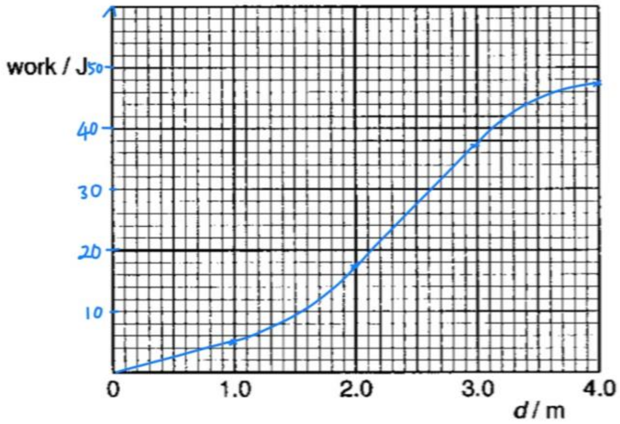
Topic 5 Work Energy Power

Suggested Solutions to Discussion Questions

<table>
<tr><th>Qn</th><th>Ans</th><th>Explanation</th></tr>
<tr><td>D1</td><td colspan="2">[2008/P3/Q1(part)]


F = gradient of W-d graph, W = area under F-d graph,

From d = 0 to 1.0 m,
W increases linearly (as F is positive & constant), from 0 to 1.0 x 5.0 = 5.0 J

From d =1.0 to 2.0 m,
W increases at increasing rate (as F is positive & increasing),
from 5.0 to [5.0 + (½)(15)(1.0)] = 17.5 J

From d = 2.0 to 3.0 m,
W increases linearly (as F is positive & constant),
from 17.5 J to [17.5 + 1.0(20)] = 37.5 J

From d = 3.0 to 4.0 m,
W increase at decreasing rate (as F is positive & decreasing),
from 37.5 J to [37.5 + (½)(1.0)(2.0)] = 47.5 J</td></tr>
<tr><td>D2</td><td>C</td><td>[CIE June 2003]
There is no change in kinetic energy, since the trolley starts and finishes at rest, so the weight is only gaining gravitational potential energy.

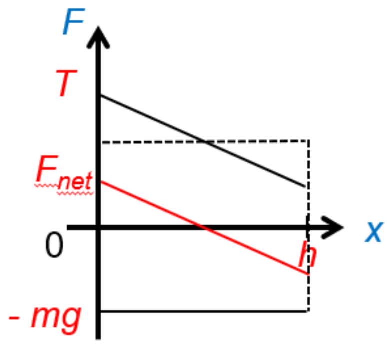
work done by tension force (trolley) $= \Delta E_P = mg\Delta h = Wq$

To better appreciate and understand the situation, one possible scenario is as shown in the F- x graphs.

Tension force, T is a varying force, weight mg acts in opposite direction.

Comparing the area under each graph,

positive work done by T = negative work done by mg
Total work done = work done by net force F_{net} = 0 J

thus, satisfying the condition that the weight is lifted from rest and finishes at rest with no change in KE.

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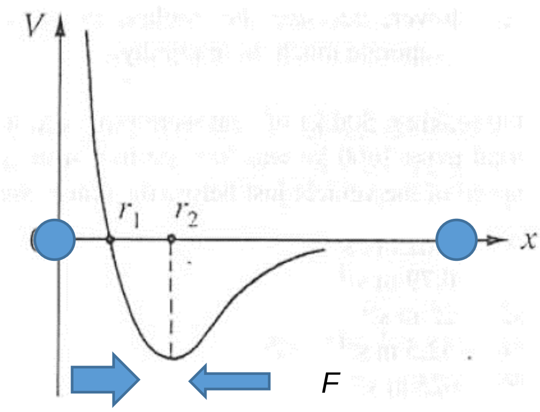
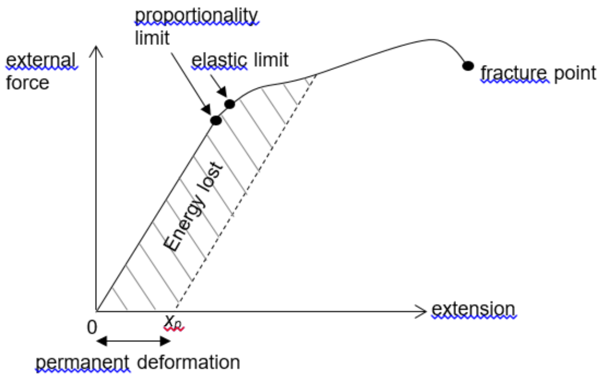
D3	**C**	*F = -dU/dx or the negative of the gradient of U-x graph.* The force on the body is the negative gradient of the potential energy-position graph. Since the gradient of the graph is positive and constant, the force is negative and constant, implying that the force acts in the opposite direction to the displacement. In other words, the force acts towards the origin, in the direction towards lower potential energy.
D4	**E**	*F = -dU/dx* The force between the molecules is the negative of the gradient of the potential energy (of the 2 molecules)-distance graph.  When $x < r_2$, the gradient of tangent line at a point on the graph is negative, the intermolecular force is thus positive (in the direction of +x), it is repulsive. When $x > r_2$, the gradient of tangent line at a point on the graph is positive, the intermolecular force is negative (in the opposite direction to +x), it is attractive. Further reference: https://www.schoolphysics.co.uk/age16-19/Properties%20of%20matter/Elasticity/text/Intermolecular_forces/index.html
D5	**D**	**[H1 N2009/P1/Q6 & 2015 P1 Q11]** Recall Work done by a force = area under the force-displacement graph. Hence option D is correct.  As for option A, work done by *F* is equal to elastic potential energy stored in the wire only when the wire is within its elastic limit. Since the wire is stretched beyond its elastic limit, (as seen on the left) not all work done by *F* becomes elastic potential energy, the energy is lost and cannot be recovered when releasing the load.
D6	**C**	**[H1 2010/P1/Q11]** Area under graph from *x* = 0 to extension up to point Q = ∫Fdx which is the work done by the applied force. When the force is released, only Z is returned as energy released by the wire. Since Z is the energy released by wire, the EPE stored in wire at Q is Z. Y is usually energy lost as heat in the material.

D7	**A**	**[J1987/P1/Q5]** When an object rises to the top, its KE will be totally converted into GPE: Loss in KE = Gain in GPE $\frac{1}{2}mv^2 - 0 = mgh \Rightarrow h = \frac{v^2}{2g}$ From this we can see that $h \alpha v^2$. $\Rightarrow \frac{h_1}{h_2} = \left(\frac{v_1}{v_2}\right)^2$ $\frac{h}{h_2} = \left(\frac{v}{\frac{1}{2}v}\right)^2$ $h_2 = \frac{1}{4}h$

when there is no work done by external force, by conservation of energy,

total initial energy = total final energy

i.e. $KE_{initial} + PE_{initial} = KE_{final} + PE_{final}$ (1st possible starting point)

i.e. $KE_{initial} - KE_{final} = PE_{final} - PE_{initial}$ or $KE_{final} - KE_{initial} = PE_{initial} - PE_{final}$

i.e Loss in KE = Gain in PE or Gain in KE = Loss in PE (2nd possible starting point)

i.e. $-\Delta KE = \Delta PE$ or $\Delta KE = -\Delta PE$

i.e. $\Delta KE + \Delta PE = 0$ (3rd possible starting point)

Realise that all the above Energy equations are equivalent, but we choose the one that is more convenient for our problem solving.

D8	**[College Physics by Serway & Faughn]**	
	Loss in GPE of mass-spring system = 2.00 x 9.81 x 0.200 x sin 37° =2.3615 J	M1
	Gain in EPE of mass-spring system = ½ $k\,x^2$ = ½ x 100 x 0.200^2 = 2.00 J	B1
	By Conservation of Energy, Loss in GPE = Gain in EPE + WD against friction WD against friction = Loss in GPE – Gain in EPE F_{fr} x 0.200 = 2.3615 – 2.00 = 0.3615 $\Rightarrow F_{fr}$ = 1.81 N.	A1
	OR Work done by friction = change in GPE + change in EPE $W = \Delta E_p + \Delta E_s$ $F_{fr} s \cos 180° = mg\Delta h + \frac{1}{2}kx^2$ $F_{fr}\left(-\frac{20}{100}\right) = (2.00)(9.81)\left(-\frac{20}{100}\right)(\sin 37°) + \frac{1}{2}(100)\left(\frac{20}{100}\right)^2$ $F_{fr} = 1.81\ \text{N}$	

D9(a) | **A** | **[J1988/P1/Q6]**

Heat generated by friction

= work done against friction

= $F x$ (since the small block would have moved up the slope by distance x when the large block had moved down by x)

D9(b)

By the Principle of Conservation of Energy,

Loss in GPE of M = Gain in KE of M & m + Gain in GPE of m + Work done against friction

Gain in KE of M & m = Loss in GPE of M – Gain in GPE of m – Work done against friction

$= Mgx - mgx \sin\theta - Fx$

D10 | **C** | **[2016 Specimen P1Q6]**

$E_k + E_p = E_{Total}$ where E_{Total} is constant

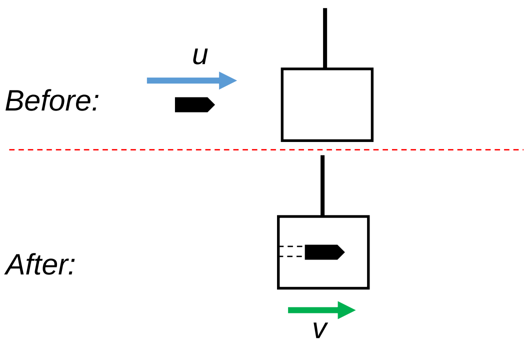
At the starting point, $E_{Total} = E_k = \frac{1}{2} mu^2$,

at the top of flight, $v = u \cos 45°$

$E_k = \frac{1}{2} m(u^2 \cos^2 45°) = \frac{1}{2} \times \frac{1}{2} mu^2 = \frac{1}{2} E_{Total}$, $E_p = E_k$

D11 | **C** | **[HCI Promo 1999/1/6]**

Consider the block and bullet before and immediately after collision:



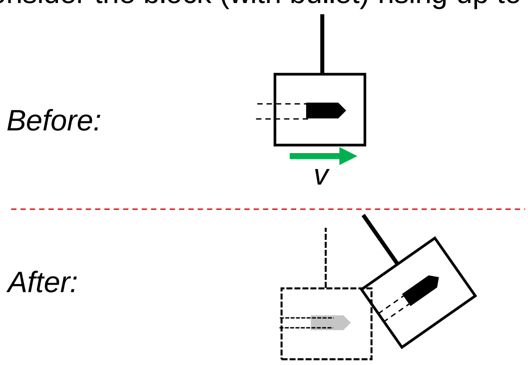
Note: KE of bullet is not conserved as the collision with the block is an inelastic collision.

By principle of conservation of momentum

$(\rightarrow)\ 0.010u + 0 = (2.010)(v)$

$u = 201v \cdots\cdots(1)$

Consider the block (with bullet) rising up to maximum height:



By principle of conservation of energy,

$\text{Loss in KE} = \text{Gain in GPE}$

$\frac{1}{2} m(v^2 - 0) = mg(0.20)$

$v^2 = 0.40g$

$v = \sqrt{0.40g} \cdots\cdots(2)$

Sub (2) to (1):

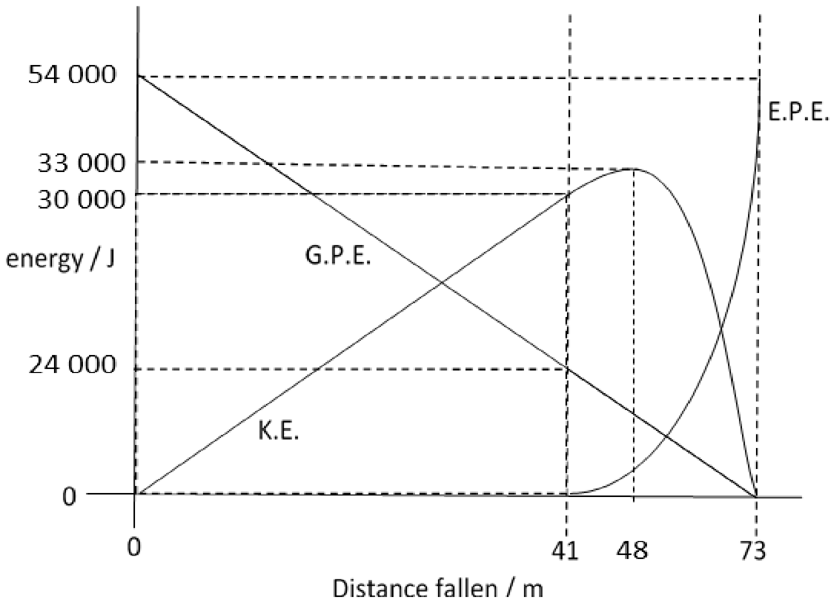
$u = 201\sqrt{0.40g} = 398 \text{ m s}^{-1}$

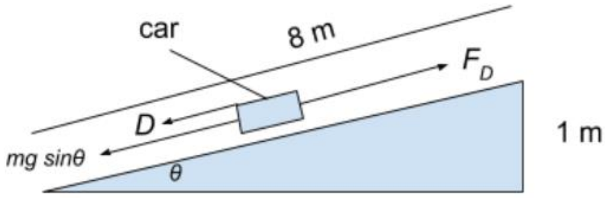
D12	**B**	**[N2012/1/10, modified]** The GPE obviously decreased as the final position is lower than the starting point. Hence the correct option is either choice A or B. We know that there must be stored energy in the spring, so it is a matter of how much. Since the spring came to rest after some time due to resistive forces, there must be some loss of energy to the surrounding (and also heat in the spring). The gain in EPE must hence be less than the loss in GPE. Hence, choice B. If we want to verify the answer, we can do so too. At equilibrium position, spring force = weight = mg Hence, $E_s = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{1}{2}(mg)x = (0.5)(4.2)(9.81)(0.29) = 6.0$ J

D13	**[H1 2009 P2 Q7]**	
(ai)	Before the rope becomes taut, man is in free fall. Assume negligible air resistance, Take vectors downwards as positive. $s = ut + ½ at^2$ $41 = 0 + ½ (9.81) t^2$ t = 2.89 s or 2.9 s (2 s.f.)	M1 A1
(aii)	There is no significant difference between the theoretical time and the 2.9 s quoted. Therefore, air resistance is insignificant	B1
(bi)	Extension = 73 – 41 = 32 m	A1
(bii)	From the area under the F-x graph from x = 0 to x = 32, E_s = ½ (32)(3400) = 54400 J	M1 A1
(ci)	(see table below)	

	At the top	After falling 41 m	After falling 73 m (i.e. when stopped)
Gravitational potential energy / J	54 000	54 000 – 30 000 = 24 000	**0**
Elastic potential energy/ J	0	0	54 000
Kinetic energy / J	0	75 x 9.81 x 41 = 30 000	0

(ii)	**1.** He has maximum KE when he reaches his equilibrium position, where net force = 0, acceleration is zero, velocity is maximum. At equilibrium point, net force = 0 Tension = Weight = (75)(9.81) = 740 N From graph, Extension, e = 7 m when tension is 740 N Total distance fallen = 41 + 7 = 48 m	 M1 A1

	2. By conservation of energy, Loss in E_p = Gain in E_k + Gain in E_s (75)(9.81)(48) = Max. KE + ½ k (7^2) Maximum KE = 33000 J	M1 B1 A1
(iii)	 To associate the graphs to theory: E_p = mgh ∴ GPE varies linearly with fallen distance E_s = 0 (before rope is taut) E_s = ½ kx^2 = ½ k (total fallen distance – $41)^2$ ∴Varies quadratically E_k = Total energy – GPE – EPE = constant – linear – quadratic ∴Varies quadratically	B4

D14	**D**	**[H2 2010/P1/Q11]** $P = Fv = mav$ a constant $\Rightarrow$ $v^2 = 2as$ since u = 0. Thus $P = ma\sqrt{2as}$ $\Rightarrow$ $P \propto \sqrt{s}$
D15	**D**	**[N2009/P1/Q12]** Since the car is moving at a constant speed along the horizontal road. acceleration and therefore net force = 0 Drag force at this speed, D = driving force = 200 N On the incline, since the car is travelling at the same speed, drag force D remains the same. Let the new driving force be F_d 

		Along the slope : $\sum F = 0$ $F_d - D - mg\sin\theta = 0$ $F_d = D + mg\sin\theta = 200 + (800)(9.81)(1/8) = 1181$ N Power supplied when traveling with this speed $= F_d v = 1181(20) = 23\,600$ W Alternatively, As the car is traveling at a constant speed on the level road resistive force, D = driving force = 200 N When the car rises 1 m for each 8 m of travel along the road on the slope, Rate of change of GPE along the slope = $mgh/t = mgv\sin\theta = \frac{1}{8}mgv$ ∴Total power = rate of work done against resistive force + rate of increase GPE $= Dv + \frac{1}{8}mgv$ $= 200 \times 20 + \frac{1}{8}(800)(9.81)(20) = 23\,600\text{W} = 24$ kW
D16	**D**	**[N2009/P1/Q13]** Since $P = Fv$, and drag force is proportional to the square of the velocity, power must be proportional to cube of the velocity, meaning $P = kv^3$, where k is some constant. The situations of 2 engines and 1 engine yield the following two equations. $72000 = k(12)^3$ $36000 = kv^3$ Solving them gives us $\frac{v}{12} = \sqrt[3]{\left(\frac{1}{2}\right)}$, or v = 9.52 m s^{-1}.
D17	**B**	**[CIE/N2011/1/16, modified]** $\langle P \rangle = \frac{\Delta E_P}{\Delta t} = \frac{mg\Delta h}{\Delta t} = mgv$
D18	**D**	**[HCI/2007/BT2/P1/Q7]** Consider a column of air of length L and cross-sectional area A approaching the windmill perpendicularly. KE per unit time contained in this column = $(\frac{1}{2}mv^2) / t = [\frac{1}{2}(\rho AL)v^2] / t = \frac{1}{2}\rho Av^3$ Since efficiency is 60%, power harnessed $= (0.60)(\frac{1}{2}\rho Av^3) = (0.60)(\frac{1}{2})(1)(60.0^2\pi)(8.00^3) = 1.74$ MW