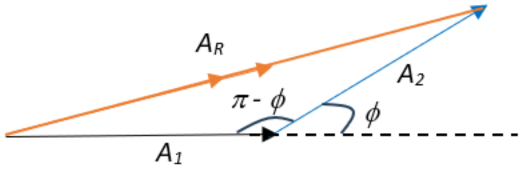
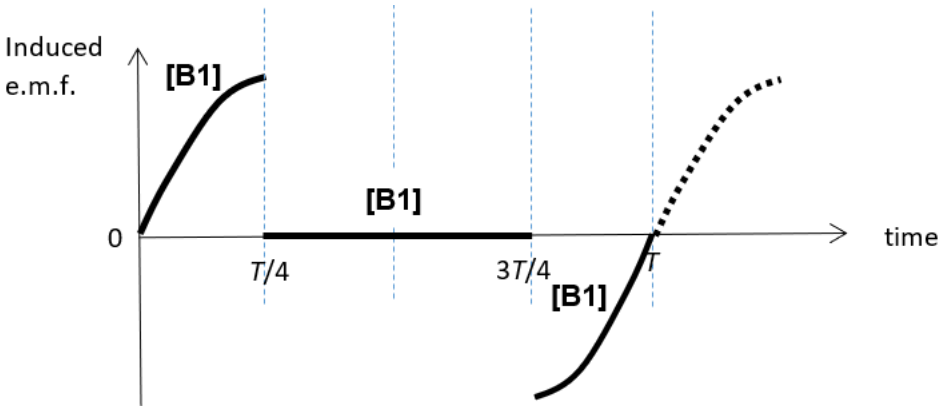
1(a)(i)	$\omega = \dfrac{80\text{ rev}}{1\text{ min}}\left(\dfrac{2\pi\text{ rad}}{\text{rev}}\right)\left(\dfrac{1\text{ min}}{60\text{ sec}}\right) = 8.4\text{ rad s}^{-1}$ $v = \omega r = (8.4)(0.60) = 5.0\text{ m s}^{-1}$	B1 A1
1(a)(ii)	$\alpha = \dfrac{\Delta\omega}{\Delta t} = \dfrac{\omega_f - \omega_i}{\Delta t} = \dfrac{0 - 8.4}{40} = -0.21\text{ rad s}^{-2}$	A1
	Comments: The angular acceleration must be negative.	
1(a)(iii)	$\Delta\theta = \omega_i t + \dfrac{1}{2}\alpha t^2 = (8.4)(40) + \dfrac{1}{2}(-0.21)(40)^2 = 168\text{ rad}$ $168\text{ rad}\left(\dfrac{1\text{ rev}}{2\pi\text{ rad}}\right) = 26.8\text{ rev}$ Accept answers with 2 s.f.	B1 A1
1(b)	Using the principle of conservation of angular momentum, $L = mvd = I_{total}\omega$ where L is the angular momentum of the blu-tac and $I_{total} = \dfrac{MR^2}{2} + mR^2$ $mvd = \left(\dfrac{MR^2}{2} + mR^2\right)\omega$ $\omega = \dfrac{2mvd}{(M + 2m)R^2}$	B1 B1 A1
	*Comments: About half of students had difficulties in expressing the distance from the blu-tac to the centre of rotational axis correctly. They cited **d** rather than **R**.* *Some could not conceptualize the conservation of angular momentum, hence they either used "conservation" of kinetic energy or linear momentum. However, since the collision is an inelastic one, the "conservation" of kinetic energy of the system will not work.*	

2(a)	In the Earth frame, after the collision, $mu = mv_1 + mv_2$ $u = v_2 - v_1 \rightarrow v_2 = u + v_1$ $u = v_1 + u + v_1$ $\rightarrow v_1 = 0$ and $v_2 = u$ Finding velocities in the zero-momentum frame before the collision, $v_{cm} = \frac{mu}{2m} = \frac{u}{2}$ $u_{1,cm} = u_{1,E} + u_{E,cm} = u - \frac{u}{2} = \frac{u}{2}$ $u_{2,cm} = u_{2,E} + u_{E,cm} = 0 - \frac{u}{2} = -\frac{u}{2}$ **After the elastic collision**, their velocities change sign in the zero-momentum frame (CM frame), $v_{1,cm} = -\frac{u}{2}$ and $v_{2,cm} = \frac{u}{2}$ Hence, the kinetic energy of the system after the collision in zero-momentum frame is $KE_{after,cm} = \frac{1}{2}m\left(-\frac{u}{2}\right)^2 + \frac{1}{2}m\left(\frac{u}{2}\right)^2 = \frac{mu^2}{4}$	M1 M1 A0
	Comments: Everyone could obtain the velocity of CM in Earth frame, well done. However, many did not figure out the velocities in the CM frame after the collision. *Some used the velocities before the collision to calculate the kinetic energy after the collision. Only a handful of them mentioned that since the collision was elastic, hence the kinetic energy would remain the same after the collision.*	
2(b)(i)	By PCOLM, $mu_{A,x} = mv_A \cos 30° + mv_B \cos 45°$ $mv_A \sin 30° - mv_B \sin 45° = 0 \rightarrow v_A = \frac{v_B \sin 45°}{\sin 30°} = \sqrt{2}v_B$ $40.0 = \sqrt{2}v_B \cos 30° + v_B \cos 45°$ $v_B = 20.7$ m s^{-1} $v_A = 29.3$ m s^{-1}	B1 A2
	Comments: Many successfully applied the PCOLM in 2D. A handful of students need to brush up their concept of the conservation of momentum.	
2(b)(ii)	$\text{fraction} = \frac{KE_{before} - KE_{after}}{KE_{before}} = \frac{(1/2)m(40.0)^2 - (1/2)m(29.3)^2 - (1/2)m(20.7)^2}{(1/2)m(40.0)^2}$ 19.6% of initial kinetic energy is **dissipated**.	B1 A1
	*Comments: Unfortunately, many did not interpret the term **fraction** correctly and went on to find the ratio of KE of asteroid A before and after the collision.*	

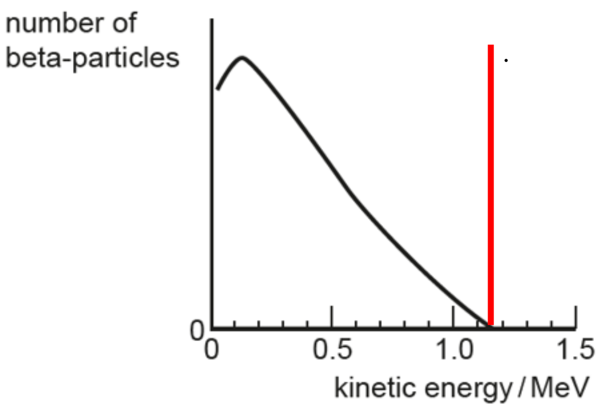
3(a)(i)	$y_2 = A_2 \cos(\omega t + \phi)$	A1
	Comments: This question was well done.	
3(a)(ii)	 Using graphical method, $A_r^2 = A_1^2 + A_2^2 - 2A_1A_2 \cos(\pi - \phi)$ $= A_1^2 + A_2^2 - 2A_1A_2(\cos \pi \cos \phi + \sin \pi \sin \phi)$ $= A_1^2 + A_2^2 + 2A_1A_2 \cos \phi$	C1 B1
	Comments: Majority have difficulty handling the vector addition. It would be helpful to draw the vector diagram involving the vector addition and obtain the required angle.	
3(b)(i)	Since intensity I is proportional to A^2, *for the combined intensity of the waves arriving at D:* $I_1 \propto A_1^2,\ I_2 \propto A_2^2,\ I_r \propto A_r^2$ $\Rightarrow A_r^2 = A_1^2 + A_2^2$ $\therefore\ 2A_1A_2 \cos \phi = 0$ $\rightarrow \cos \phi = 0 \Rightarrow \phi = \frac{\pi}{2} + 2\pi n$	B1 A1
	Comments: Many students were lost in the mathematics and failed to arrive at a logical deduction of the phase difference.	
3(b)(ii)	Path difference between the two waves arriving at Q_1 and Q_2: $LQ_2 - LQ_1 = \sqrt{1+d^2} - 1$ $\frac{LQ_2 - LQ_1}{\lambda} = \frac{\Delta\phi}{2\pi}$ $\sqrt{1+d^2} - 1 = \frac{\frac{\pi}{2}}{2\pi}\lambda = \frac{\lambda}{4}$ $1 + d^2 = \left(1 + \frac{\lambda}{4}\right)^2$ $1 + d^2 = 1 + 2\frac{\lambda}{4} + \frac{\lambda^2}{16}$ $16d^2 = \lambda(8 + \lambda)$	B1 B1 B1 A0
	Comments: Those who were able to apply the answer from (b)(i) to this part were most likely to succeed in answering this question and they also presented their workings clearly. Students were also more successful in handling the mathematics for this question as they persevered to arrive at the given relationship.	

4(a)	From Faraday's Law $\varepsilon = \frac{d\Phi}{dt} = \frac{d}{dt}(BAN\cos\omega t)$ $= BNA\frac{d\cos\omega t}{dt} = -BNA\omega\sin\omega t$ $N = \frac{L}{4\alpha}$, $A = \alpha^2$ $\varepsilon = -B(\frac{L}{4\alpha})(\alpha^2)(2\pi f)\sin\omega t = -\frac{1}{2}BL\alpha\pi f\sin\omega t$ Magnitude of maximum induced e.m.f. = $\frac{1}{2}BL\alpha\pi f$	B1 B1 B1
	Comments: Note that in derivation as in this case-should start with an expression for the magnetic flux linkage and then relate it to the induced e.m.f. A few students who did not start off with the correct expression for the magnetic flux linkage being a cosine expression rather than sine probably did not take note of the initial position of the coil relative to the magnetic field. A few also forgot to express all terms in the final expression in terms of the stated physical quantities.	
4(b)	As the coil rotates through *T*/4, the magnetic flux linkage through the coil decreases to zero (induced e.m.f reaches maximum) and remains zero till 3*T*/4. After 3*T*/4, the flux linkage through the coil increases to a maximum (induced e.m.f decreases to zero) till *T*. 	
	Comments: A common mistake: did not label the time for which the induced e.m.f. changes.	

5(a)(i)	$\frac{dU}{dr} = -\frac{2A}{r^3} + \frac{B}{r^2} = 0$ Only one value of *r* for circular orbit, $\frac{dU}{dr} = 0$ $r_c = \frac{2A}{B} = \frac{2(8.39 \times 10^{22})}{(3.98 \times 10^{15})}$ $r_c = 4.22 \times 10^7$ m	B1 B1 A0
	Comments: Most students can do it correctly.	
5(a)(ii)	$U = \frac{A}{r_c^2} - \frac{B}{r_c} = \frac{B^2}{4A} - \frac{B^2}{2A} = -\frac{B^2}{4A}$ $U = -\frac{(3.98 \times 10^{15})^2}{4(8.39 \times 10^{22})}$ $U = -4.72 \times 10^7$ J TE = U_{eff} = - 4.72×10^7 J, because in circular orbits the radial component of velocity is zero, hence the **radial kinetic energy** is zero.	B1 A1 B1
	Comments: Most students can do it correctly.	
5(a)(iii)	$\frac{A}{r_c^2} = \frac{L^2}{2mr_c^2} \rightarrow L = \sqrt{2mA}$ $L = mvr = m\omega_c r_c^2 = \sqrt{2mA}$ $\omega_c = \frac{\sqrt{2mA}}{mr_c^2} = \frac{\sqrt{2(10)(8.39 \times 10^{22}}}{10(4.22 \times 10^7)^4}$ $\omega_c = 7.27 \times 10^{-5}$ rad s^{-1}	B1 B1 A0
	Comments: Most students can do it correctly.	
5(b)(i)	$1.00 \times 10^7 - 4.72 \times 10^7 = \frac{A}{r^2} - \frac{B}{r}$ $-3.72 \times 10^7 = \frac{A}{r^2} - \frac{B}{r}$ At perihelion (closest distance) and aphelion (farthest distance), the radial component of velocity is zero. $-3.72 \times 10^7 r^2 = 8.39 \times 10^{22} - 3.98 \times 10^{15} r$ $-3.72 \times 10^7 r^2 + 3.98 \times 10^{15} r - 8.39 \times 10^{22} = 0$ $r_{closest} = 2.89 \times 10^7$ m $r_{farthest} = 7.81 \times 10^7$ m	 B1 B1 A1
	Comments: Some students could not get the answers because they forgot about using the given equation.	

5(b)(ii)	$a = \left(\dfrac{r_{far} + r_{near}}{2}\right)$ By Kepler's third law, $\dfrac{T^2_{circular}}{T^2_{elliptical}} = \dfrac{r_c^3}{a^3}$ $\dfrac{4\pi^2}{T^2_{elliptical}\omega_c^2} = \dfrac{r_c^3}{a^3}$ $\omega_c = 7.27 \times 10^{-5}$ rad s^{-1} from part **(a)(iii)** $T^2_{elliptical} = \dfrac{4\pi^2 a^3}{\omega_c^2 r_c^3} = \dfrac{4\pi^2\left(\dfrac{7.81\times10^7 + 2.89\times10^7}{2}\right)^3}{\left(7.27\times10^{-5}\right)^2\left(4.22\times10^7\right)^3}$ $T_{elliptical} = 1.23 \times 10^5$ s	B1 B1 B1 A1
	Comments: some students incorrectly used the answer in Q5 (b)(i) for this part of question without getting the average of perihelion and aphelion.	

6(a)(i)	The track is curved/deflected, only electrically charged particles that move in magnetic field experience a magnetic force and get deflected.	B1
	Comments: A handful of students did not mention of the curved track in the photo.	
6(a)(ii)	Fleming's left-hand rule suggests that the resultant/magnetic force for the moving charge in the magnetic field **is towards left only** if particle is positively charged/positron	B1
	Comments: About 1/3 of the students did not talk about the direction of magnetic force.	
6(a)(iii)	• track becomes more curved hence radius of curvature at B decreases/smaller *r* • speed *v* decreases because $r = mv / BQ$ • kinetic energy decreases as the energy is absorbed by lead	B1 B1 B1
	Comments: Many students provided detailed explanations. The relationship between radius and velocity (r = mv / BQ) and "kinetic" energy were expected to be seen in the answers. *A number of them did not link the curvature to the radius hence, they had difficulties in explaining the effects on speed and kinetic energy of the particle.* *A handful of students could not decipher the physics from the tracks seen.*	
6(b)(i)	Initial momentum of parent nucleus (polonium) is zero. By the principle of conservation of linear momentum, the sum of final momentum of daughter nuclei must be zero. $(v_X / v_\alpha) = 4/206$, which is 0.0194 **or** slightly less than 0.02 / 2%	B1 B1
	Comments: A sizeable number of students did not talk about which momentum to be conserved which was the stationary polonium nucleus. They conveniently equated the momentum of lead to the momentum of alpha particles without appropriately mentioning why it was so. *Some did not provide evidence where the idea of less than 2% comes from.*	
6(b)(ii)	$(209.93676\text{u}) - (205.92945\text{u} + 4.00151\text{u}) = 0.0058\text{u}$ $0.0058 \times (1.66 \times 10^{-27}) \times (3.0 \times 10^{8})^2 = 8.6652 \times 10^{-13}$ J $\dfrac{8.6652 \times 10^{-13}\ \text{J}}{\dfrac{1.6 \times 10^{-19}\ \text{J}}{\text{eV}}}$ $= 5.4 \times 10^{6}$ eV $= 5.4$ MeV	B1 B1 A0
	Comments: The last step of dividing by 1.6 x 10^{-19} J was expected to be seen in this 'show that' question. *A handful of students could not conceptualize the energy released in nuclear reactions. It is important to have a strong understanding of the H2 syllabus first. Before sitting for your H3 paper, ensure that your H2 concepts are firm and established.*	

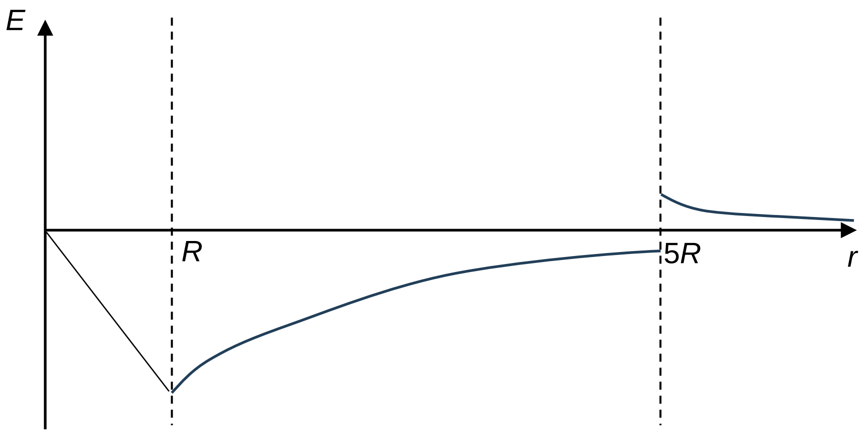
6(c)	 **Fig. 6.3** All would have the same energy of 1.16 MeV.	C1
	Comments: This part is very well done.	
6(d)(i)	$\sum p_{vertical} = 0$ $-m_n v_n \sin 20° + m_p v_p \sin 70° = 0$ $v_n = \frac{m_p v_p \sin 70°}{m_n \sin 20°} = \frac{9.11\times10^{-31}(2.2\times10^{8})\sin 70°}{1.67\times10^{-27}\sin 20°}$ $v_n = 3.3\times10^{5}$ m s^{-1}	B1 A1
	Comments: A handful of students erroneously equated the component of velocities only. Another handful of students either could not conceptualize the interaction as applying PCOLM or did not consider two-dimensional momenta.	
6(d)(ii)	Assume that all the rest mass energy of the positron and electron are converted to produce two gamma-ray photons. Rest mass $E_{positron}$ + Rest mass $E_{electron} = 2\times\frac{hc}{\lambda_{max}}$ $\lambda_{max} = \frac{2hc}{2mc^2} = \frac{h}{mc} = \frac{6.63\times10^{-34}}{9.11\times10^{-31}(3.00\times10^{8})}$ $\lambda_{max} = 2.43\times10^{-12}$ m	B1 B1 A1
	Comments: Plenty of students did not appreciate the pair annihilation process in which the positron and electron disappear to produce gamma ray photons. They did not use the info "slow" given in the question appropriately. They thought both the positron and electron have kinetic energies before annihilating and did not consider rest-mass energies of them. *This was the hardest part in this question. About 1/3 of the students could not conceptualize it or leave this part blank.*	
6(d)(iii)	The initial total momentum of the position-electron is zero. By the principle of conservation of linear momentum, the **total momentum of the photons must be zero** after the pair annihilation process. Hence, the two gamma particles move in opposite directions with the **same magnitude of momentum**.	B1 B1
	Comments: A handful of students attributed "kinetic energy" for the photons and went on to use opposite velocities in their explanations rather than momenta. Photons only have electromagnetic energy and are massless. Their momentum is $\frac{h}{\lambda}$.	

	Some did not mention what was the initial momentum to be conserved. In fact, similar concept was tested in part (b)(i) and similar mistakes were made. This underscores the importance of treating "zero momentum" with respect. If the initial total momentum is any number ***including zero****, you must clearly mention it because this is the momentum to be conserved when there is no net external force acting on the system.*	

7(a)(i)	For thin conducting spherical shell, $q_{net} = q_{inner} + q_{outer}$ $q_{outer} = q_{net} - q_{inner}$ $q_{outer} = +4Q - (+2Q) = +2Q$ (Award mark even if no working is shown)	B1
	Comments: Most students were able to get this mark.	
7(a)(ii)	The Gaussian surface has a radius of r , where $r < R$. $\oint E \bullet dA = \frac{q_{enc}}{\varepsilon_0}$ where $q_{enc} = \rho V_r$ $E(4\pi r^2) = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0}$ $E = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0 (4\pi r^2)} = \frac{\rho r}{3\varepsilon_0} = \frac{\frac{-2Q}{\frac{4}{3}\pi R^3} r}{3\varepsilon_0}$ $E = -\frac{Q}{2\pi\varepsilon_0 R^3} r$	B1 B1 A1
	Comments: A number of students lost a mark because they did not include the negative sign when substituting the negative 2Q charge. *Also some students were not able to find the q_{enc} when for the uniformly distributed insulator. Note that when the distribution is uniform (in this question the distribution is referring to charge), there is no need to do any integration. The charge density ρ is just the Total charge/ Total volume of insulator. Contrast this with 7(b)(i) and (ii)where the current density J is not uniform but given by an expression $J=kr^2$. For this case there will be a need to integrate to find the current enclosed.*	
7(a)(iii)	Between $R < r < 5R$, if we consider another Gaussian surface of r, the total charge enclosed q_{enc} is -2Q. $E_R(4\pi R^2) = \frac{-2Q}{\varepsilon_0} \rightarrow E_R = 10 \text{ N C}^{-1}$ $E_{2R}(4\pi (2R)^2) = \frac{-2Q}{\varepsilon_0}$ $E_{2R} = \frac{10}{4} = 2.5 \text{ N C}^{-1}$	B1 A1
	Comments: Most students were able to attain this mark albeit with a complicated working when proportionality would suffice.	
7(a)(iv)	$V_f - V_i = -\int_R^{5R} E dr$ $\Delta V = -\int_R^{5R} \frac{-2Q}{4\pi\varepsilon_0 r^2} dr = \frac{Q}{2\pi\varepsilon_0}\int_R^{5R} \frac{1}{r^2} dr$ $\Delta V = \frac{Q}{2\pi\varepsilon_0}\left(-\frac{1}{r}\right]_R^{5R} = \frac{Q}{2\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{5R}\right) = \frac{2Q}{5\pi\varepsilon_0 R}$	B1 B1 A1

<table>
<tr><td></td><td>Alternative Method
$\Delta V = \dfrac{-2Q}{4\pi\varepsilon_0 r_f} - \dfrac{-2Q}{4\pi\varepsilon_0 r_i}$ where $r_f = 5R$ and $r_i = R$ (for using -2Q and not 2Q or 4Q)

$\Delta V = \dfrac{2Q}{4\pi\varepsilon_0 R} - \dfrac{2Q}{4\pi\varepsilon_0 5R} = \dfrac{2Q}{5\pi\varepsilon_0 R}$</td><td>B1

B1
A1</td></tr>
<tr><td>7(a)(v)</td><td>1.
For indicating that E is negative and it changes linearly from 0 to R
For a curve that is continuous at R and asymptotically approaches zero in the region $R < r < 5R$
For a positive, concave-up, and decreasing curve for $r > 5R$ (with a magnitude which is equal to the magnitude of E just before 5R.)

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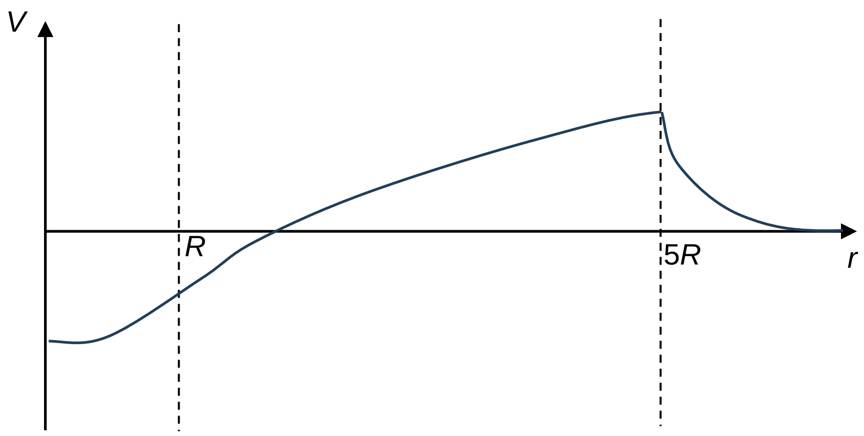
B1
B1</td></tr>
<tr><td></td><td>Comments: Most students were unable to get full mark. In particular the mark for the segment of the graph from r greater or equal to 5R. At r = R to r < 5R, the system behaves like a -2Q point charge centred at r = 0. Once outside thin spherical shel r > 5R, the system behaves similar to a +2Q point charge whose centre is at r = 0. As such, the magnitudes at the 5R should be equal but opposite in sign.</td><td></td></tr>
<tr><td>7(a)(v)</td><td>2.
For a potential curve that is always increasing from 0 to 5R for a continuous graph between 0 to 5R
For a potential curve that is decreasing for $r > 5R$

</td><td>B1
B1</td></tr>
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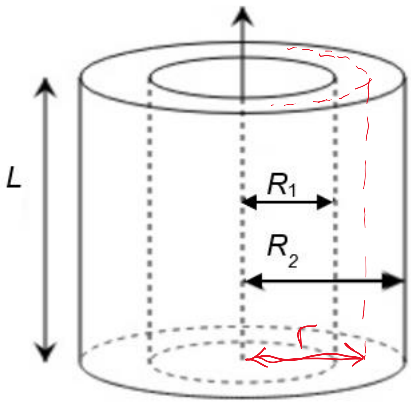
	Comments: Most students were unable to get full mark.The simplest way is to remember that E = -dV/dr. Checking that the gradient of this graph is equal to the magnitude of the E-field vs r graph.	
7(b)(i)	**1.** Draw a circular Amperian loop of radius *r* centred on the axis of the wire. The magnetic field is constant along this loop. Applying Ampere's Law, within the conductor, $r < R$ $\oint B \bullet dl = \mu_0 I_{enc} = \mu_0 \int_0^r J(2\pi r')dr'$ $B(2\pi r) = 2\pi\mu_0 \int_0^r (kr'^2)r'dr'$ $B = \frac{\mu_0 k}{r}\int_0^r r'^3 dr' = \frac{\mu_0 k r^3}{4}$	B1 A1
	Comments: Some students were unable to find the I_{enc} because of lack of familiarity with the integral expression.	
	2.Outside the conductor, $r > R$, $\oint B \bullet dl = \mu_0 I_{enc} = \mu_0 \int_0^R J(2\pi r')dr'$ $B(2\pi r) = 2\pi\mu_0 \int_0^R (kr'^2)r'dr'$ $B = \frac{\mu_0 k}{r}\int_0^R r'^3 dr' = \frac{\mu_0 k R^4}{4r}$	B1 A1
	Comments: See above.	
7(b)(ii)	The largest magnitude of *B* occurs at the surface of copper wire. Using either expression from **(i)1** or **2**, For $r = R,\ B = \frac{\mu_0 k R^3}{4}$	B1 A1
	Comments: Most students were able to get this mark.	

8(a)(i)	When the switch S and Q are in contact, the p.d. in the RC circuit is zero. $0 = \frac{Q}{C} + \frac{dQ}{dt}R$ $\int_{Q_0}^{Q} \frac{dQ}{Q} = \int_0^t -\frac{dt}{RC}$ $\ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC}$ $Q = Q_0 e^{-t/RC}$ $CV = CEe^{-t/RC}$ $V = Ee^{-t/RC}$	B1 A1 A1
8(a)(ii)	Smaller capacitance [1] Faster decay [1]	B1 B1
8(b)(i)	$L\frac{dI}{dt} = V_L = \frac{d\Phi}{dt}$ $L = \frac{\Phi}{I} = \frac{NBA}{I} = \frac{N(\mu_0 n I)A}{I}$ $L = \frac{N\left(\mu_0 \frac{N}{l} I\right)A}{I}$ $L = \frac{\mu_0 N^2 A}{l}$	B1 B1 A1
	Comments: Some students do not know that the magnetic flux in solenoid can be approx. by the formula $\mu_0 n I$.	
8(b)(ii)	Larger inductance [1] Slower decay [1]	B1 B1
8(c)(i)	$0 = \frac{Q}{C} + L\frac{di}{dt} = \frac{Q}{C} + L\frac{d}{dt}\left(\frac{dQ}{dt}\right)$ $\frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0 \rightarrow \omega^2 = \frac{1}{LC}$ $Q = Q_0 \cos(\omega t)$ $V = E\cos(\omega t) = E\cos\left(\frac{t}{\sqrt{LC}}\right)$	B1 B1 A1
8(c)(ii)	$V = E\cos(\omega t)$, sketch a cosine curve indicate $T = 2\pi\sqrt{LC}$ in the graph	B1 B1

8(c)(iii)1	$U = \frac{1}{2}CE^2$ $\frac{U}{4} = \frac{1}{2}CV^2$ $V = E/2 = 8.0\text{ V}$	B1 A1
8(c)(iii)2	When energy stored in inductor is ¾ of its max, energy stored in capacitor is 1/4 of its max. $\frac{E}{2} = E\cos(\omega t) \rightarrow \cos(\omega t) = \frac{1}{2}$ $\omega t = \frac{\pi}{3}$ $t = \frac{\pi}{3}\sqrt{LC} = 2.09 \times 10^{-3}\text{ s}$	B1 B1 A1
	Comments: Some students incorrectly thought that the energy stored in capacitor is 0.75 of its max.	

9(a)	Roll A will land first. Both rolls lose the same amount of gravitational potential energy. However, for roll B, it gains rotational kinetic energy in addition to translational kinetic energy. Thus, at any given time, it will have a lower speed as compared to roll A.	B1 B1
9(b)	 $dI = r^2 dm$ $dm = \rho dV = \rho(2\pi r dr)L$ where ρ is the density of the object, L is the length of the roll. $dI = r^2\rho(2\pi r dr)L = 2\pi\rho L \int_{R_{in}}^{R_{out}} r^3 dr$ $I = 2\pi\rho L \left.\frac{r^4}{4}\right\|_{R_{in}}^{R_{out}} = 2\pi\rho L\left(\frac{R_{out}^4 - R_{in}^4}{4}\right) = 2\pi L\frac{M}{\pi\left(R_{out}^2 - R_{in}^2\right)L}\left(\frac{R_{out}^4 - R_{in}^4}{4}\right)$ where $R_{out}^4 - R_{in}^4 = \left(R_{out}^2 - R_{in}^2\right)\left(R_{out}^2 + R_{in}^2\right)$ $I = \frac{M\left(R_{out}^2 + R_{in}^2\right)}{2}$	 B1 B1 B1 A0
9(c)	Consider the roll after it has fell through a height of h and turned through an angular displacement θ. Then, $h = R_{out}\theta$ By conservation of energy, Loss in GPE = Gain in KE = Gain in rotational KE + Gain in translational KE $$Mgh = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}Mv_{cm}^2$$ Since the roll is exhibiting pure rolling, $v_{cm} = R_{out}\omega$. Substituting and simplifying,	 B1 B1

	$$Mg(R_{out}\theta) = \frac{1}{2}\left(\frac{1}{2}M\left(R_{in}^2 + R_{out}^2\right)\right)\omega^2 + \frac{1}{2}M(R_{out}\omega)^2$$ $$Mg(R_{out}\theta) = \frac{1}{2}\left(\frac{1}{2}M\left(R_{in}^2 + 3R_{out}^2\right)\right)\omega^2$$ Since the roll starts from rest, $\omega^2 = 2\alpha\theta$. $$Mg(R_{out}\theta) = \frac{1}{2}\left(\frac{1}{2}M\left(R_{in}^2 + 3R_{out}^2\right)\right)(2\alpha\theta)$$ Simplifying for α, we get our expression $$\alpha = \frac{2gR_{out}}{R_{in}^2 + 3R_{out}^2}$$	B1 B1 A0
9(d)	For roll A in free fall, applying kinematics equations: $$H = \frac{1}{2}gt^2$$ Taking roll B which has rotated through an angle of θ, $$\theta = \frac{1}{2}\alpha t^2$$ Substituting and simplifying, $$H = \frac{1}{2}g\left(\frac{2\theta}{\alpha}\right)^2 = \frac{g\theta}{\alpha}$$ Using $h = R_{out}\theta$ and the expression in **(a)(iii),** $$H = \frac{g\left(\frac{h}{R_{out}}\right)}{\left(\frac{2gR_{out}}{R_{in}^2 + 3R_{out}^2}\right)} = \frac{h\left(R_{in}^2 + 3R_{out}^2\right)}{2R_{out}^2} = \frac{1}{2}\left(3 + \frac{R_{in}^2}{R_{out}^2}\right)h$$ Substituting values, $$H = \frac{1}{2}\left(3 + \frac{1}{4^2}\right)h = 1.531h = 1.5h$$ B1 – correct expression for free fall of A M1 – eliminating common quantity *t* B1 – using $h = R_{out}\theta$ A1 – correct final answer (with units)	B1 M1 B1 A1

9(e)(i)	Using $\tau = I\alpha$ and considering the rotation of the tube, $$\alpha = \frac{\tau}{I} = \frac{(\mu m_{tube} g)R_{in}}{m_{tube}R_{in}^2} = \frac{\mu g}{R_{in}}$$ (Note that α is a constant)	B1 B1 for I A1
9(e)(ii)	By parallel axis theorem, $I = MR_{in}^2 + MR_{in}^2 = 2MR_{in}^2$ Since there is no net torque, angular momentum of the cylinder is conserved about the point of contact. $$(MR_{in}^2)\omega_i = (2MR_{in}^2)\omega_f$$ $$\omega_f = \frac{1}{2}\omega_i$$	B1 B1 A1