Xinmin Secondary School
Math Department
Additional Mathematics: Calculus Practice

Name: ______________________________ (   )   Date: ___________

Differentiation and Integration

1 XMS 2020 AM P2, Q4 [2]

(a) Find $\int \frac{1-e^{2x}}{1-e^{x}} dx$.

(b) It is given that $\int_1^3 ax^2 dx = 7$, where a is a constant. Find the value of $\int_2^6 ax^2 dx$. [3]

2 XMS 2020 AM P2, Q6

(a) Differentiate $4e^{-x}(\cos x + \sin x)$ with respect to x. [3]

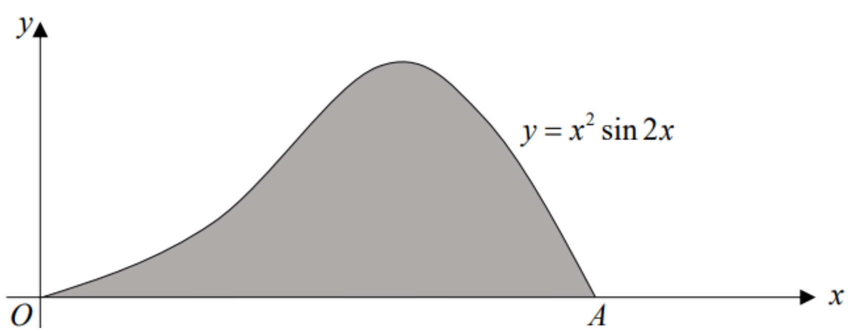
(b) Hence evaluate $\int_0^{\frac{\pi}{2}} e^{-x} \sin x \, dx$. [2]

3 XMS 2022 AM P1, Q12

(a) Find $\frac{d}{dx}(x \sin 2x)$. [1]

(b) Hence, find $\int 2x \cos 2x \, dx$. [1]

(c) The diagram below shows the curve $y = x^2 \sin 2x$, which passes through the origin and A. Differentiate $x^2 \cos 2x$ with respect to x and using the result found in part **(b)**, find the area of the shaded region, which is bounded by the curve $y = x^2 \sin 2x$ and the x-axis.



[6]

4 **MFSS 2022 AM P2, Q7**

(i) Given that $y = \frac{x}{(3x+2)^2}$, show that $\frac{dy}{dx} = \frac{2-3x}{(3x+2)^3}$. [4]

(ii) Hence, find the value of $\int_0^1 \frac{-3x}{(3x+2)^3}\,dx$. [4]

5 **FMSS 2022 AM P2, Q2**

(i) Differentiate $y = e^{2x}(5x-4)$ with respect to x. [3]

(ii) Hence find $\int 3xe^{2x}\,dx$. [3]

Integration to find equation of a graph

1 **XMS 2022 AM P1, Q3**

Given that $f'(x) = \frac{6}{3x+5}$ and that $f(x)$ passes through the origin, find the equation of $f(x)$. [3]

2 **MFSS 2022 AM P1 Q8**

The equation of the **gradient** of a curve is $f'(\theta) = \cos 2\theta - 3\sin 2\theta$. Given that the curve $y = f(\theta)$ passes through the point $\left(\frac{\pi}{2}, -\frac{1}{2}\right)$, find

(i) the equation of the curve, [3]

(ii) the coordinates of the turning points of the curve for $0 \le \theta \le \pi$. [4]

3 **FMSS 2022 AM P2, Q4**

A curve is such that $\frac{d^2y}{dx^2} = 6x - 2$. The gradient of the curve at the point $(2, -9)$ is 3.

(i) Find the equation of the curve. [6]

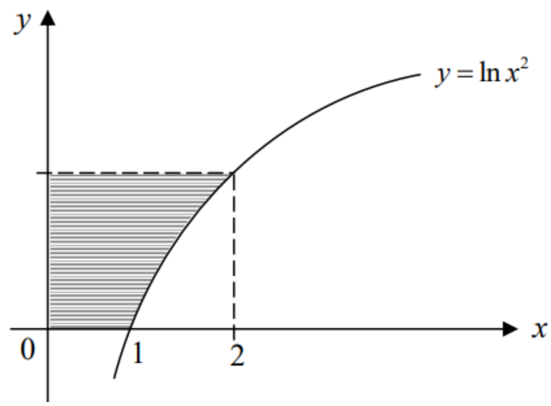
(ii) Show that the gradient of the curve is never less than $-\frac{16}{3}$. [3]

Integration: Area under Curve

1 **XMS 2020 AM P1, Q1**

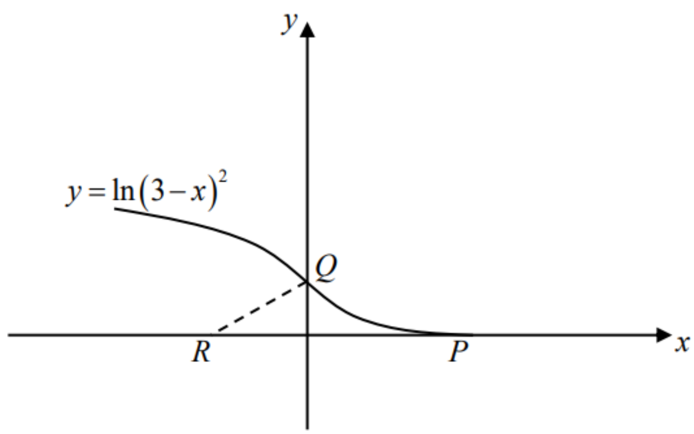
The diagram shows the graph of $y = \ln x^2$ for $x > 0$. [4]

Find the area of the shaded region.



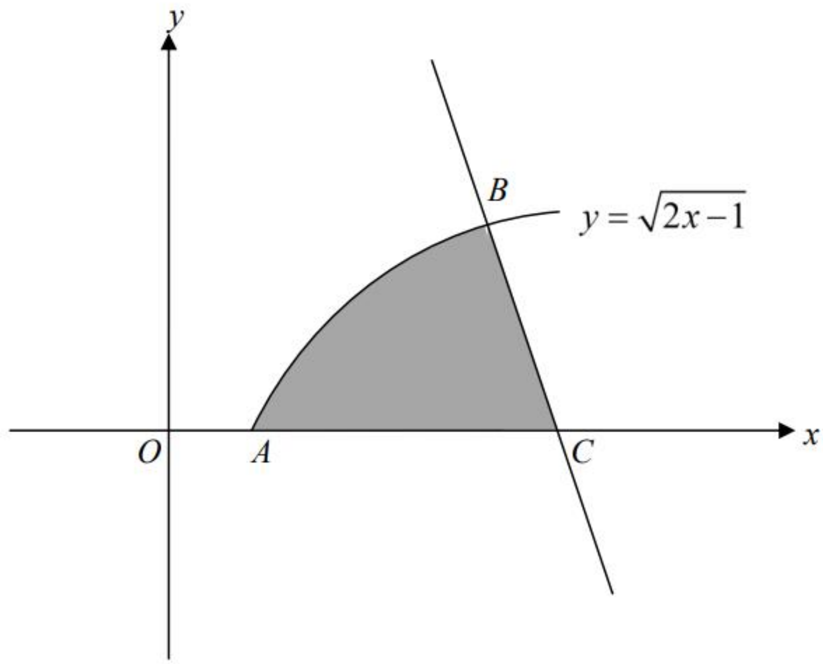
2 **XMS 2022 AM P2, Q11**

The diagram shows part of the curve $y = \ln(3-x)^2$ which crosses the axes at P and Q.



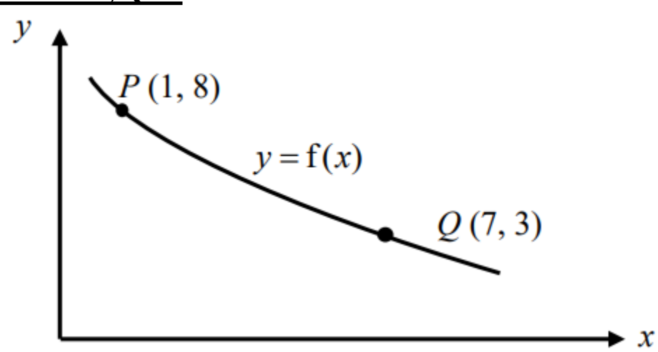
The normal to the curve at Q meets x-axis at R. Find the area of the region bounded by the curve, RQ and the x-axis. [12]

3



The diagram shows part of the graph of $y=\sqrt{2x-1}$ intersecting the x-axis at A and has a gradient of $\frac{1}{3}$ at B. The normal of the graph at B intersects the x-axis at C. By first finding the coordinates of A, B and C or otherwise, show that the area of the shaded region is $10\frac{1}{2}$ units^2. [12]

4 **<u>FMSS 2022 AM P2, Q9b</u>**



The diagram shows part of the curve $y=f(x)$. Given that the points P (1, 8) and Q(7, 3) lie on the curve and that $\int_1^7 y\ \mathrm{d}x = 29$, find the numerical value of $\int_3^8 x\,\mathrm{d}y$.

[3]

Kinematics

1 XMS 2020 AM P2, Q8

A particle moves in a straight line, so that t seconds after leaving a fixed point O, its velocity, v m/s, is given by $v = t^2 - kt + 24$. The particle passes O with an acceleration of $-10\,\text{m/s}^2$.

(i) Show that $k = 10$. [2]

(ii) Find the time when the particle reaches minimum velocity. [3]

(iii) Find the distance travelled by the particle in the tenth second. [4]

(iv) Explain with workings, whether the particle will return to O for $t > 9$. [2]

2 XMS 2022 AM P1, Q13

A motorcycle is rode along a straight horizontal road. As it passes a point A, the brakes are applied and the motorcycle slows down, with its velocity halved as it passes B.
For the journey from A to B, the displacement, s m, of the motorcycle from A, t seconds after passing A, is given by $s = 400\left(1 - e^{-\frac{t}{10}}\right)$.

(a) Find the exact time taken for the journey from A to B. [4]

(b) Find the average speed of the motorcycle for the journey from A to B. [2]

(c) Find the exact acceleration of the motorcycle at B. [2]

3 MFSS 2022 AM P1, Q12

A particle moves in a straight line, such that, t seconds after passing a fixed point O, its velocity, v m/s, is given by $v = \frac{2}{5}e^{3t} - 6e^{\frac{1}{2}-t}$. The particle comes to an instantaneous rest at point A.

(i) Show that the particle reaches A when $t = \frac{1}{8}(1 + 2\ln 15)$. [2]

(ii) Find the acceleration of the particle at A. [2]

(iii) Find the distance OA. [4]

(iv) Explain whether the particle will pass through point O **again** at some instant during the 2nd second. [2]

4 FMSS 2022 AM P1, Q11

A particle moves in a straight line. At time t seconds, after the beginning of the motion, 3m from the fixed point O, its velocity v m/s, is given by $v = 16 + 6t - at^2$, where a is a constant.

(a) Given that the maximum velocity is 25 m/s, show that $a = 1$. [3]

Hence, calculate

(b) the value of t when the velocity of the particle is equal to its initial velocity, [2]

(c) the value of t when the particle is momentarily at rest, [2]

(d) the distance travelled by the particle in the first nine seconds. [3]