2023 TJC H2 FM Prelim Paper 2 (Suggested Solution)

Question 1

(a)

$$\frac{dy}{dx} = (y-1)^2$$

Let $f(x, y) = (y-1)^2$

Using the Euler Method with step size 0.5,

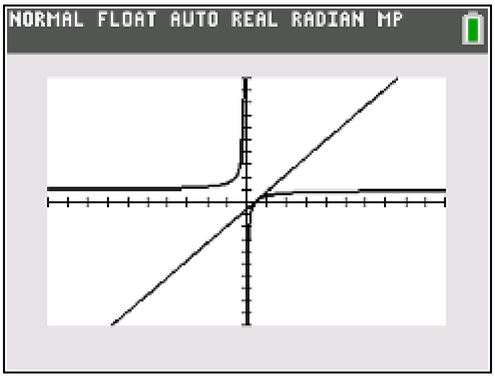
x_n	y_n	$\frac{dy}{dx}$
0	-1.6	6.76
0.5	$-1.6 + 0.5\times 6.76 = 1.78$	0.6084
1	$1.78 + 0.5\times 0.6084 = 2.0842$	

Using Euler method, $y = 2.8042 \approx 2.80$ (3 s.f.) when $x = 1$

(b)

The particular solution is $-\frac{1}{y-1} = x + \frac{5}{13}$

As seen in the diagram below, the tangent to the curve at $x = 0.5$ is steep which results in the first approximation to be already exceed the asymptotic value of y.



(c)

Using the improved Euler method,

$x_0 = 0$, $y_0 = -1.6$, $h = 0.5$, $f(x, y) = (y-1)^2$

$$y_1 = y_0 + h\left[\frac{f(x_0, y_0) + f(x_0 + h, y_0 + hf(x_0, y_0))}{2}\right]$$

$$= -1.6 + 0.5\left[\frac{6.76 + 0.6084}{2}\right]$$

$$= 0.2421$$

$$y_2 = y_1 + h\left[\frac{f(x_1, y_1) + f(x_1 + h, y_1 + hf(x_1, y_1))}{2}\right]$$

$$= 0.441 \text{ (3 s.f.)}$$

The accuracy of the estimate can be improved by reducing the step size.

Question 2	
$u_{n+1}\sqrt{n} = au_n\sqrt{n+1} - b\sqrt{n^2+n}$ $\frac{u_{n+1}}{\sqrt{n+1}} = a\frac{u_n}{\sqrt{n}} - b$ (dividing throughout by $\sqrt{n(n+1)}$) $v_{n+1} = av_n - b$	
(a)(i)	$v_{n+1} = 2v_n - b$ General solution, $v_n = A(2^n) + B$ Note that $u_1 = \frac{v_1}{\sqrt{1}} = v_1$ Thus $v_1 = 2A + B \Rightarrow u_1 = 2A + B = 10000$ $B = 2B - b \Rightarrow B = b, A = \frac{1}{2}(10000 - b)$ Particular solution, $v_n = (10000-b)2^{n-1} + b$ $\frac{u_n}{\sqrt{n}} = (10000-b)2^{n-1} + b$ $u_n = \left[(10000-b)2^{n-1} + b\right]\sqrt{n}$
(a)(ii)	This will eventually go to zero when $b > 10000$. The population of virus will first increase to a maximum value and then decrease to zero.
(b)	$v_{n+1} = 2v_n - 2$ $u_n = \left[9998(2^{n-1}) + 2\right]\sqrt{n}$ Using GC: $u_n - 1000000 > 0$ when $n = 7$ i.e. after 1 week

Question 3
(i) (a) Characteristic equation: $\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$ $(a-\lambda)(d-\lambda) - bc = 0$ $\lambda^2 - (a+d)\lambda + ad - bc = 0$ $\lambda = \frac{a+d \pm \sqrt{(a+d)^2 - 4(ad-bc)}}{2}$ $= \frac{a+d \pm \sqrt{(a+d)^2 - 4ad + 4(1-d)(1-a)}}{2}$ $= \frac{a+d \pm \sqrt{(a+d)^2 + 4(1-a-d)}}{2}$ $= \frac{a+d \pm \sqrt{(a+d)^2 - 4(a+d) + 4}}{2}$

$= \dfrac{a+d \pm \sqrt{(a+d-2)^2}}{2}$ $= \dfrac{a+d \pm (a+d-2)}{2}$ $= \dfrac{2(a+d)-2}{2}$ or $\dfrac{a+d-a-d+2}{2}$ $= a+d-1$ or 1 (shown) Or $\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & 1-d \\ 1-a & d \end{pmatrix}$, Characteristic equation: $\lvert M-\lambda I \rvert = 0$ $\begin{vmatrix} a-\lambda & 1-d \\ 1-a & d-\lambda \end{vmatrix} = 0$ $(a-\lambda)(d-\lambda)-(1-d)(1-a)=0$ $\lambda^2-(a+d)\lambda+ad-(1-a-d+ad)=0$ $\lambda^2-(a+d)\lambda+a+d-1=0$ $(\lambda-1)(\lambda-(a+d-1))=0$ $\lambda=1$ or $a+d-1$ (shown)

$\lambda=1$: $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$
$\left.\begin{matrix} (a-1)x+by=0 \\ cx+(d-1)y=0 \end{matrix}\right\\} by=(a-1)x \Rightarrow (d-1)y=(a-1)x$
$\therefore \begin{pmatrix} x \\ y \end{pmatrix} // \begin{pmatrix} d-1 \\ a-1 \end{pmatrix}$
$\lambda=a+d-1$: $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = (a+d-1)\begin{pmatrix} x \\ y \end{pmatrix}$
$\left.\begin{matrix} by+(1-d)x=0 \\ cx+(1-a)y=0 \end{matrix}\right\\} by=(d-1)x \Rightarrow by=-bx$
$\therefore \begin{pmatrix} x \\ y \end{pmatrix} // \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
The eigenvectors corresponding to 1 and $a+d-1$ are $\begin{pmatrix} d-1 \\ a-1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ respectively.

Question 4

(a) $a = 2$, $b + c = 6$

Equation of top ellipse: $\frac{y^2}{b^2}+\frac{x^2}{2^2}=1 \Rightarrow x^2 = 4\left(1-\frac{y^2}{b^2}\right)$

Equation of bottom ellipse: $\frac{y^2}{c^2}+\frac{x^2}{2^2}=1 \Rightarrow x^2 = 4\left(1-\frac{y^2}{c^2}\right)$

$$\text{Volume of capsule} = \pi\int_{-c}^{0} 4\left(1-\frac{y^2}{c^2}\right)\mathrm{d}y + \pi\int_{0}^{b} 4\left(1-\frac{y^2}{b^2}\right)\mathrm{d}y$$

$$= 4\pi\left(\left[y-\frac{y^3}{3c^2}\right]_{-c}^{0} + \left[y-\frac{y^3}{3b^2}\right]_{0}^{b}\right)$$

$$= 4\pi\left(\left[c-\frac{c^3}{3c^2}\right]+\left[b-\frac{b^3}{3b^2}\right]\right)$$

$$= 4\pi\left(\frac{2(b+c)}{3}\right)$$

$$= 4\pi\left(\frac{2}{3}(6)\right)$$

$$=16\pi$$

(b) $\frac{y^2}{(2a)^2}+\frac{x^2}{a^2}=1 \Rightarrow x^2 = a^2\left(1-\frac{y^2}{4a^2}\right) = a^2 - \frac{y^2}{4}$

$$2x\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{-2y}{4} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = -\frac{y}{4x}$$

$$1+\left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2 = \frac{16x^2+y^2}{16x^2} = \frac{16\left(a^2-\frac{y^2}{4}\right)+y^2}{16x^2} = \frac{16a^2-3y^2}{16x^2}$$

Surface area, $S = 2\pi\int_{-c}^{0} x\sqrt{1+\left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2}\,\mathrm{d}y = 2\pi\int_{-2a}^{0} x\sqrt{\frac{16a^2-3y^2}{16x^2}}\mathrm{d}y = \frac{\pi}{2}\int_{-2a}^{0}\sqrt{16a^2-3y^2}\,\mathrm{d}y$

Consider $\int_{-2a}^{0}\sqrt{16a^2-3y^2}\,\mathrm{d}y$

$$\int_{-2a}^{0}\sqrt{16a^2-3y^2}\,dy=\left[y\sqrt{16a^2-3y^2}\right]_{-2a}^{0}-\int_{-2a}^{0}y\frac{1}{2\sqrt{16a^2-3y^2}}(-6y)\,dy$$

$$=\left[2a\sqrt{16a^2-3(4a^2)}\right]+\int_{-2a}^{0}\frac{3y^2}{\sqrt{16a^2-3y^2}}\,dy$$

$$=4a^2+\int_{-2a}^{0}\frac{-(16a^2-3y^2)+16a^2}{\sqrt{16a^2-3y^2}}\,dy$$

$$=4a^2-\int_{-2a}^{0}\sqrt{16a^2-3y^2}\,dy+\int_{-2a}^{0}\frac{16a^2}{\sqrt{16a^2-3y^2}}\,dy$$

$$\therefore\quad 2\int_{-2a}^{0}\sqrt{16a^2-3y^2}\,dy=4a^2+\left[\frac{16a^2}{\sqrt{3}}\sin^{-1}\left(\frac{\sqrt{3}y}{4a}\right)\right]_{-2a}^{0}$$

$$\Rightarrow\quad \int_{-2a}^{0}\sqrt{16a^2-3y^2}\,dy=2a^2-\frac{8a^2}{\sqrt{3}}\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)=2a^2+\frac{8a^2}{\sqrt{3}}\left(\frac{\pi}{3}\right)$$

Hence surface area $=\dfrac{\pi}{2}\left(2a^2+\dfrac{8a^2\pi}{3\sqrt{3}}\right)$

$$=a^2\pi+\frac{4a^2\pi^2}{3\sqrt{3}}\ \text{cm}^2$$

Alternative method of finding $\int_{-2a}^{0}\sqrt{16a^2-3y^2}\,dy$ using substitution

$$\int_{-2a}^{0}\sqrt{16a^2-3y^2}\,dy$$

$$=\int_{-\frac{\pi}{3}}^{0}\sqrt{16a^2(1-\sin^2\theta)}\left(\frac{4a}{\sqrt{3}}\cos\theta\right)d\theta$$

$$=\frac{16a^2}{\sqrt{3}}\int_{-\frac{\pi}{3}}^{0}\cos^2\theta\,d\theta$$

$$=\frac{8a^2}{\sqrt{3}}\int_{-\frac{\pi}{3}}^{0}(\cos2\theta+1)\,d\theta$$

$$=\frac{8a^2}{\sqrt{3}}\left[\frac{\sin 2\theta}{2}+\theta\right]_{-\frac{\pi}{3}}^{0}$$

$$=\frac{8a^2}{\sqrt{3}}\left[\frac{\sqrt{3}}{4}+\frac{\pi}{3}\right]$$

$$=2a^2+\frac{8a^2\pi}{3\sqrt{3}}$$

Let $y=\dfrac{4a}{\sqrt{3}}\sin\theta$

$\dfrac{dy}{d\theta}=\dfrac{4a}{\sqrt{3}}\cos\theta$

When $y=-2a,\theta=-\dfrac{\pi}{3}$

When $y=0,\theta=0$

Question 5	
(a)	$y=\frac{u}{x}\Rightarrow xy=u\overset{diff}{\Rightarrow}x\frac{\mathrm{d}y}{\mathrm{d}x}+y=\frac{\mathrm{d}u}{\mathrm{d}x}$ $\overset{diff}{\Rightarrow}x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}^2u}{\mathrm{d}x^2}$ $\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{x}\frac{\mathrm{d}^2u}{\mathrm{d}x^2}-\frac{2}{x}\frac{\mathrm{d}y}{\mathrm{d}x}$ $=\frac{1}{x}\frac{\mathrm{d}^2u}{\mathrm{d}x^2}-\frac{2}{x}\left[\frac{1}{x}\left(\frac{\mathrm{d}u}{\mathrm{d}x}-y\right)\right]$ $=\frac{1}{x}\frac{\mathrm{d}^2u}{\mathrm{d}x^2}-\frac{2}{x^2}\frac{\mathrm{d}u}{\mathrm{d}x}+\frac{2}{x^2}\left(\frac{u}{x}\right)$ $=\frac{1}{x}\frac{\mathrm{d}^2u}{\mathrm{d}x^2}-\frac{2}{x^2}\frac{\mathrm{d}u}{\mathrm{d}x}+\frac{2u}{x^3}$
	Substituting the above into the given DE gives $\frac{1}{x}\frac{\mathrm{d}^2u}{\mathrm{d}x^2}-\frac{2}{x^2}\frac{\mathrm{d}u}{\mathrm{d}x}+\frac{2u}{x^3}+\frac{2}{x}\left[\frac{1}{x}\left(\frac{\mathrm{d}u}{\mathrm{d}x}-\frac{u}{x}\right)\right]-\frac{25u}{x}=0$ $\Rightarrow\frac{\mathrm{d}^2u}{\mathrm{d}x^2}-25u=0$
	AE: $k^2-25=0\Rightarrow k=\pm5$ GS: $u=A\mathrm{e}^{5x}+B\mathrm{e}^{-5x}\Rightarrow y=\frac{1}{x}\left(A\mathrm{e}^{5x}+B\mathrm{e}^{-5x}\right)$.
(b)	$y=px\sin2x+qx\cos2x$ $\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=p\left(2x\cos2x+\sin2x\right)+q\left(-2x\sin2x+\cos2x\right)$ $=\left(p-2qx\right)\sin2x+\left(2px+q\right)\cos2x$ $\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\left(p-2qx\right)\cos2x-2q\sin2x$ $-2\left(2px+q\right)\sin2x+2p\cos2x$ $=-4\left(px+q\right)\sin2x+4\left(p-qx\right)\cos2x$
	Substitute the above into the given DE gives $-4\left(px+q\right)\sin2x+4\left(p-qx\right)\cos2x$ $+4\left(px\sin2x+qx\cos2x\right)=\sin2x$ $\Rightarrow-4q\sin2x+4p\cos2x=\sin2x$ $\Rightarrow-4q=1,4p=0$ $\Rightarrow q=-\frac{1}{4},p=0$ PI is $y=-\frac{1}{4}x\cos2x$

	AE: $k^2+4=0 \Rightarrow k=\pm 2\text{i}$ CF: $y=A\cos 2x+B\sin 2x$ GS: $y=A\cos 2x+B\sin 2x-\frac{1}{4}x\cos 2x$
	Substitute $x=n\pi,\ n\in\mathbb{Z}^+$ into above GS gives $y=A\cos 2n\pi+B\sin 2n\pi-\frac{1}{4}n\pi\cos 2n\pi$ $y=A-\frac{n\pi}{4}$ $\frac{y}{n}=\frac{A}{n}-\frac{\pi}{4}\approx-\frac{\pi}{4}$ for large n

Question 6

H_0 : gender of customers and colour preference of Jpad pro tablet are independent.

H_1 : gender of customers and colour preference of Jpad pro tablet are not independent.

Level of significance: 5%

Computation: Under H_0, Expected frequencies, $E_{ij}=\frac{r_i \times c_j}{N}$ (N = grand total) are shown below:

		Colour preference			
		White	Black	Grey	Total
Gender	Male	10.8	21.6	21.6	54
	Female	19.2	38.4	38.4	96
	Total	30	60	60	150

Test statistic: $\chi^2=\sum_i\sum_j\frac{\left(O_{ij}-E_{ij}\right)^2}{E_{ij}}$

Calculation of contribution to test statistic:

		Colour preference		
		White	Black	Grey
Gender	Male	$\frac{(-0.8)^2}{10.8}$	$\frac{(k-20.6)^2}{21.6}$	$\frac{(21.4-k)^2}{21.6}$
	Female	$\frac{(0.8)^2}{19.2}$	$\frac{(20.6-k)^2}{38.4}$	$\frac{(k-21.4)^2}{38.4}$

Therefore $\chi^2 = \frac{5}{54} + \frac{38.4(k-20.6)^2 + 21.6(20.6-k)^2}{829.44} + \frac{38.4(21.4-k)^2 + 21.6(k-21.4)^2}{829.44}$

$$= \frac{5}{54} + \frac{60(k-20.6)^2}{829.44} + \frac{60(k-21.4)^2}{829.44}$$

$$= \frac{5}{54} + \frac{125}{1728}\left[(k-20.6)^2 + (k-21.4)^2\right]$$

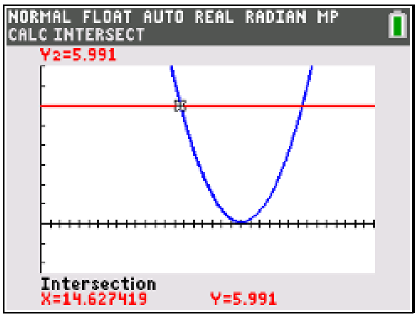
$$= \frac{5}{54} + \frac{125}{1728}\left[(k-20.6)^2 + (k-21.4)^2\right]$$

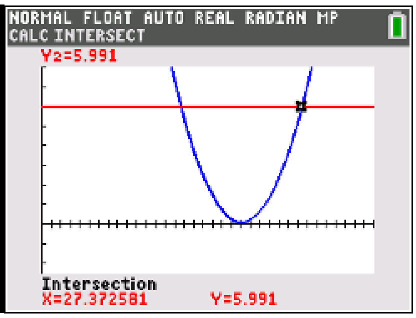
Degrees of freedom: $\nu = (2-1)(3-1) = 2$

Rejection region: $\chi^2 \geq 5.991$

Given that there is no association at 5% level between gender and colour preference of Jpad pro tablet, $\Rightarrow$ H_0 is not rejected at 5% level.

$$\therefore \frac{5}{54} + \frac{125}{1728}\left[(k-20.6)^2 + (k-21.4)^2\right] < 5.991$$





Using GC, $14.6 < k < 27.3$

Therefore, possible values of k are $15 \leq k \leq 27$ where $k \in \mathbb{Z}$

When $k = 30$

$$\chi^2 = \frac{5}{54} + \frac{125}{1728}\left[(30-20.6)^2 + (30-21.4)^2\right]$$

$\chi^2 = 11.83448$

$p-\text{value} = P\left(\chi^2 \geq 11.83448\right) = 0.0026926216 = 0.00269$ (3.sf)

$p-\text{value} = 0.00269 < 0.003$ $\therefore$ H_0 is rejected at 0.3% level of significance. This suggests that there is strong evidence of association between gender of customers and colour preference of Jpad pro tablet.

Question 7

(a) Let X and Y be the random variable denoting the scores of child after taking and before taking Vitalin respectively.
Let $D = X - Y$ and m be the median of the difference D.

H_0: $m = 0$
H_1: $m > 0$
at 5% significance level.

Child	1	2	3	4	5	6	7	8	9	10
Before	15	15	13	10	8	8	6	4	5	4
After	19	12	13	11	8	13	16	17	14	2
d_i	4	–3	0	1	0	5	10	13	9	–2
$\|d_i\|$	4	3	0	1	0	5	10	13	9	2
Rank	4	3	-	1	-	5	7	8	6	2

Sum of positive ranks, $P = 31$
Sum of negative ranks, $Q = 5$
$T_{calc} = \min(P, Q) = 5$

Now $n = 8$, reject H_0 if $T_{calc} \leq 5$.

Since $T_{calc} = 5$, we reject H_0. Thus, at 5% level of significance, the data provide sufficient evidence that the median score for children after taking Vitalin is higher.

(b)
If a sign test is conducted instead, $S_- = 2$ where S_- is the number of negative signs of D.

Under H_0, $S_- \sim B(8, 0.5)$.

p-value $= P(S_- \leq 2) = 0.14453 > 0.05$

Since p-value > 0.05, we do not reject H_0. Thus, at 5% level of significance, the data provide insufficient evidence to say that the median score for children after taking Vitalin is higher. The conclusion is reversed.

The conclusion from Wilcoxon Signed Rank Test should be taken as it takes into account the **magnitude of the difference** rather than just the signs.

Question 8

(i) $P(X = r) = p^r(1-p)$

(ii) $P(X < r) = 1 - P(X \geq r)$

$= 1 - P(\text{more than } r \text{ steps})$

$= 1 - p^r$

Or $P(X < r) = P(X = 0) + P(X = 1) + \ldots P(X = r-1)$

$= (1-p) + p(1-p) + \ldots p^{r-1}(1-p)$

$=\dfrac{(1-p)(1-p^r)}{1-p}$ $=1-p^r$
(iii) Let Y be the number of steps taken by the robot till it stops. $Y \sim \text{Geo}(1-p)$ $\text{E}(Y) = \dfrac{1}{1-p}$ $\text{E}(X) = \text{E}(Y-1) = \dfrac{1}{1-p}-1$ $9=\dfrac{1}{1-p}-1 \Rightarrow \dfrac{1}{1-p}=10 \Rightarrow p=1-\dfrac{1}{10}=0.9$
(b) (i) $\text{P}(X=r)=\binom{r+1}{1}p^r(1-p)^2 = (r+1)p^r(1-p)^2$
(ii) $\text{P}(X>3)=1-\text{P}(X\le 3)$ $=1-[\text{P}(X=0)+\text{P}(X=1)+\text{P}(X=2)+\text{P}(X=3)]$ $=1-\left[(1-p)^2+2p(1-p)^2+3p^2(1-p)^2+4p^3(1-p)^2\right]$ $=1-(1-p)^2(1+2p+3p^2+4p^3)$ $(=5p^4-4p^5)$ OR $\text{P}(X>3)=$ P(more than 5 steps required) = P(5 steps right) + P(4 steps right + 1 step up) $=p^5+\binom{5}{1}p^4(1-p)$ $=p^5+5p^4(1-p)$ $=5p^4-4p^5$

Question 9	
(a)	$1=\int_1^\infty \dfrac{k}{x^n}\,\text{d}x$ $1=\left[\dfrac{k}{x^{n-1}(-n+1)}\right]_1^\infty$ $1=k\left(0-\dfrac{1}{1-n}\right)$ $k=n-1$

(b)	$\mathrm{Var}(X)=\int_1^\infty x^2\frac{n-1}{x^n}\mathrm{d}x-\left(\frac{n-1}{n-2}\right)^2$ $=(n-1)\left[\frac{x^{2-n+1}}{3-n}\right]_1^\infty-\frac{(n-1)^2}{(n-2)^2}$ $=\frac{n-1}{3-n}(0-1)-\frac{(n-1)^2}{(n-2)^2}$ $=\frac{n-1}{n-3}-\frac{(n-1)^2}{(n-2)^2}$ (or simply to ..) $=\frac{(n-1)(n-2)^2-(n-1)^2(n-3)}{(n-3)(n-2)^2}$ $=\frac{(n-1)}{(n-3)(n-2)^2}=\frac{1}{(n-3)(n-2)}\mathrm{E}(X)$ $\frac{\mathrm{Var}(X)}{\mathrm{E}(X)}=\frac{1}{(n-3)(n-2)}$ Since $n>3$ and n is an integer, $\therefore n\ge 4\Rightarrow(n-3)\ge 1$ and $(n-2)\ge 2$ $(n-3)(n-2)\ge 2$ $\frac{1}{(n-3)(n-2)}\le\frac{1}{2}<1$ Hence $\frac{\mathrm{Var}(X)}{\mathrm{E}(X)}<1$ $\mathrm{Var}(X)<\mathrm{E}(X)$ Or $\mathrm{Var}(X)-\mathrm{E}(X)$ $=\frac{n-1}{n-3}-\frac{(n-1)^2}{(n-2)^2}-\frac{n-1}{n-2}$ $=(n-1)\left[\frac{(n-2)^2-(n-1)(n-3)-(n-2)(n-3)}{(n-3)(n-2)^2}\right]$ $=\frac{n-1}{(n-3)(n-2)^2}\left[n^2-4n+4-(n^2-4n+3)-(n^2-5n+6)\right]$ $=\frac{n-1}{(n-3)(n-2)^2}(-n^2+5n-5)$ $=\frac{n-1}{(n-3)(n-2)^2}\left[-\left(n-\frac{5}{2}\right)^2+\frac{5}{4}\right]$ Since n > 3 (integer), $\frac{n-1}{(n-3)(n-2)^2}\left[-\left(n-\frac{5}{2}\right)^2+\frac{5}{4}\right]<0$,

	For checking: $\text{E}(X)=\int_1^{\infty}(n-1)x^{1-n}\text{d}x=(n-1)\left[\dfrac{x^{2-n}}{2-n}\right]_1^{\infty}=\dfrac{n-1}{n-2}$
(c)(i)	$\text{P}(X\le x)=\int_1^{x}\dfrac{3}{x^4}\text{d}x=\left[-\dfrac{1}{x^3}\right]_1^{x}=-\dfrac{1}{x^3}+1$ c.d.f = F(x) = $\begin{cases}0, & x<1\\ 1-\dfrac{1}{x^3}, & x\ge 1\end{cases}$
(ii)	$\text{P}(X>x)=1-\left[-\dfrac{1}{x^3}+1\right]=\dfrac{1}{x^3}$ $\text{P}(X>7\mid X>5)$ $=\dfrac{\text{P}(X>7)}{\text{P}(X>5)}$ $=\dfrac{1-\text{F}(7)}{1-\text{F}(5)}$ $=\dfrac{\frac{1}{7^3}}{\frac{1}{5^3}}=\dfrac{125}{343}$

Question 10	
(a)	Let μ_x mm Hg and μ_y mm Hg be the population mean seated systolic blood pressure taken before the "CGarlic diet" and after the "CGarlic diet" respectively. $\text{H}_0:\mu_x-\mu_y=0$ $\text{H}_1:\mu_x-\mu_y>0$ Level of significance: 5% Assumptions: (1) The seated systolic blood pressure taken before and after the "CGarlic diet" are normally distributed. (2) The 2 samples of seated systolic blood pressure taken before and after the "CGarlic diet" are independent of each other. (3) The population variances of the seated systolic blood pressure taken before and after the "CGarlic diet" are equal. Test statistic: Use 2-sample t -test When H_0 is true, $T=\dfrac{\bar{X}-\bar{Y}}{S_p\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}}\sim t_{n_1+n_2-2}$ where $S_p^2=\dfrac{(n_x-1)S_x^2+(n_y-1)S_y^2}{n_x+n_y-2}$ Degrees of freedom: $\nu=10+10-2=18$ Rejection region: $t\ge 1.73406$

	Computation: $n_x = 10, \quad n_y = 10$ $\bar{x} = \frac{149}{10} + 100 = 114.9, \; s_x^2 = \frac{1}{10-1}\left[3277 - \frac{(149)^2}{10}\right] = 10.83666615^2$ $\bar{y} = \frac{99}{10} + 100 = 109.9, \; s_y^2 = \frac{1}{10-1}\left[1527 - \frac{(99)^2}{10}\right] = 7.795297728^2$ $s_p^2 = 9.439279631^2$ $t = 1.1844$ $p - \text{value} = 0.1258$ Conclusion: Since $p - \text{value} = 0.1258 > 0.05$ (or $t = 1.1844 < 1.73406$), $\therefore \text{H}_0$ is not rejected at 5% level of significance. Hence there is insufficient evidence to conclude that the "CGarlic diet" can reduce the seated systolic blood pressure.
(b)	Assumption: The decrease in seated systolic blood pressure is normally distributed. Let d denotes the decrease in seated systolic blood pressure. Let μ_D denotes the mean decrease in seated systolic blood pressure. Unbiased estimate of population mean decrease is $\bar{d} = \frac{50}{10} = 5$ Unbiased estimate of population variance is $s^2 = \frac{1}{9}\left[\sum d^2 - \frac{\left(\sum d\right)^2}{10}\right] = \frac{1}{9}\left[938 - \frac{(50)^2}{10}\right] = 8.743251366^2$ Since sample size is small and population variance is unknown, $\frac{\bar{d} - \mu}{S/\sqrt{n}} \sim t_{n-1}$ Therefore a 95% confidence interval for μ_D is $\left(5 - 2.262157\left(\frac{s}{\sqrt{10}}\right), \quad 5 + 2.262157\left(\frac{s}{\sqrt{10}}\right)\right)$ $= (-1.2545, \quad 11.2545)$ $= (-1.25, \quad 11.25)$
(c)	Let $D = X - Y$ $\text{H}_0 : \mu_D = 0$ $\text{H}_1 : \mu_D \neq 0$ From (ii) since the interval contains $\mu_D = 0$, Hence H_0 is not rejected at 5% level of significance. Hence there is insufficient evidence to conclude that there is a change in the seated systolic blood pressure before and after the "CGarlic diet."