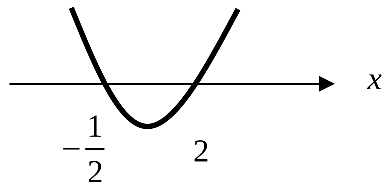
1	(i) $2x^2-3x-2\geq 0$ $(2x+1)(x-2)\geq 0$
	Sketch the graph:
	
	$x\geq 2$ or $x\leq -\frac{1}{2}$
	(ii) $-\frac{1}{2}<e^x<2$
	$x<\ln 2$
2	(a) Let $y=\left(3x^2-\frac{2}{\sqrt{x}}\right)^2$
	$y=9x^4-12x^{\frac{3}{2}}+\frac{4}{x}$
	$\frac{dy}{dx}=36x^3-18\sqrt{x}-\frac{4}{x^2}$
	(b) $\int\frac{2}{\sqrt[3]{5-4x}}\,dx=2\int(5-4x)^{-\frac{1}{3}}dx$
	$=2\frac{(5-4x)^{\frac{2}{3}}}{\frac{2}{3}(-4)}+C=-\frac{3}{4}(5-4x)^{\frac{2}{3}}+C$ where C is an arbitrary constant.

3 (a) $\dfrac{dy}{dx} = 8e^{1-4x}$

When $x = 1$, $y = 8 - 2e^{-3}$

$\dfrac{dy}{dx} = 8e^{1-4(1)} = 8e^{-3}$

Equation of tangent at the point $x = 1$:

$$y - (8 - 2e^{-3}) = 8e^{-3}(x-1)$$

$$y = 8e^{-3}x - 8e^{-3} + 8 - 2e^{-3}$$

$$y = 8e^{-3}x - 10e^{-3} + 8 \quad \text{where } m = 8e^{-3} \text{ and } c = 8 - 10e^{-3}$$

(b) $A = \displaystyle\int_{\frac{1}{4}}^{1} 8 - 2e^{1-4x}\,dx$

$$= \left[8x + \frac{1}{2}e^{1-4x} \right]_{\frac{1}{4}}^{1}$$

$$= \left[8 + \frac{1}{2}e^{-3} \right] - \left[2 + \frac{1}{2} \right]$$

$$= 5\frac{1}{2} + \frac{1}{2e^{3}}$$

$p = 5\dfrac{1}{2}$ and $q = \dfrac{1}{2}$

4 (i) $y = x^4 - 10x^3 + 36x^2 - 54x + 17$

$\dfrac{dy}{dx} = 4x^3 - 30x^2 + 72x - 54$

$\dfrac{dy}{dx} = 0$

$4x^3 - 30x^2 + 72x - 54 = 0$

Using GC, $x = 1.5$ or $x = 3$

When $x = 1.5$, $y = -11.6875$

When $x = 3$, $y = -10$

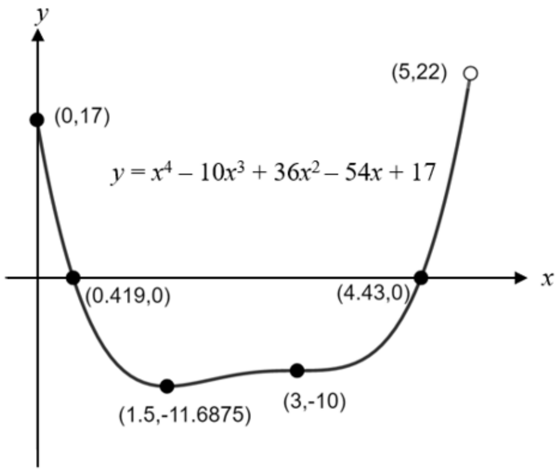
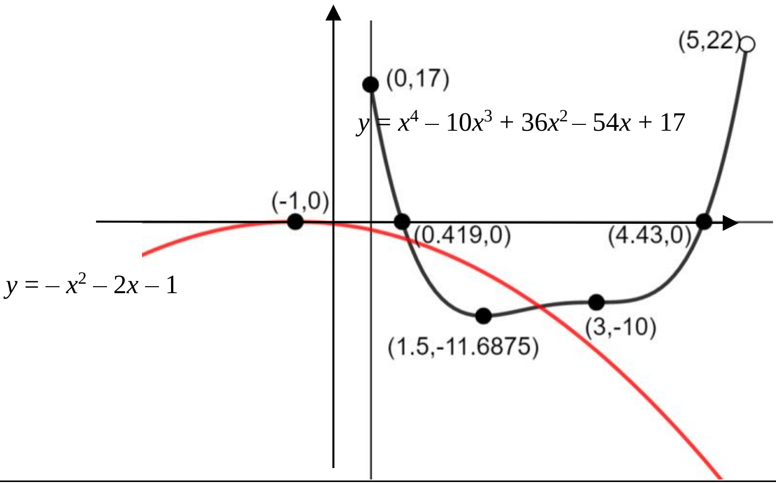
Coordinates of stationary points are $(1.5, -11.6875)$ and $(3, -10)$

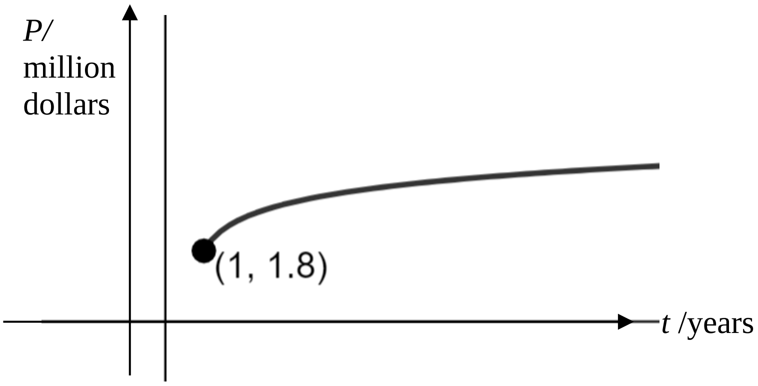
(ii)

x	1.4	1.5	1.6
$\dfrac{dy}{dx}$	-1.0240044	0	0.7839964
Slope			

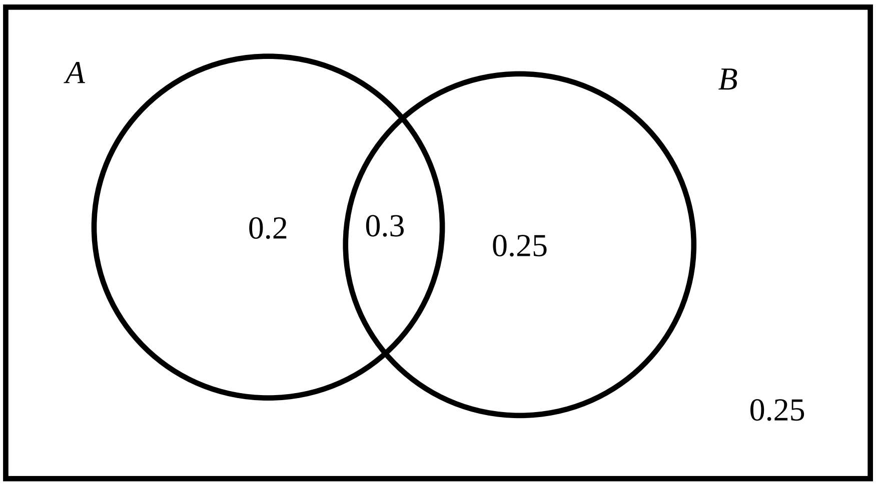
$(1.5, -11.6875)$ is a minimum point.

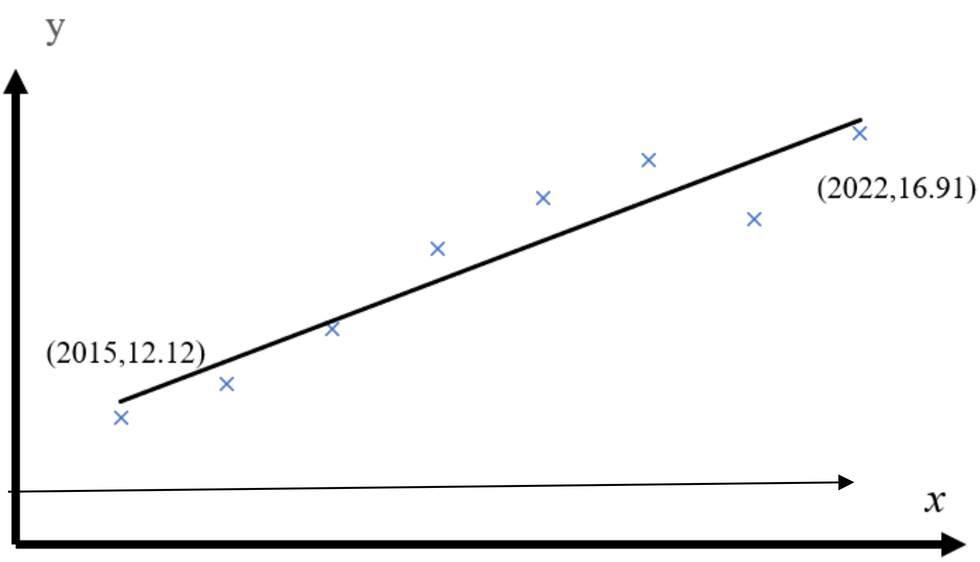
x	2.9	3	3.1
$\dfrac{dy}{dx}$	0.0560016	0	0.0640024
Slope			

	(3, – 10) is an inflexion point.
	(iii) 
	(iv) $x^4 - 10x^3 + 37x^2 - 52x + 18 = 0$ $x^4 - 10x^3 + 36x^2 - 54x + 17 + x^2 + 2x + 1 = 0$ $x^4 - 10x^3 + 36x^2 - 54x + 17 + x^2 + 2x + 1 = 0$ $x^4 - 10x^3 + 36x^2 - 54x + 17 = - x^2 - 2x - 1$ 
	(v) Using GC, $x = 0.503$ or $x = 2.246$ (to 3 dp)
5	(a) Let x, y and z be the price of a Brand A, Brand B and Brand C laptop respectively. $16x + 23y + 20z = 109\ 741$ ------------ (1) $20z = 16x + 23\ 596$ $- 16x + 20z = 23\ 596$ ------------ (2) $2x + 2y + 2z = 10\ 994$ ------------ (3) Using GC, $x = 1399$, $y = 1799$ and $z = 2299$. 2 equations correct – 1 mark 3 equations correct – 2 marks The total price of two Brand A laptops and three Brand C laptops = \$[2(1399) + 3(2299)]

	= \$9695
	(b) **(i)** 
	(ii) P = 1.8 + 0.6ln (9 – 2) = 2.9675 = 2.97 million dollars The value 2.97 million dollars represents the annual profits in the year up to time t = 3 years.
	(iii) P = 1.8 + 0.6ln (3t – 2) $\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1.8}{3t-2}$ or $\frac{9}{5(3t-2)}$
	(iv) For $t \geq 1$, $\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1.8}{3t-2} = \frac{9}{5(3t-2)} > 0$ As the years t increases, the annual profits are expected to increase at a decreasing rate.
	(v) When t = 5, $\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1.8}{3(5)-2} = 0.13846 = 0.138$ million dollars/year

6	(i) The customers can be numbered from 1 to 10 000. Using a random number generator, 1000 numbers can be generated. The customers whose number matches the number generated, will be asked to do the survey.
	(ii) The advantage of random sampling is that everyone has equal chance of being selected. The disadvantage of random sampling is that it is time consuming.
7	(i) Number of committees that can be formed = ${}^7C_2 = 21$
	(ii) Required Probability = $\frac{{}^7C_3}{{}^9C_4} + \frac{{}^7C_3}{{}^9C_4} = \frac{70}{126} = \frac{5}{9}$
	(iii)Number of ways = $4! - 3! \times 2 = 12$
8	(i) $\mathrm{P}(F \cup B') = \frac{300-23}{300} = \frac{277}{300}$
	(ii) $\mathrm{P}(B \mid M) = \frac{23}{23+80+35} = \frac{1}{6}$
	(iii) $\mathrm{P}(M \cap E) = \frac{80}{300} = \frac{4}{15}$
	(iv) Since $\mathrm{P}(M)\mathrm{P}(E) = \frac{138}{300} \times \frac{137}{300} = 0.210 \neq \mathrm{P}(M \cap E)$, M and E are not independent events.
	(v) Probability = $\frac{63}{300} \times \frac{62}{299} \times \frac{237}{298} \times 3 = 0.10389 \approx 0.104$
9	(i) $\mathrm{P}(B \mid A') = \frac{1}{2}$ $\frac{\mathrm{P}(B \cap A')}{\mathrm{P}(A')} = \frac{1}{2}$ $\mathrm{P}(B \cap A') = \frac{1}{2} \times \left(1 - \frac{1}{2}\right) = 0.25$
	$\mathrm{P}(A \cup B) = 0.5 + 0.25 = 0.75$
	(ii) $\mathrm{P}(A \mid B') = \frac{4}{9}$ $\frac{\mathrm{P}(A \cap B')}{1 - \mathrm{P}(B)} = \frac{4}{9}$
	Let $\mathrm{P}(A \cap B) = x$
	$\frac{0.5 - x}{1 - (0.25 + x)} = \frac{4}{9}$ $9(0.5 - x) = 4 - 4(0.25 + x)$ $x = 0.3$

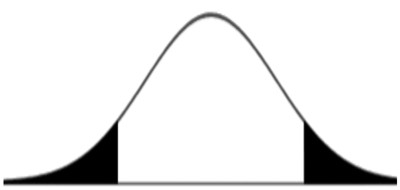


10	(i) Let X be the number of trees that will produce pears in the first year after planting, out of 5. $X \sim \mathrm{B}(5, 0.85)$
	$\mathrm{P}(X < 3) = \mathrm{P}(X \le 2) = 0.026611$
	$\mathrm{P}(\text{no replacement needed}) = 1 - 0.026611 = 0.97338 \approx 0.973$
	(ii) $0.026611 \times 3000 \approx 79.8$ customers
	(iii) Let Y be the number of customers that require a replacement. $Y \sim \mathrm{B}(3000, 0.026611)$
	$\mathrm{P}(Y > 100) = 1 - \mathrm{P}(Y \le 100) = 1 - 0.98847 \approx 0.0115$
	(iv) Since n = 3000 is large, by Central Limit Theorem, $\overline{X} \sim \mathrm{N}\left(5 \times 0.85, \frac{5 \times 0.85 \times 0.15}{3000}\right)$ approximately
	$\mathrm{P}(\overline{X} > 4.28) = 0.019795 \approx 0.0198$
11	 (i)

	(ii) Product Moment Correlation Coefficient, r-value is 0.940. There is a strong positive linear correlation between the year and the total cards billing in Singapore in the 1st Quarter. The total cards billing increases year after year.
	(iii) $y = 0.676071x - 1349.898929$ $y = 0.6761x - 1349.8989$ (4 dec places)
	(iv) $y = 0.676071(2023) - 1349.898929 = 17.79 \approx 17.8$ The total cards billing is estimated to be \$17.8 thousand million in year 2023. Although the product moment correlation coefficient r-value = 0.940 is close to 1, $x = 2023$ is outside of data range of [2015 to 2022], the linear relationship between the total cards billing and year may not hold (extrapolation). Hence the estimate is unreliable.

12	(i) Let X be the random variable denoting "the time in minutes taken by Tom to service a randomly chosen car". Let Y be the random variable denoting "the time in minutes taken by Tom to service a randomly chosen van". $X \sim \mathrm{N}(90, 5^2)$, $Y \sim \mathrm{N}(130, 10^2)$
	(i) $\mathrm{P}(Y \le t) = 0.95$
	$t = 146.45 \approx 146$
	(ii) $Y - 2X \sim \mathrm{N}\left(130 - 2\times 90, 10^2 + 2^2 \times 5^2\right)$ $Y - 2X \sim \mathrm{N}(-50, 200)$
	$\mathrm{P}(-60 \le Y - 2X \le 60) = 0.76025 \approx 0.760$
	Let V be the random variable denoting "the time in minutes taken by Tom to take a break".
	$V \sim \mathrm{N}(5.5, 1.4^2)$
	(iii) $X_1 + X_2 + Y + V_1 + V_2 \sim \mathrm{N}\left(2\times 90 + 130 + 2\times 5.5, 2\times 5^2 + 10^2 + 2\times 1.4^2\right)$ $X_1 + X_2 + Y + V_1 + V_2 \sim \mathrm{N}(321, 153.92)$
	$\mathrm{P}(X_1 + X_2 + Y + V_1 + V_2 < 5\times 60) = 0.045259 \approx 0.0453$

	(iv) Let $C=\frac{50}{60}\left(X_1+X_2\right)+\frac{90}{60}\left(Y\right)$ $C\sim \mathrm{N}\left(\frac{5}{6}\times 2\times 90+\left(\frac{9}{6}\right)\times 130,\left(\frac{5}{6}\right)^2\times 2\times 5^2+\left(\frac{9}{6}\right)^2\times 10^2\right)$ $C\sim \mathrm{N}(345, 259.722)$
	$\mathrm{P}\left(C>350\right)=0.37818\approx 0.378$

13	(i) Unbiased estimate of population mean $= \dfrac{303.4}{64} = 4.740625$ (exact) Unbiased estimate of population variance $= \dfrac{1}{63}\left(1615.96 - \dfrac{303.4^2}{64}\right)$ $= 2.8199\left(\text{to 5sf}\right)$ $= 2.82\left(\text{to 3sf}\right)$
	(ii) Let X be the random variable denoting "the crop weight per plant" and μ be "the population mean crop weight per plant" To test $H_0 : \mu = 5$ against $H_1 : \mu < 5$ (horticulturist's claim)
	(iii) Under H_0, $\bar{X} \sim N(5, \dfrac{2.8199}{64})$ approximately by Central Limit Theorem since $n = 64$ is large, Using GC, the test statistics $\bar{x} = 4.740625$ gives $z_{calc} = -1.2357$ and p-value = 0.10829 = 0.108(to 3 sig fig) Since p-value = 0.108 > 0.1, we do not reject H_0 and there is insufficient evidence to conclude that the mean crop weight per plant is less than 5kg at 10% level of significance. The horticulturist's claim is not supported by the data.
	(iv) Reject H_0 when p-value < level of significance $\dfrac{\alpha}{100} > 0.10829$ $\alpha > 10.8\%$
	(v) Level of significance is the probability of concluding the mean crop weight per plant is less than 5 kg when in fact it is not.
	(vi) Let Y be the random variable denoting "the crop weight per plant in region B" To test $H_0 : \mu = 5$ $H_1 : \mu \neq 5$ Two tail–tailed test at 1% level of significance Under H_0, $\bar{Y} \sim N(5, \dfrac{1.5^2}{20})$ H_0 is not rejected $\bar{y}$ lies outside the critical region.  $4.14 < m < 5.86$
	(vi) The crop weight per plant in region B follows a normal distribution.