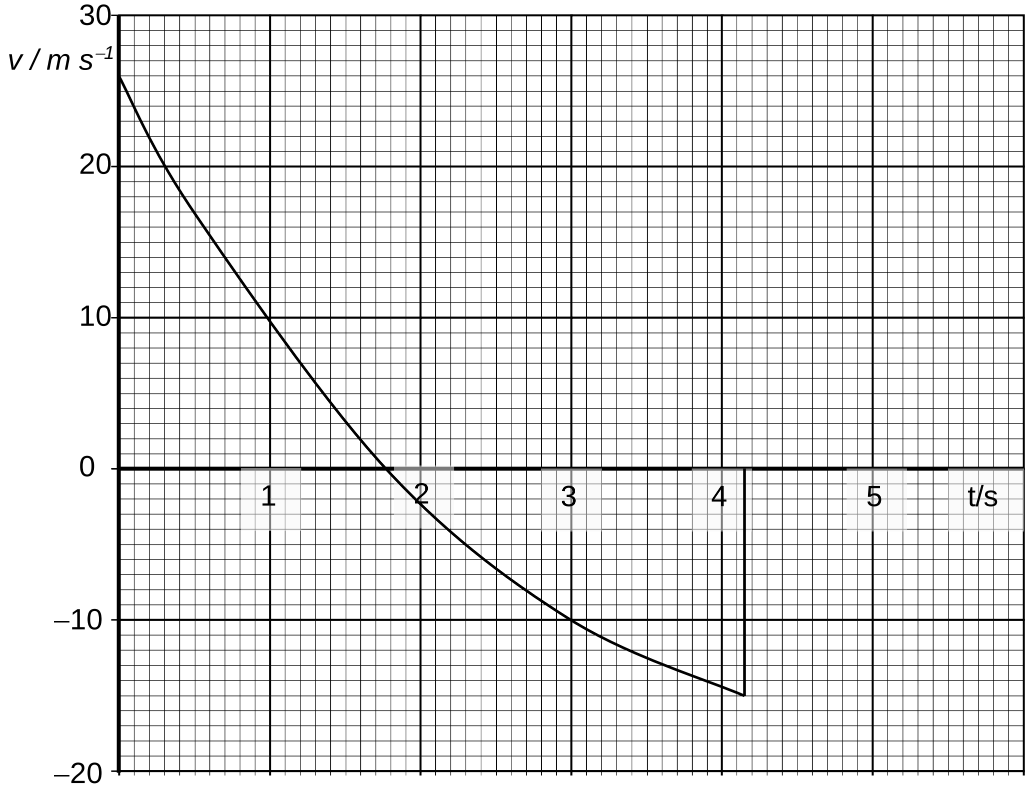


Discussion Questions

12. In a traffic accident, a motorist claims that he applied his full brakes when he saw the pedestrian but it was too late therefore the accident occurred. Traffic police arrested the motorist for speeding and driving recklessly. Based on the information given below, explain how the police arrived at the conclusion and estimate the speed of the motorist before he saw the pedestrian.
    - The speed limit for the road is 50 km h^{-1}.
    - Skid marks found on the road was about 100 m long
    - The maximum retarding force from the vehicle gives a deceleration of 5 m s^{-2}
    - Human reaction time cannot be faster than 0.20 seconds

13. A ball is dropped from a height of 6.0 m. Assume that there is negligible air resistance.
    (a) (i) find the velocity of the ball just before it hits the ground
    (ii) find the time taken by the ball to reach the ground after release.
    (b) Assuming that no energy is lost on rebound, sketch a graph of velocity against time for the ball for 3 impacts with the ground and its corresponding displacement-time graph and distance-time graph.
    (c) If the ball only retains 80 % of its kinetic energy on each impact, sketch a new graph of velocity against time for the ball for 3 impacts with the ground and its corresponding distance-time graph.

14. *The graph below shows the variation with time* t *of the velocity* v *of a ball from the moment it is thrown with a velocity of 26 m s⁻¹ vertically upwards.*



(a) State the time at which the ball reaches its maximum height.

(b) State the feature of a velocity-time graph that enables the acceleration to be determined.

(c) Hence determine the initial deceleration of the ball. Explain why it can be greater than 9.81 m s^{-2}.

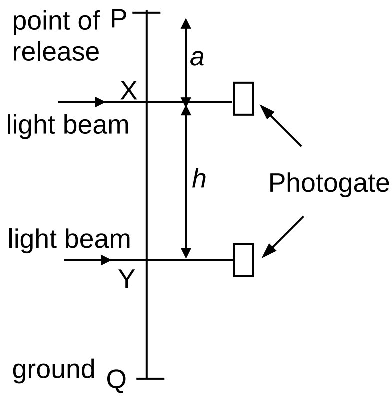
(d) State and explain the time at which the acceleration is g.

(e) Sketch an acceleration-time graph of the ball, indicating clearly the time given in (d).

(f) Sketch on the graph above, a v-t graph if air resistance is negligible.

[N03/III/1 part]

15. The acceleration g, of free fall is determined by timing the fall of a steel ball by using light-gates as shown in the diagram. The ball passes points X at time t_x, and point Y at time t_y after release from P.



Show that g can be expressed as $\frac{2h}{(t_y^2 - t_x^2)}$.

[Modified from N10/I/5]

16. Cindy throws a ball vertically upwards with a velocity of 8.0 ms^{-1} from the edge of a cliff
(a) Find the maximum height of the ball
(b) If the cliff is 100 m deep, find the time taken for the ball to hit the bottom of the cliff
(c) Crystal throws a rock (of the same size as the ball) vertically downwards with velocity of 8.0 ms^{-1}, would the rock or the ball has a greater speed when it reaches the bottom of the cliff?
(i) If we ignore air resistance
(ii) If air resistance cannot be ignored

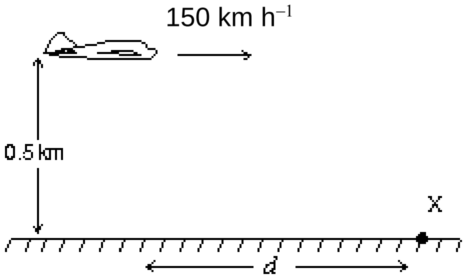
Non-Rectilinear Motion: Projectile

Self-Attempt Questions

17. In the absence of air resistance, a bullet shot horizontally from a gun will
A strikes the ground much later than one dropped vertically from the same point at the same instant
B strikes the ground vertically downwards
C strikes the ground at the same time as one dropped vertically from the same point at the same instant
D strikes the ground much sooner than one dropped from the same point at the same instant

Ans: *C*

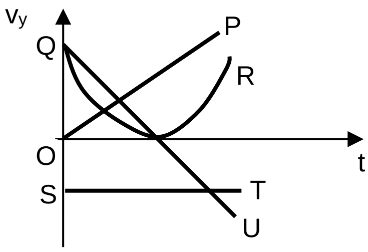
18. The airplane shown is in level flight at an altitude of 0.50 km and a speed of 150 km h^{-1}. At what distance d should it release a heavy bomb to hit the target X?
Take $g = 10$ m s^{-2}.



A 150 m
B 295 m
C 417 m
D 2550 m

Ans: *C*

19. Which of the curves on the graph below best represents the vertical component v_y versus t for a projectile fired at an angle of 45° above the horizontal?



A OP
B QU
C QR
D ST

Ans: *B*

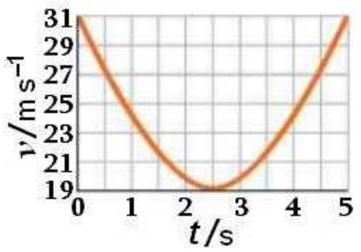
20. (a) A small ball rolls horizontally off the edge of a tabletop that is 1.20 m high. It strikes the floor at a point 1.52 m horizontally from the edge of the table. Neglecting air resistance,
(i) How long is the ball in the air?
(ii) Find the speed at the instant it leaves the table.
(iii) Calculate the horizontal and vertical components of the ball's velocity just before striking the floor?
(iv) Hence, determine its velocity just before striking the floor.
(b) If a similar ball is dropped vertically from the same level as the tabletop, state and explain your answer clearly, for this ball, the duration it is in the air.

Ans: *0.495s. 3.07 m s^{-1},v_x = 3.07 m s^{-1}, v_y = 4.86 m s^{-1}, 57.7° from horizontal, no change*

Discussion Questions

21. An airplane, diving at an angle of 53° with the vertical, releases a bomb at an altitude of 730 m. The bomb hits the ground 5.0 s after release. Neglecting air resistance,
    (a) Determine the speed of the aircraft?
    (b) How far did the bomb travel horizontally during its flight?
    (c) What were the horizontal and vertical components of its velocity just before striking the ground?
    (d) Hence state the velocity of the bomb just before striking the ground.
    (e) Explain the effect on your answers in (b), (c) and (d) if air resistances were not neglected.

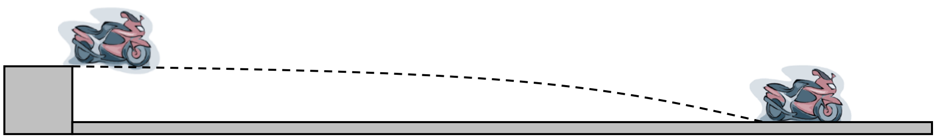
22. A golf ball is struck at ground level and follows a parabolic trajectory. The speed of the golf ball as a function of time is shown below, where t = 0 s is at the instant the ball is struck.



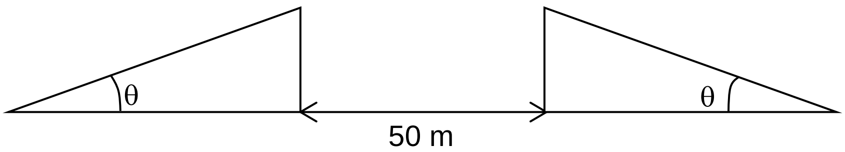
Neglecting air resistance,

(a) State the horizontal velocity of the golf ball.
(b) How far does the golf ball travel horizontally before returning to ground level?
(c) What is the initial vertical component of the golf ball's velocity?
(d) What is the maximum height above ground level attained by the ball?

23. a) A motorcycle stunt-rider moving horizontally takes off from a point 1.25m above the ground, landing 10 m away as shown in the diagram. What was the speed at take off?



b) A stuntman on a motorcycle plans to ride up a ramp in order to jump over a river 50 m wide. If he intends to ride at the maximum speed of 30 ms^{-1}, what is the range of angles θ of ramp he should use to jump over the river?



c) What minimum velocity must the stuntman have in order to jump over the river?
[J93/I/3 modified]

Data Based Question

24 A table from a car driver's handbook reads as follows:

On a dry road, a car in good condition driven by an alert driver will stop in the distances shown in the table.

Speed / ms^{-1}	**Thinking distance* / m**	**Braking distance^ / m**	**Overall distance / m**
5.0	3.0	1.9	4.9
10	6.0	7.5	13.5
15	9.0	17	26
20	12	30	42
25	15	47	62
30	18	68	86
35	21	92	113

*Thinking distance is the distance travelled by the car during the driver's reaction time

^Braking distance is the distance in which the car stops after the brakes have been applied

(a) Explain why thinking distance is directly proportional to speed whereas braking distance is not. Describe in words the relationship between braking distance and speed.

(b) What constant value of negative acceleration has the author of the table used in calculating the braking distances?

(c) Calculate the overall stopping distance for a car travelling at 50 ms^{-1}

(d) What would be the effects on the thinking distance and the braking distance of each of the following conditions?

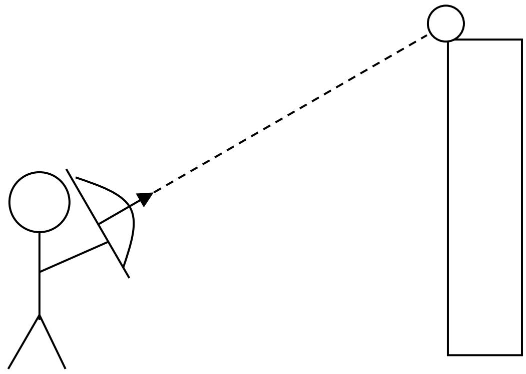
(i) The road is wet

(ii) The driver is not fully alert

[J89/II/9]

Challenging Questions

25 Robin Hood believed that he could hit a falling object if he aimed directly at the object and fire his arrow at the same time as the object is dropped. Prove or disprove Robin Hood



26 Robin Hood was out in the hills chasing a bandit running away from him when he fired an arrow that hit him as shown below. Given that the bandit was 25 m away from Robin Hood when Robin Hood fired his arrow, find the speed of the bandit.