Name:_____ **Class:**_____

JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2024

MATHEMATICS
Higher 2

9758/02
13 September 2024

Paper 2

3 hours

Candidates answer on the Question Paper.

Additional materials: List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given by [ ] at the end of each question or part question.

For Candidate's Use	For Examiner's Use
Question Number	**Marks Obtained**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total Marks	/ 100

This document consists of **6** printed pages.

[Turn over

Section A: Pure Mathematics [40 marks]

1 **(a)** Sketch, on the same axes, the graphs of $y = 3x$ and $y = |x^2 - a^2|$ where $a > 0$. [2]

(b) Find the exact solution of $|x^2 - a^2| = 3x$ for $0 < x < a$. [2]

2 **(a)** Find $\int \sin px \cos qx \, dx$, where p and q are constants such that $p \neq q$ and $p \neq -q$. [2]

(b) Given that $m \neq 0$, find $\int x \sin mx \, dx$. [3]

(c) Using the result in part **(b)**, for all positive integers m, evaluate $\int_0^{\pi} x \sin mx \, dx$, giving your answers in the form $\frac{k}{m}\pi$ where the possible value(s) of k are to be determined. [2]

3 It is given that $f(x) = e^{\tan^{-1} x}$, where $\tan^{-1} x$ denotes the principal values.

(i) Show that $(1+x^2)f'(x) = f(x)$. [1]

(ii) By further differentiation of the above result, find the Maclaurin series for $f(x)$, up to and including the term in x^3. [5]

(iii) Using the series in part **(ii)** find an approximate value of $\int_0^{0.5} f(x) \, dx$, giving your answer to 4 significant figures. [1]

(iv) Comment on the suitability of substituting $x = 1$ into the series in part **(ii)** to estimate the value of $e^{\frac{\pi}{4}}$. [1]

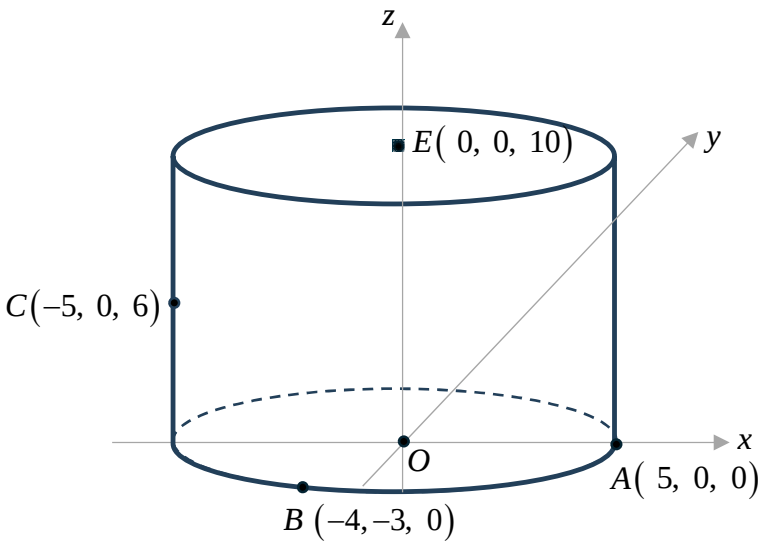
4 **(i)** Show that $\frac{7}{n-2} - \frac{5}{n-1} - \frac{2}{n} = \frac{9n-4}{(n-2)(n-1)n}$. [2]

(ii) Hence find $\sum_{n=3}^{N} \frac{9n-4}{(n-2)(n-1)n}$, giving your answer in the form $k - f(N)$, where k is a constant. [3]

(iii) Show that $\sum_{n=3}^{N} \frac{9n-4}{(n-2)(n-1)n}$ is convergent and state the sum to infinity of this series. [2]

(iv) Use your answer in **(ii)** to find $\sum_{n=2}^{N} \frac{9n+14}{n(n+1)(n+2)}$. [3]

5



A closed cylinder has a base in the shape of a circle with centre O. The coordinates of points A, B and C are $(5,\ 0,\ 0)$, $(-4, -3,\ 0)$ and $(-5,\ 0,\ 6)$ respectively. Point E is directly above O with coordinates $(0,\ 0,\ 10)$.

(i) Point D lies in the cylinder such that $ABCD$ is a parallelogram. Find the position vector of D and determine the shape of $ABCD$, justifying your answer. [3]

(ii) Find the cartesian equation of the plane ABC. [2]

(iii) Find the acute angle between the plane ABC and the base of the cylinder. [2]

(iv) Point F lies on AE such that $AF : AE = 1:5$. Find the length of projection of AF onto the plane ABC. [4]

Section B: Probability and Statistics [60 marks]

6 The individual letters of the word APPROPRIATE are printed on identical cards and arranged in a straight line.

(a) Find the number of arrangements of all 11 letters of the word such that

(i) the letters are **not** in alphabetical order, [2]

(ii) all the vowels are together and only two of the P′s are together. [3]

(b) The cards are now placed in a bag and 3 cards are drawn without replacement. Find the probability that there are at least two vowels drawn. [2]

7 The number of years (x) an employee has worked for the company and the corresponding salary increment, in dollars (y), received by the employee are given in the table.

Years, x	6	7	9	11	13	15	18
Amount, y	155	170	211	230	248	260	265

(i) Draw the scatter diagram for these values, labelling the axes clearly. [1]

It is thought that the salary increment, \$$y$, can be modelled by one of the formulae

$$y = ax + b \qquad \text{or} \qquad y = c\ln x + d$$

where a, b, c and d are constants.

(ii) Find, correct to 4 decimal places, the value of the product moment correlation coefficient between

(a) x and y,

(b) $\ln x$ and y. [2]

(iii) Use your answers to parts **(i)** and **(ii)** to explain which of $y = ax + b$ or $y = c\ln x + d$ is the better model. [2]

It is required to estimate the value of x for which $y = 200$.

(iv) Explain why neither the regression line of x on y nor the regression line of $\ln x$ on y should be used. [1]

(v) Find the equation of a suitable regression line and use it to find the required estimate, commenting on its reliability. [3]

8 A store owner receives a shipment of stationery items, including notebooks, pens, and correction tapes. Historical data indicates that 1% of the notebooks, 2% of the pens, and 4% of the correction tapes are defective. The quality of notebooks, pens and correction tapes is independent of one another. The store owner decides to sell the stationery in 180 packets, each containing one notebook, two pens and one correction tape. A packet is deemed unsatisfactory if any of the four items is defective.

(i) Show that the probability that a randomly selected packet is unsatisfactory is 0.0872, correct to 3 significant figures. [1]

The number of packets that are unsatisfactory is denoted by X. You may assume that X can be modelled by a binomial distribution.

(ii) Find the probability that there are at least 5 but less than 10 packets that are unsatisfactory. [2]

(iii) Find the least value of r such that the probability that there are more than r packets that are unsatisfactory is at most 0.12. [2]

Before selling the packets of stationery, he decided to select a sample of 9 packets to check for unsatisfactory packets.

(iv) How should the packets be selected? Give a reason for this method of selection. [2]

(v) Find the probability that the ninth packet is the third unsatisfactory packet selected. [2]

9 Box *A* contains five cards numbered 1, 2, 2, 3 and 3.

Box *B* contains three cards numbered 4, 5 and 5.

Cards that are numbered 2, 3 and 5 are red, while cards that are numbered 1 and 4 are blue. A card is drawn from each of the two boxes. If both cards are of the same colour, then the score will be the sum of the numbers on the two cards. If both cards are of different colours, then the score will be the product of the numbers on the two cards. Let X be the score obtained.

(i) Show that $P(X = 8) = \frac{2}{5}$. [2]

(ii) Find the probability distribution of X. [3]

(iii) Find $E(X)$ and $Var(X)$. [3]

(iv) Find the probability that the mean score of 50 independent observations of X lies between 7.5 and 8.5. [2]

10 In a drinks factory, a machine is programmed to dispense 500 ml of green tea into empty bottles.

(a) After a routine check of the machine, the production manager suspects that the machine is dispensing more green tea than expected. A random sample of 50 bottles is taken for a hypothesis test at $\alpha\%$ level of significance and the data is as follows:

Volume of green tea (correct to nearest ml)	498	499	500	501	502	503	504
Number of bottles	5	11	10	11	10	1	2

Taking $\alpha = 2$, test the production manager's suspicion and state the meaning of the *p*-value in the context of the question. [5]

(i) Determine the set of values of α for which the production manager's suspicion is valid. [1]

(ii) Explain why the production manager is able to carry out the hypothesis test without knowing anything about the distribution of the volume of green tea dispensed by the machine. [1]

(b) The machine is being recalibrated to dispense 500 ml of green tea. Another random sample of 50 bottles of green tea is taken and the mean and standard deviation of this sample are 502 ml and k ml respectively. A hypothesis test, at the 5% significance level, concluded that the recalibration is done accurately. Find the set of values that k can take. [5]

11 [For this question, you should state clearly the values of the parameters of any normal distribution you use.]

After dinner every day, Benedict gives himself some time for relaxation before starting his revision at 8 pm. He finishes his dinner at D minutes past 7 pm, where D follows the distribution $N(10, 2^2)$. After dinner, he usually spends M minutes on social media, which follows the distribution $N(k, 12^2)$.

(i) Given that Benedict starts his revision late 12.5% of the time if he spends time on social media after his dinner, show that $k = 36.0$. [3]

For the rest of this question, assume that $k = 36$.

(ii) Sketch the distribution of the time that Benedict spends on dinner after 7 pm and on social media till 8 pm. [2]

On some occasions after dinner, when he does not spend time on social media, he plays online games. The time he takes to finish the games, G minutes, follows the distribution $N(45, 10^2)$.

(iii) On a particular day after dinner, Benedict spent time on social media. On another day, he played online games. Find the probability that the time he starts revision on the two days differ by at most 5 minutes. [3]

On average, Benedict plays online games on 20% of the evenings after dinner.

(iv) On a particular evening, Benedict started his revision late. Find the probability that he played online games that evening. [4]

(v) State a necessary assumption for your calculations in **(ii), (iii)** and **(iv)** to be valid. [1]