INDEX NUMBER

Anglo - Chinese School
(Independent)

FINAL EXAMINATION 2021

YEAR THREE EXPRESS

ADDITIONAL MATHEMATICS

PAPER 2 **4049/02**

Tuesday **12 October 2021** **1 hour 30 minutes**

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number in the space at the top of this page.
Write in dark blue or black pen.
You may use an HD pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 60.

For Examiner's Use
/ 60

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where *n* is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions.

1 A right angled triangle has an area of $16+6\sqrt{2}\ \text{cm}^2$ and a base length of $4-2\sqrt{2}$ cm. Without using a calculator, find the perpendicular height of the triangle in the form of $a+b\sqrt{2}$ where a and b are integers. [4]

2 Given that $8x^2 + Ax - 5 \equiv (3-2x)^2 + B(x-1)^2 - 3(C-x)$ for all real values of x, find the value of each of the constants A, B and C. [4]

3 **(a)** Prove that $(\sec\theta - \tan\theta)^2 \equiv \dfrac{1-\sin\theta}{1+\sin\theta}$. [4]

(b) Hence, solve the equation $(\sec\theta - \tan\theta)^2 = 3$ for $0^\circ \le \theta \le 360^\circ$. [3]

4 **(a)** Solve the equation $10\ \sin x + 9 = -2\,\text{cosec}\ x$ for $0^{\circ} \leq x \leq 360^{\circ}$ [5]

(b) Solve the equation $\sin\left(2\theta + \dfrac{\pi}{4}\right) = \cos\left(2\theta + \dfrac{\pi}{4}\right)$ for $0 \leq \theta \leq \pi$.

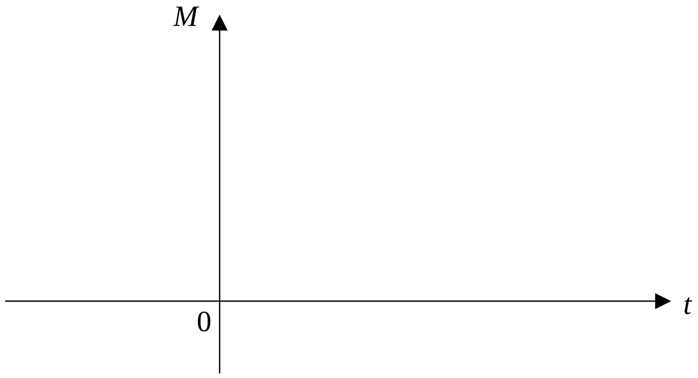
Leave your answer (s) in terms of π. [3]

4 **(c)** On the same axes, sketch the graphs of $y = 2\cos 2x$ and $y = 2\sin x + 1$ for $0^\circ \le x \le 360^\circ$.

Hence state the number of solutions for which $\frac{1}{2} - \cos 2x = -\sin x$ [5]

5 A radioactive substance, Radium (Ra^{226}), of mass 100g is decaying such that after t years, the amount remaining, M, is given by the equation $M = 100e^{-0.0004279t}$.

(a) Sketch the graph of M against t, indicating any axes intercepts. [1]



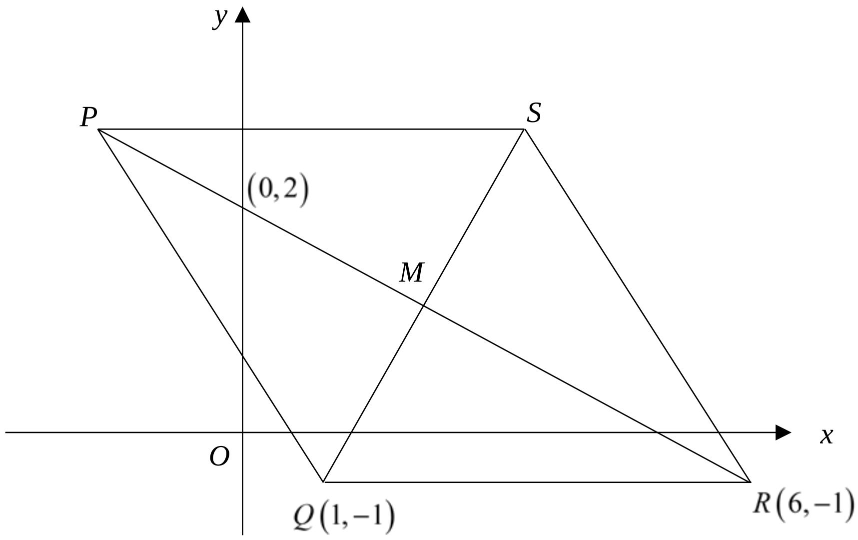
(b) Find the amount of the Radium remaining after 20 000 years. [2]

(c) Find the half-life of the Radium (i.e. the time taken to decay to half the initial mass). [3]

6 **(a)** Given that the curve $y = 3x^2$ lies completely above the line $y = px - 3$. Find the range of values of p. [5]

(b) It is given that $-\frac{1}{2} < x < \frac{2}{3}$ is the solution set to the quadratic inequality $px^2 + qx - 2 < 0$ where p and q are integers. Find the value of p and of q. [3]

7 $Q(1,-1)$ and $R(6,-1)$ are two vertices of a rhombus *PQRS* as shown in the diagram below. *PR* and *QS* intersect at *M*. *PR* cuts the y – axis at $(0,2)$.



(a) Find the equation of *PR* and *QS*. [4]

7 **(b)** Calculate the coordinates of M, P and S. [6]

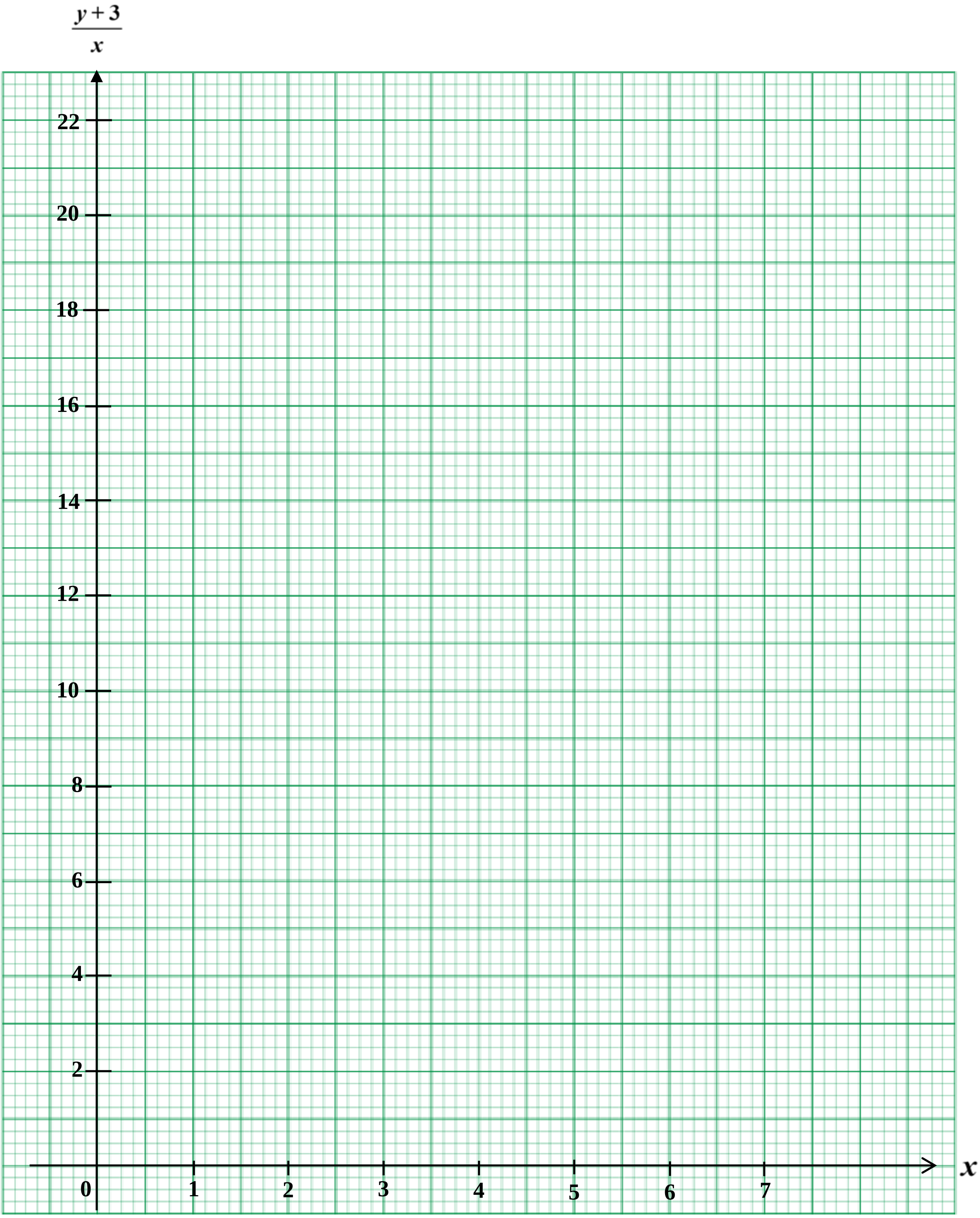
8 The table below shows experimental values of two variables x and y. The variables x and y are related by the equation $y = ax^2 + bx - 3$, where a and b are constants.

x	1	2	3	4	5
y	2	15	36	65	102

(a) On the grid opposite, draw the graph of $\frac{y+3}{x}$ against x to obtain a straight line graph. [3]

(b) Use your graph to estimate the value of a and of b. [3]

(c) Explain how you would use your graph to solve the equation $ax^2 + (b - 20)x = 0$. [2]



END OF PAPER

Answers

1. $22+14\sqrt{2}$ cm

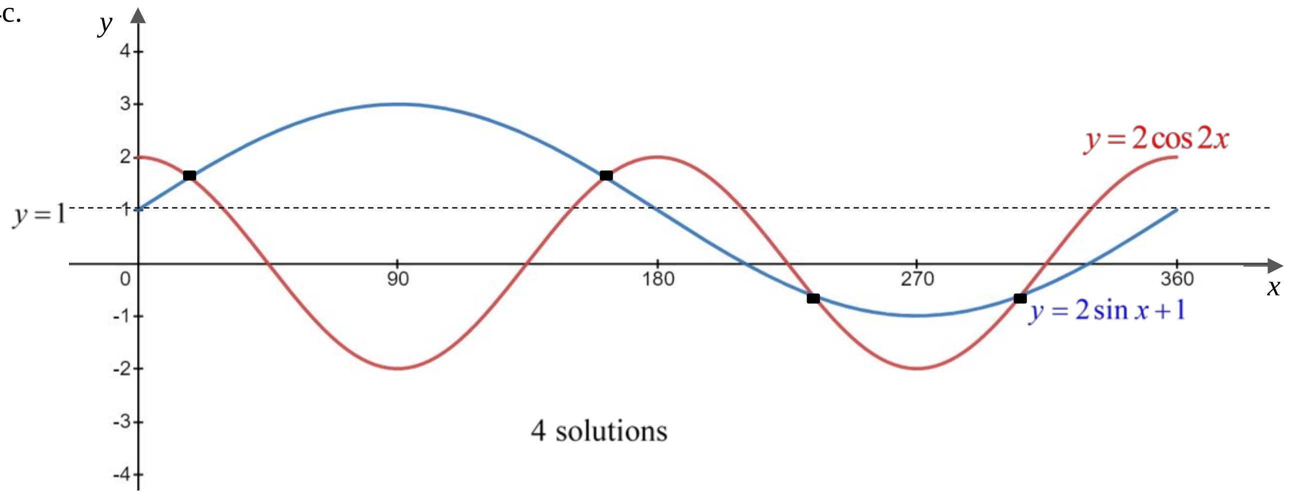
2. $A=-17, B=4, C=6$

3b. $\theta=210^\circ, 330^\circ$

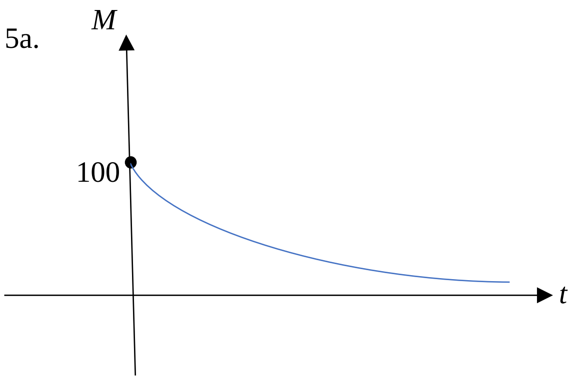
4a. $x=203.6^\circ,\quad 210^\circ,\quad 336.4^\circ,\quad 330^\circ$

4b. $\theta=0,\ \frac{\pi}{2},\ \pi$ rad

4c.



5a.



5b. 0.192 g

5c. 1620 years

6a. $-6<p<6$

6b. $p=6,\ q=-1$

7a. Equation of $PR: y=2x-3$, Equation of $QS: y=-\frac{1}{2}x+2$

7b. $M(2,1), P(-2,3), S(3,3)$

8b. $a\simeq 4.0, b\simeq 1.0$

8c. Insert the horizontal line $\frac{y+3}{x}=20$, or $Y=20$ then read off the corresponding value of x from the point of intersection