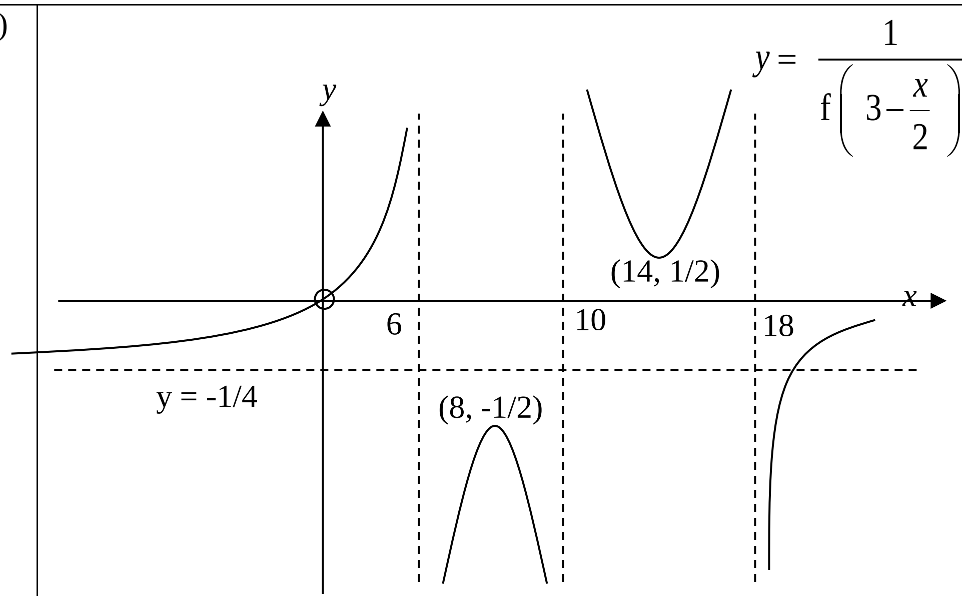
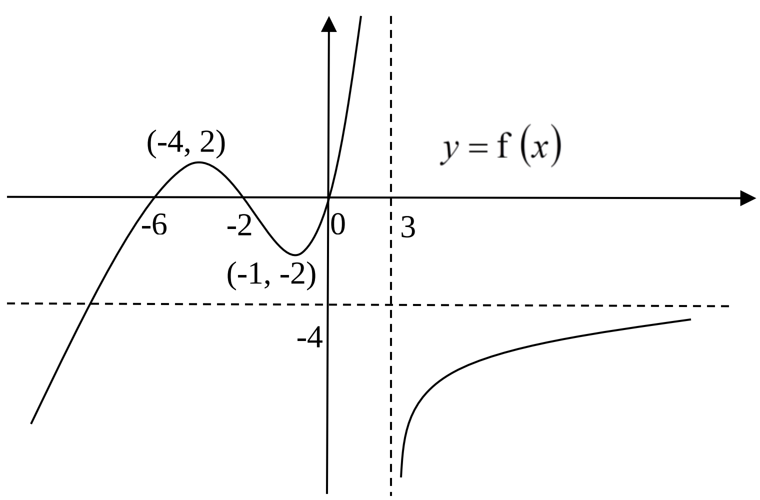
Solutions

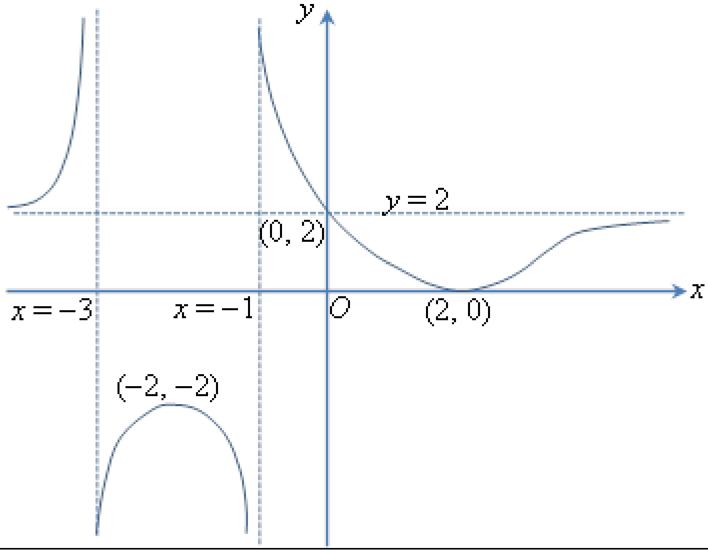
Transformation

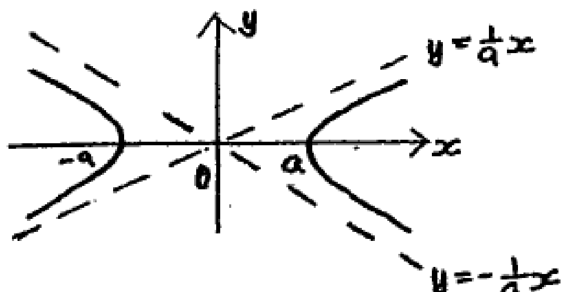
1(a)

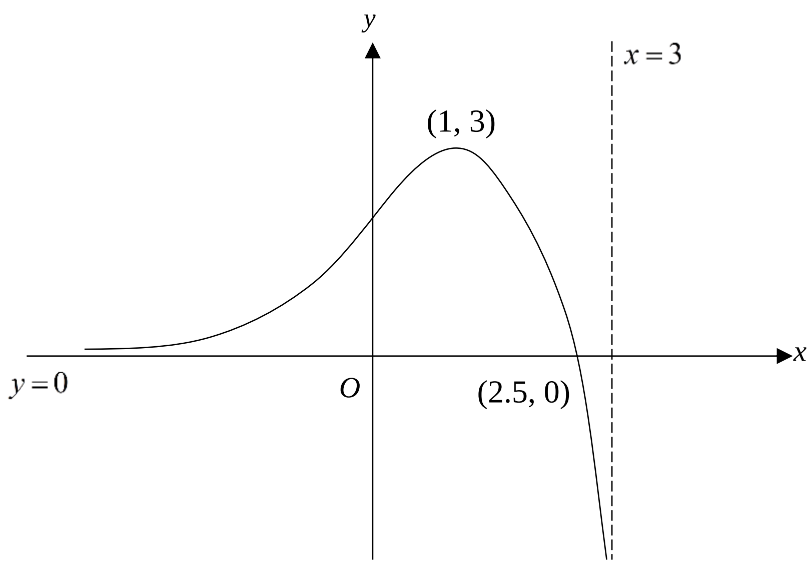
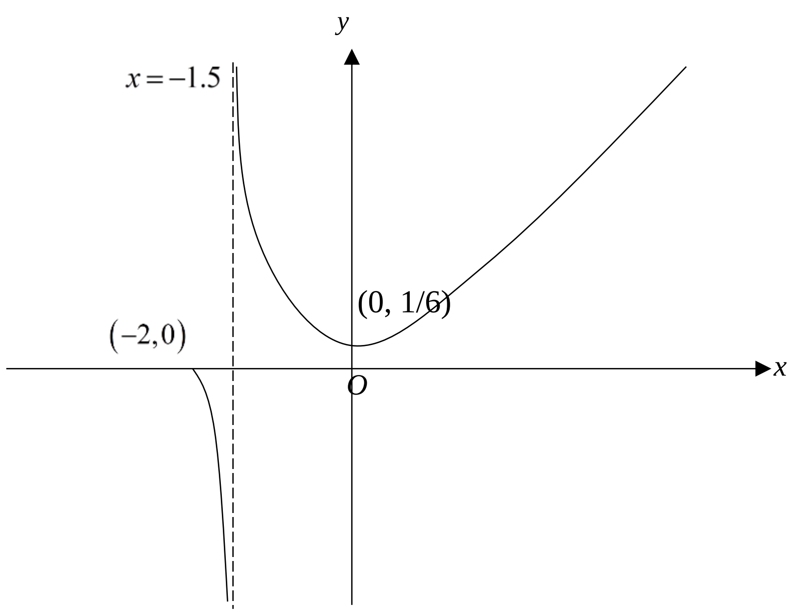


(b)

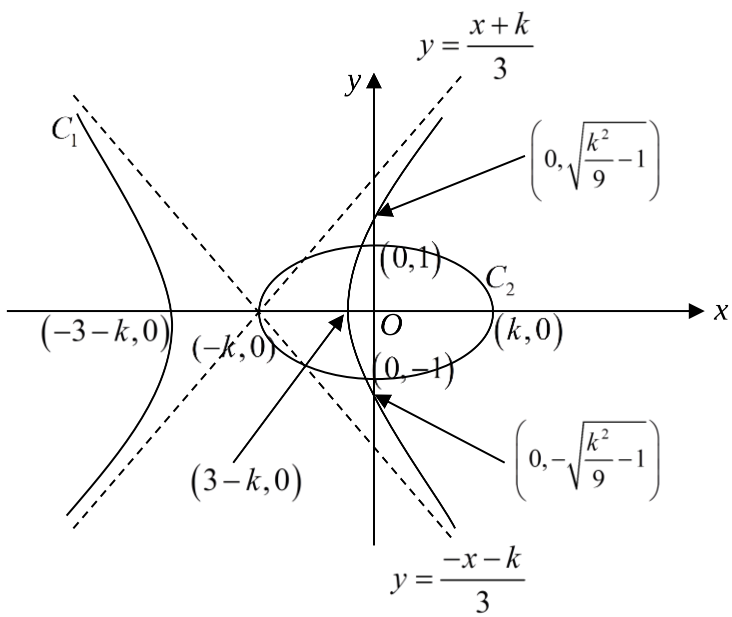


2(a)	

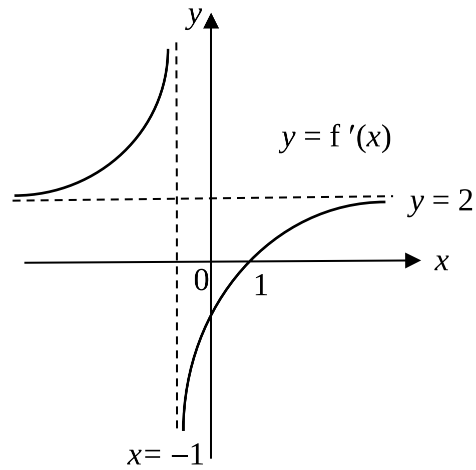
3	b)  C_2 has equation $x^2 - a^2(y-4)^2 = a^2$. $x = 4, y = 3 \Rightarrow 16 - a^2 = a^2 \Rightarrow a^2 = 8 \Rightarrow a = 2\sqrt{2}$ $(\because a > 1)$

4. Suggested solution

Suggested solution


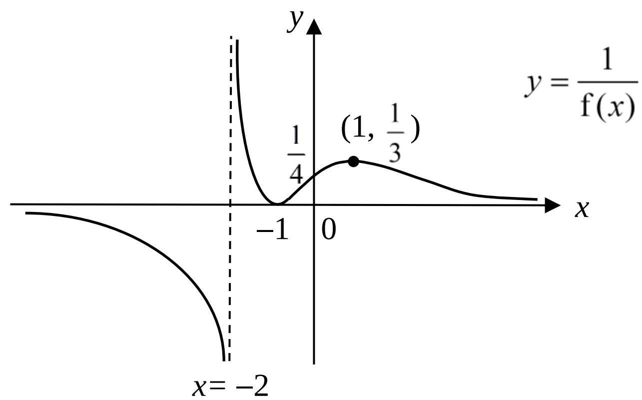
5(a)	$C_1: \; 9y^2 = (x+k)^2 - 9$ $\Rightarrow \frac{(x+k)^2}{3^2} - y^2 = 1$ Sequence of transformations: EITHER (1) Scaling parallel to the *x*-axis by factor 3 (2) Translation in the negative *x*-axis direction by *k* units OR (1) Translation in the negative *x*-axis direction by $\frac{k}{3}$ units (2) Scaling parallel to the *x*-axis by factor 3

(b)(i)	$C_1: \dfrac{(x+k)^2}{3^2} - y^2 = 1$ Intercepts: $(3-k, 0)$ $(-3-k, 0)$ $\left(0, \pm\sqrt{\dfrac{k^2}{9}-1}\right)$ Asymptotes: $y = \pm\dfrac{(x+k)}{3}$ i.e. $y = \dfrac{x+k}{3}$, $y = \dfrac{-x-k}{3}$ $C_2: \dfrac{x^2}{k^2} + y^2 = 1$ Intercepts: $(\pm k, 0)$, $(0, \pm 1)$ 
b(ii)	$\dfrac{x^2}{a^2} + \dfrac{(x+k)^2 - 9}{9} = 1$ $\dfrac{(x+k)^2}{9} - 1 = 1 - \dfrac{x^2}{a^2}$ ------ (*) Sketch the graph of $C_1: \dfrac{(x+k)^2}{3^2} - y^2 = 1$ (i.e. $y^2 = \dfrac{(x+k)^2}{9} - 1$) on the same axes as graph of $C_2: \dfrac{x^2}{a^2} + y^2 = 1$ (i.e. $y^2 = 1 - \dfrac{x^2}{a^2}$). For equation (*) to have 4 real roots, the two graphs intersect at 4 points. Since a is a positive constant, $a > k+3$.

6(a) **(i)**



(ii)



(b)

$y = 2x + 9 - \frac{2}{x+4}$

↓ replace y by $y + 1$

$y = 2x + 8 - \frac{2}{x+4}$

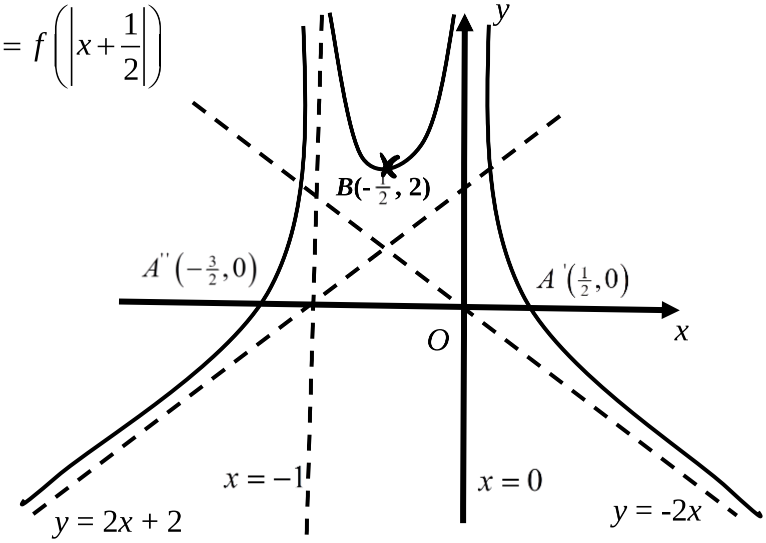
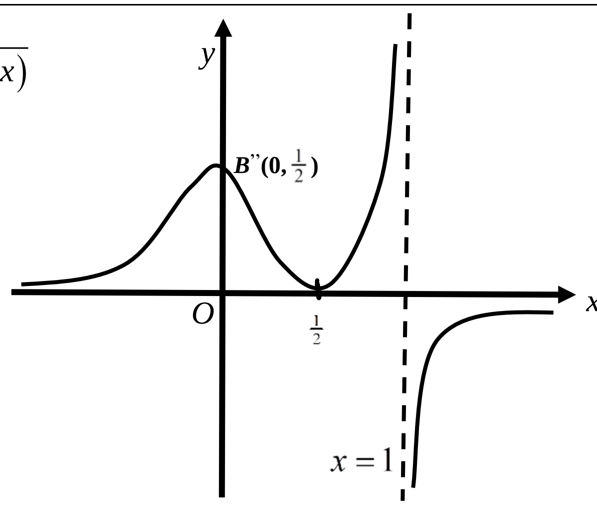
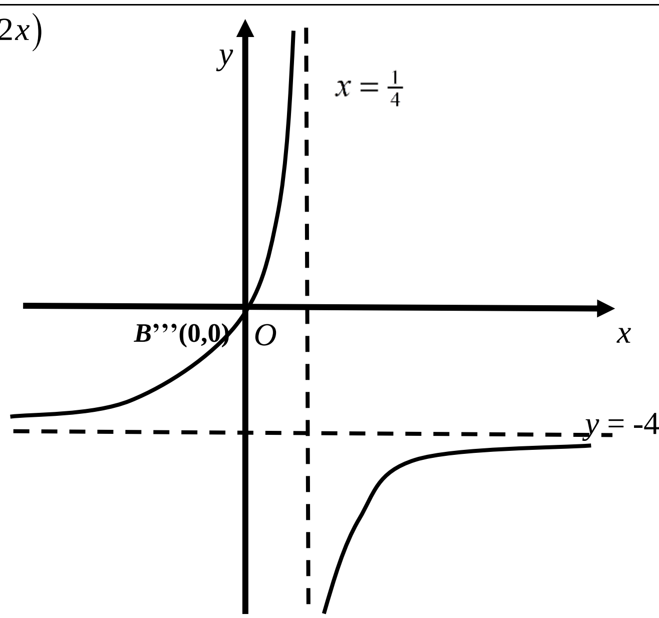
↓ replace x by $2x$

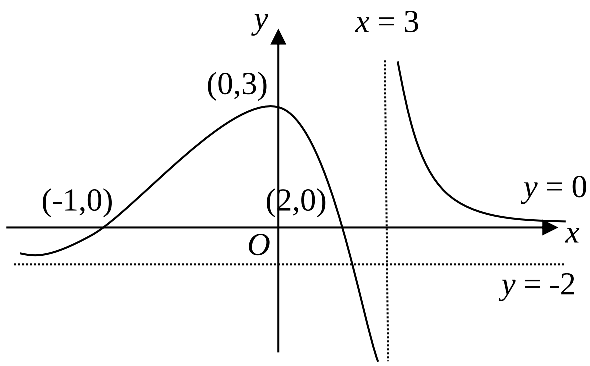
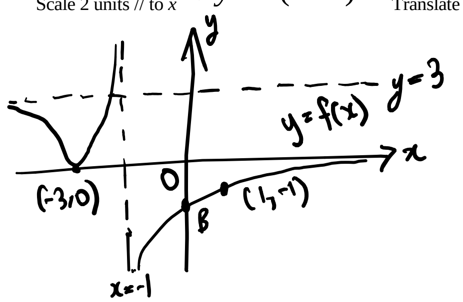
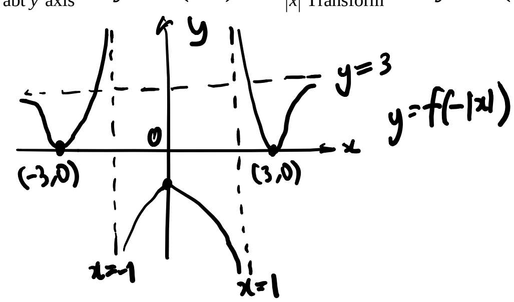
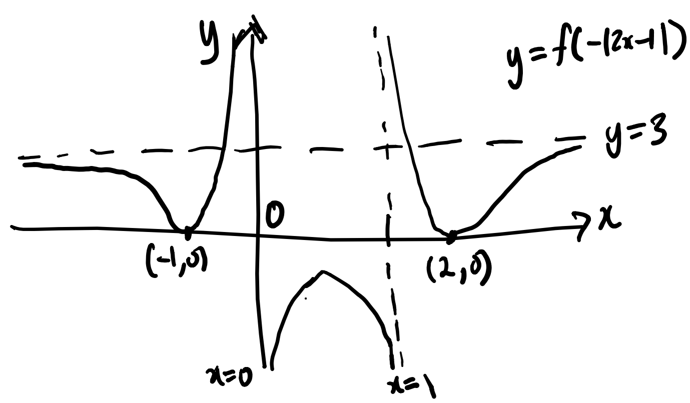
$y = 2(2x) + 8 - \frac{2}{2x+4} = 4x + 8 - \frac{1}{x+2}$

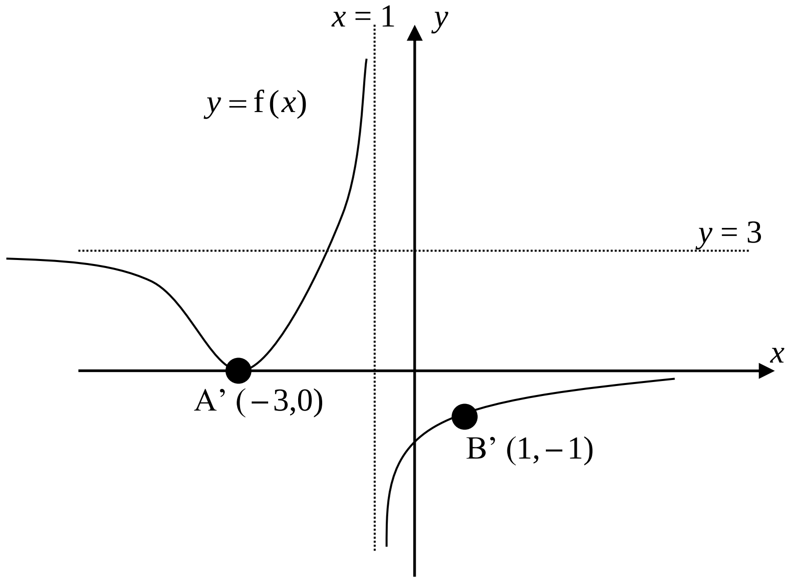
↓ replace x by $(x–2)$

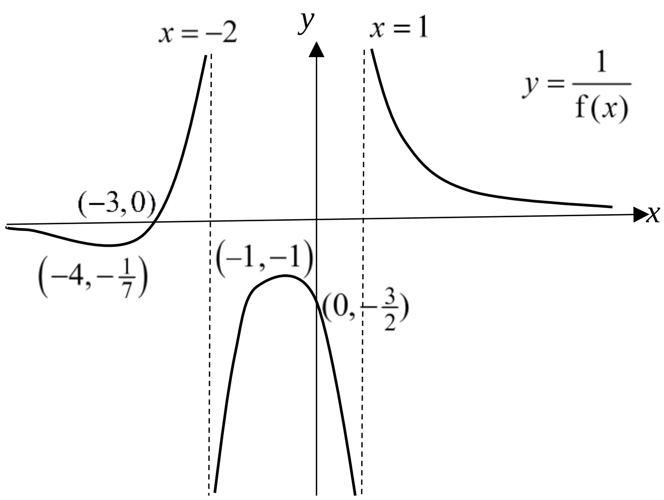
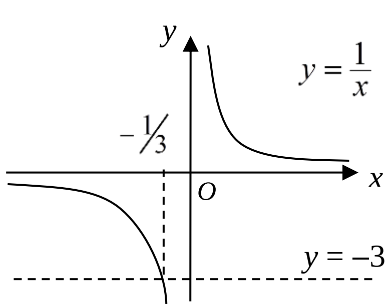
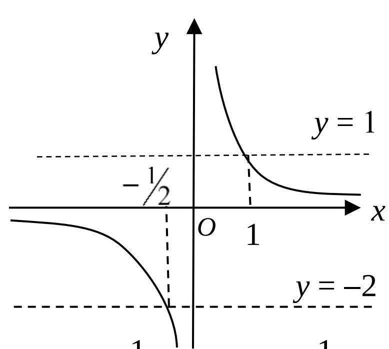
$y = 4(x-2) + 8 - \frac{1}{(x-2)+2} = 4x - \frac{1}{x}$

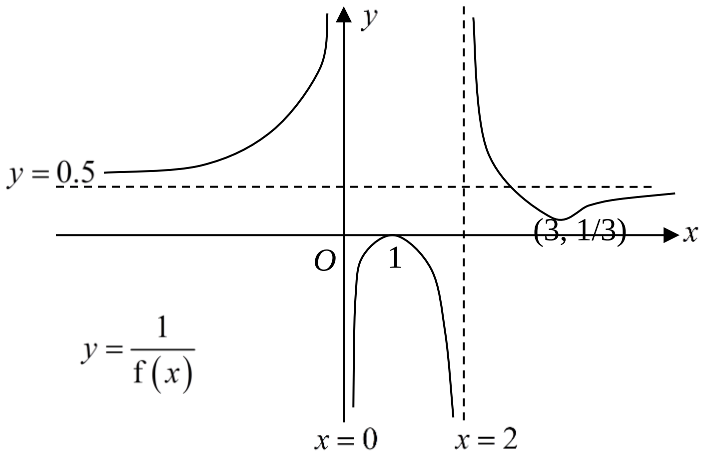
The equation of the curve before the transformations were effected is $y = 4x - \frac{1}{x}$

7	(i) $y = f\left(\left\|x+\frac{1}{2}\right\|\right)$ 
	(ii) $y = \frac{1}{f(x)}$ 
	(iii) $y = f'(2x)$ 

8a	 range of values of *k* is $k \le -2$ or $k > 3$.
b	Original solutoin was incorrect. Question was a lot more challenging than intended. Side working: $y = \mathrm{f}(2x-1) \xrightarrow[\text{Scale 2 units // to } x]{\text{replace } x \text{ with } x/2} y = \mathrm{f}(x-1) \xrightarrow[\text{Translate } -1 \text{ unit in } x \text{ dirn}]{\text{replace } x \text{ with } x+1} y = \mathrm{f}(x)$  $y = \mathrm{f}(x) \xrightarrow[\text{Reflect abt } y \text{ axis}]{\text{replace } x \text{ with } -x} y = \mathrm{f}(-x) \xrightarrow[\lvert x \rvert \text{ Transform}]{\text{replace } x \text{ with } \lvert x \rvert} y = \mathrm{f}(-\lvert x \rvert)$  $y = \mathrm{f}(-\lvert x \rvert) \xrightarrow[\text{Translate } +1 \text{ units in } x \text{ dirn}]{\text{replace } x \text{ with } x-1} y = \mathrm{f}(-\lvert x-1 \rvert) \xrightarrow[\text{Scale factor of 0.5 // } x]{\text{replace } x \text{ with } 2x} y = \mathrm{f}(-\lvert 2x-1 \rvert)$ 

	(ii)  Scale by factor 2 parallel to the *x*- axis followed by translation by -1 unit in the direction of *x*-axis.

9(i)	
(ii)	From the given graph, $f(x) < 0$ for $x < -3$ or $-2 < x < 1$ Replacing *x* by $\frac{1}{x}$,   $\frac{1}{x} < -3 \Rightarrow -\frac{1}{3} < x < 0$ or $-2 < \frac{1}{x} < 1 \Rightarrow x < -\frac{1}{2}$ or $x > 1$

<table>
<tr><td>10a</td><td>
</td></tr>
<tr><td>b</td><td>

Let $\mathrm{h}(x)=\frac{4}{4x^2+4x+1}=\frac{4}{(2x+1)^2}$

Before *C*, $y=\mathrm{h}(-x)=\frac{4}{\left[2(-x)+1\right]^2}=\frac{4}{(1-2x)^2}$

Let $\mathrm{p}(x)=\frac{4}{(1-2x)^2}$

Before *B*, $y=\mathrm{p}\left(\frac{x}{2}\right)=\frac{4}{\left[1-2\left(\frac{x}{2}\right)\right]^2}=\frac{4}{(1-x)^2}$

Let $\mathrm{g}(x)=\frac{4}{(1-x)^2}$

Before *A*, $y=\mathrm{f}(x)=\mathrm{g}(x+1)=\frac{4}{\left[1-(x+1)\right]^2}=\frac{4}{x^2}$

Let $\mathrm{h}(x)=\frac{4}{4x^2+4x+1}$.

Before *C*, $y=\mathrm{h}(-x)=\frac{4}{4(-x)^2+4(-x)+1}=\frac{4}{4x^2-4x+1}$

Let $\mathrm{p}(x)=\frac{4}{4x^2-4x+1}$

Before *B*, $y=\mathrm{p}\left(\frac{x}{2}\right)=\frac{4}{4\left(\frac{x}{2}\right)^2-4\left(\frac{x}{2}\right)+1}=\frac{4}{x^2-2x+1}$

Let $\mathrm{g}(x)=\frac{4}{x^2-2x+1}$

Before *A*, $y=\mathrm{f}(x)=\mathrm{g}(x+1)=\frac{4}{(x+1)^2-2(x+1)+1}=\frac{4}{x^2}$

</td></tr>
</table>

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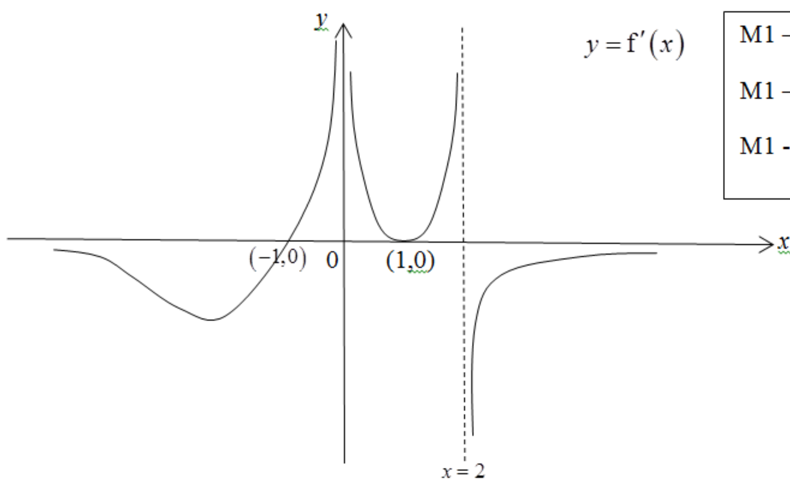
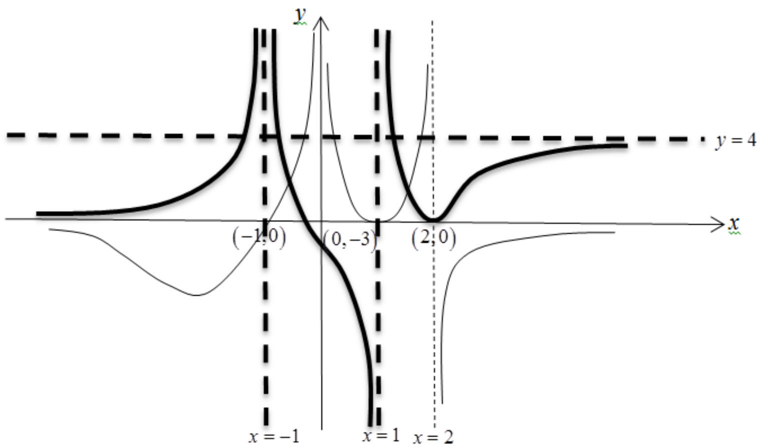
ii)



M1 – shape

M1 – labeling

M1 - Asymptotes

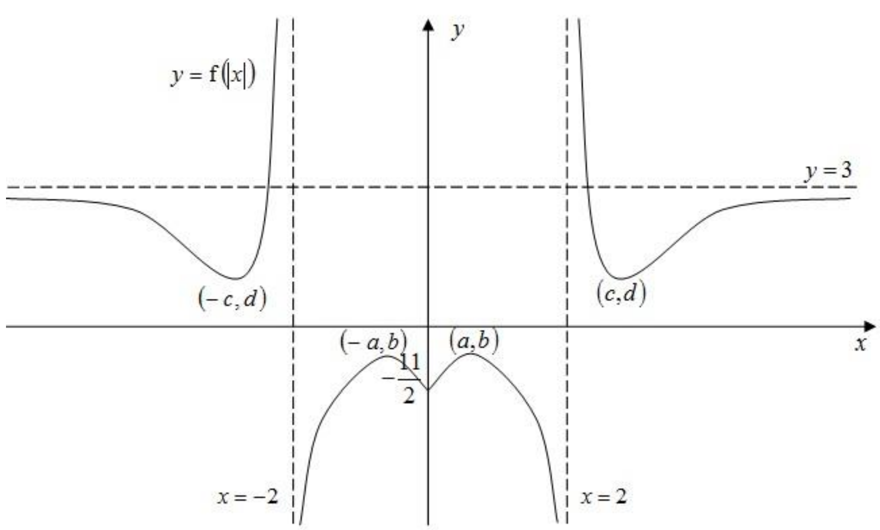


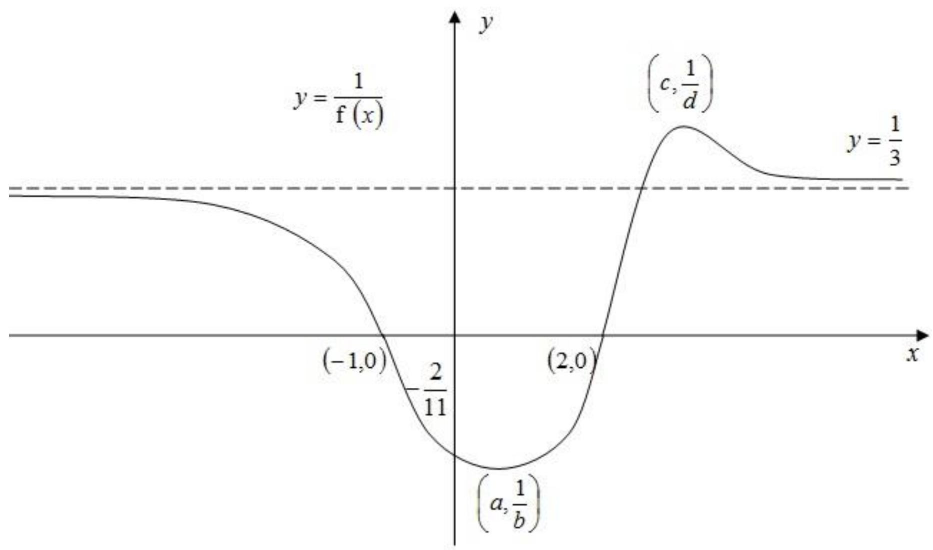
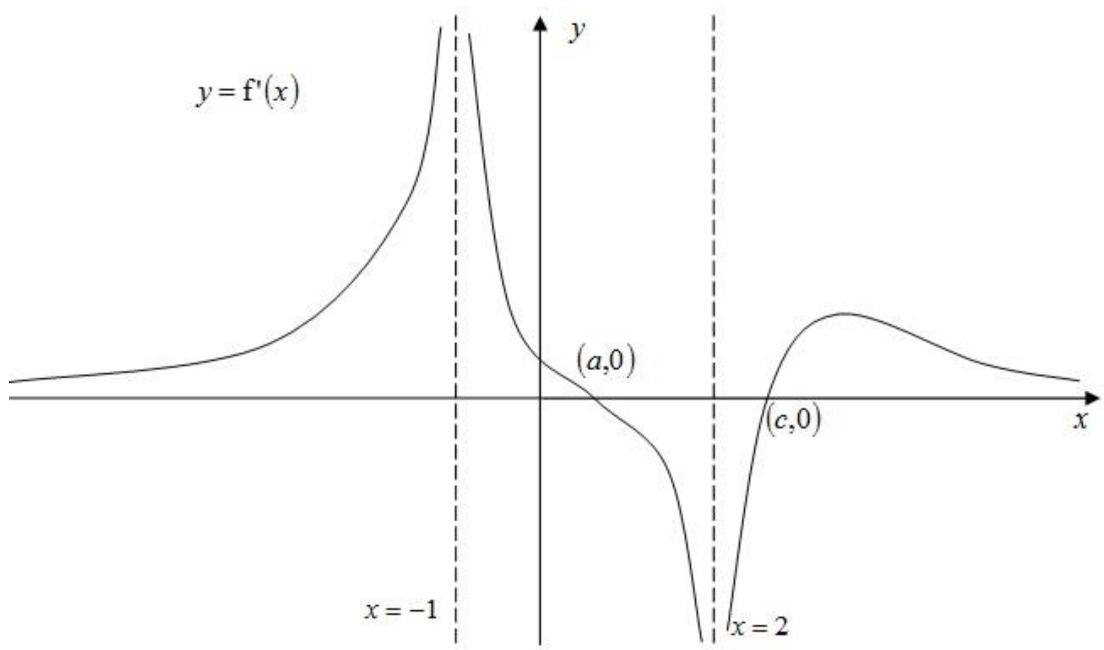
M1 – shape

M1 – labeling & asymptotes

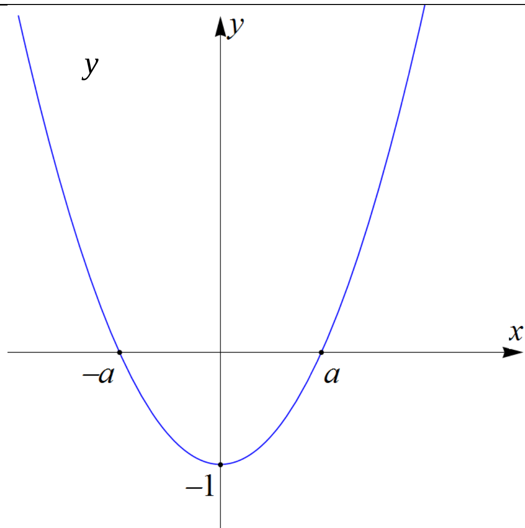
From graph, there are 2 points of intersection. Thus, there are 2 solutions.

12(i)

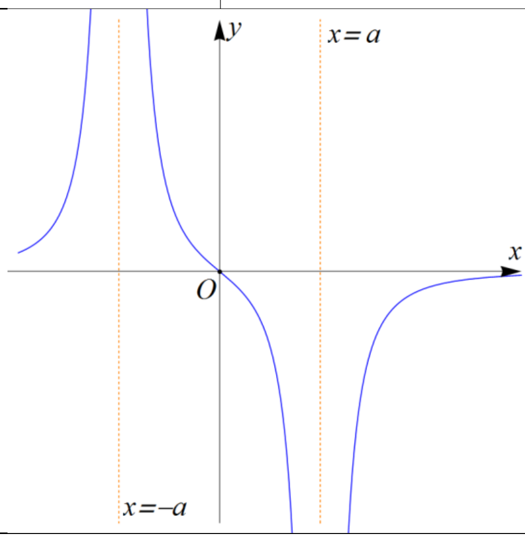


(ii)	
(iii)	
13 (b)	Using long division $\frac{x^2-4x}{(x-2)^2}=1-\frac{4}{(x-2)^2}$ So A=1 and B = -4 Series of transformations: 1. Translate the graph of $y=\frac{1}{x^2}$ by 2 units in the positive *x* direction 2. Stretch the resulting graph parallel to the *y*-axis with a scale factor of 4. 3. Reflect about the *x*-axis. 4. Translate the resulting graph by 1 unit in the positive *y* direction.

14(a)



(b)



To obtain the original equation, we reverse the transformation. I.e, we apply the following transformation in succession to $y=\dfrac{2}{1-(x+1)^2}$.

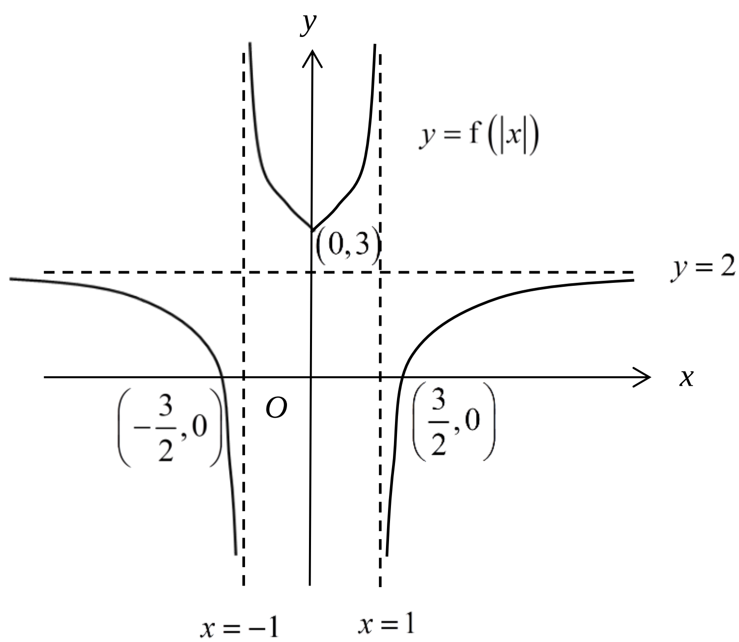
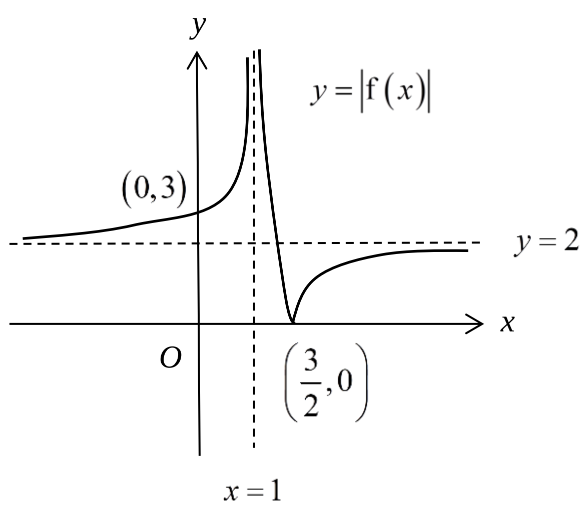
C' : A scaling parallel to the y -axis by a factor of $\dfrac{1}{2}$,

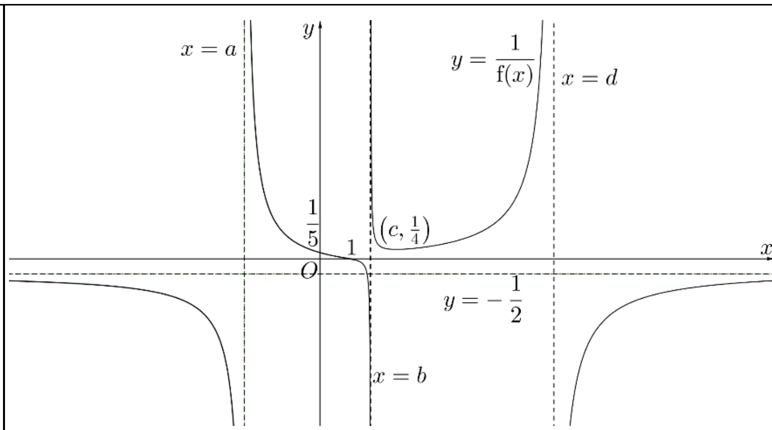
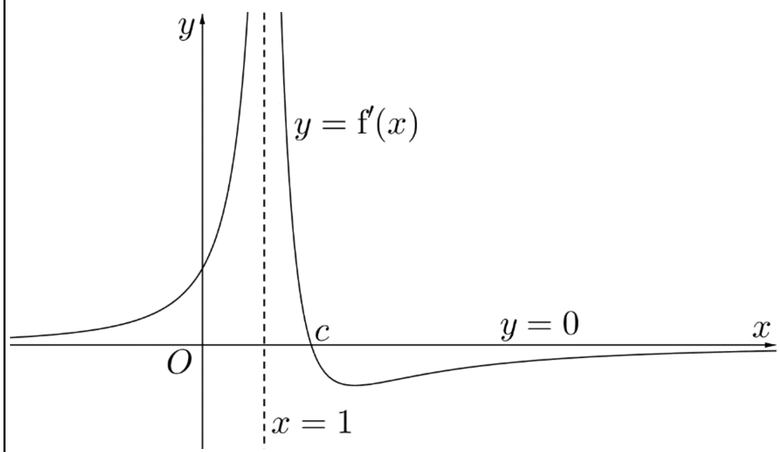
B' : A reflection in x -axis,
A' : A translation of 1 unit in the positive x -direction.

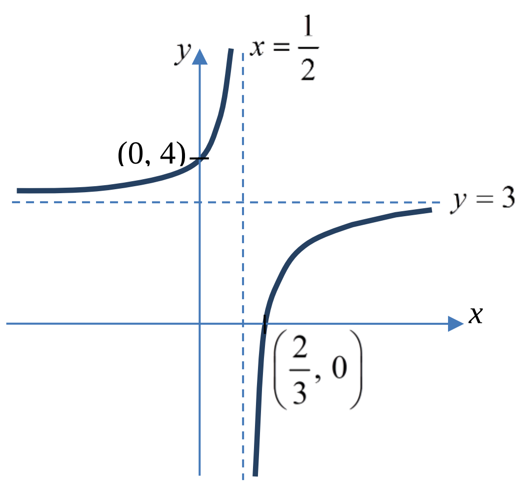
Therefore, we have

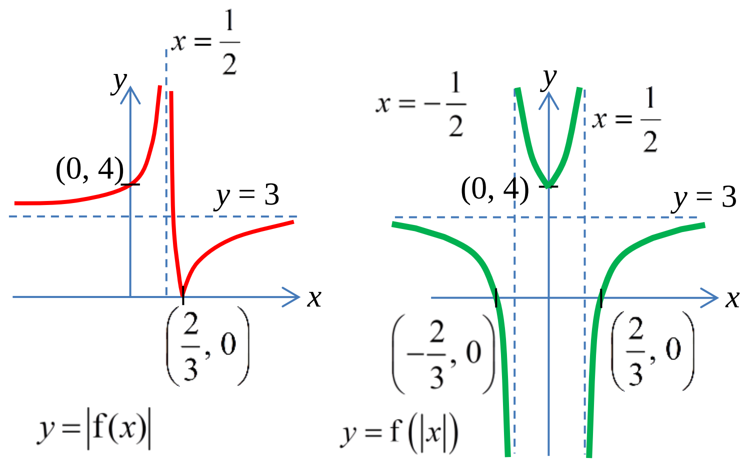
$$\frac{2}{1-(x+1)^2}\xrightarrow{C'}\frac{1}{1-(x+1)^2}\xrightarrow{B'}\frac{1}{(x+1)^2-1}\xrightarrow{A'}\frac{1}{x^2-1}$$

Since $\mathrm{f}(x)$ has vertical asymptotes at $x=a$ and $x=-a$, we conclude that $a=1$ as $a>0$

15	 y $y = f(\lvert x \rvert)$ (0,3) y = 2 x O $\left(-\frac{3}{2}, 0\right)$ $\left(\frac{3}{2}, 0\right)$ x = −1 x = 1  y $y = \lvert f(x) \rvert$ (0,3) y = 2 x O $\left(\frac{3}{2}, 0\right)$ x = 1

16 (a)	
(b)	

17	Equation of the new curve is $y=3-\frac{1}{2x-1}$. 



From the graph, for $|f(x)| > f(|x|)$, $x < -\frac{1}{2}$ or $\frac{1}{2} < x < \frac{2}{3}$.

SAJC Prelim 9758/2018/02/Q1	
18	$y = a(x-1) + \frac{b}{x+c}$ $\downarrow$ A (replace y with $-y$) $-y = a(x-1) + \frac{b}{x+c}$ $y = -a(x-1) - \frac{b}{x+c}$ $\downarrow$ B (replace x with $x+1$) $y = -a(x+1-1) - \frac{b}{x+1+c}$ $y = -ax - \frac{b}{x+1+c}$ Since $x = 0$ is a vertical asymptote, $1 + c = 0 \Rightarrow c = -1$. Therefore, $y = -ax - \frac{b}{x} = f(x)$. Since $\left(1, \frac{1}{6}\right)$ is a turning point on $y = \frac{1}{f(x)}$, $(1, 6)$ is a turning point on $y = f(x)$. $-a - b = 6$ --- (1) $\frac{dy}{dx} = -a + \frac{b}{x^2}$ Since when $x = 1$, $\frac{dy}{dx} = 0$, $-a + b = 0$ --- (2)

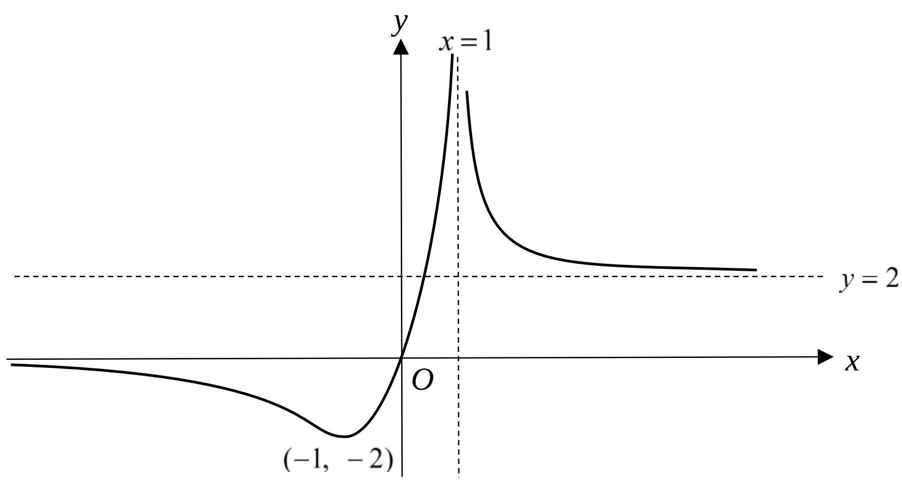
	Solving (1), (2) using a GC, $a = -3,\ b = -3$ Therefore, $a = -3,\ b = -3$ and $c = -1$

19(a)	$\mathrm{f}(3-x) = k$ Range of values of k is $-\frac{9}{8} < k < -1$.
(b)	The graph of $y = 2\mathrm{f}(3-x)$ undergoes 1) A reflection in the y-axis followed by 2) A scaling of ½ unit parallel to the y-axis $A\left(-3, -\frac{9}{4}\right) \xrightarrow{\text{(1) reflection in the } y\text{-axis}} A_1\left(3, -\frac{9}{4}\right)$ $\xrightarrow{\text{(2) scaling of a factor } \frac{1}{2} \text{ parallel to } y\text{-axis}} A_2\left(3, -\frac{9}{8}\right)$ Coordinates of the minimum point on the graph of $y = \mathrm{f}(3+x)$ are $\left(3, -\frac{9}{8}\right)$. Equation of vertical asymptote: $x = -1$ Equation of horizontal asymptote : $y = -1$

20	$y = p + \frac{1}{x+q}$ 1st : x replaced by $\frac{x}{4}$ to get $y = p + \frac{1}{\frac{x}{4}+q} = p + \frac{4}{x+4q}$ 2nd: y replaced by $y+3$ to get $y + 3 = p + \frac{4}{x+4q} \Rightarrow y = p - 3 + \frac{4}{x+4q}$ 3rd : x replaced by $-x$ to get $y = p - 3 + \frac{4}{-x+4q}$ Horizontal asymptote: $y = p - 3 = 1 \Rightarrow p = 4$ Vertical asymptote: $-x + 4q = 0$ $\Rightarrow x = 4q = 2 \Rightarrow q = \frac{1}{2}$

21. EJC/2022/2/Q2

The graph of $y = \mathrm{f}(x)$ is given below. It has one vertical asymptote at $x = 1$ and two horizontal asymptotes $y = 0$ and $y = 2$. The graph passes through the origin O and has a turning point at $(-1,\ -2)$.



(a) On separate diagrams, sketch the following graphs, indicating the coordinates of the points where the graphs cross the axes, the turning points, and the equations of any asymptotes if possible.

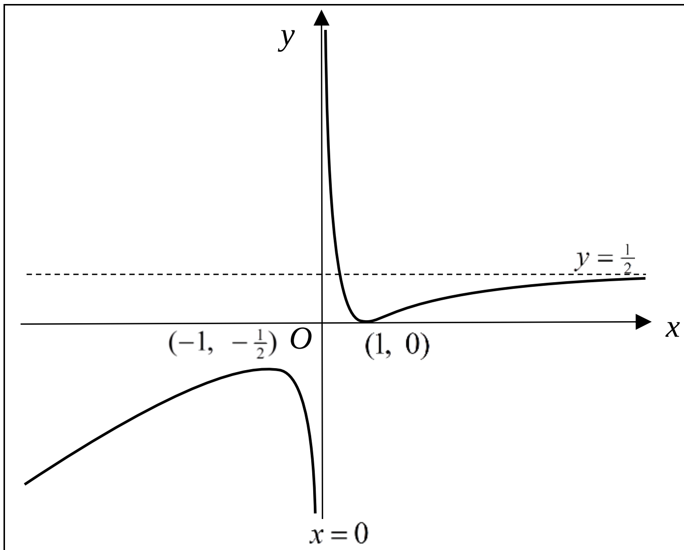
(i) $y = \dfrac{1}{\mathrm{f}(x)}$ [3]

(ii) $y = \mathrm{f}'(x)$ [3]

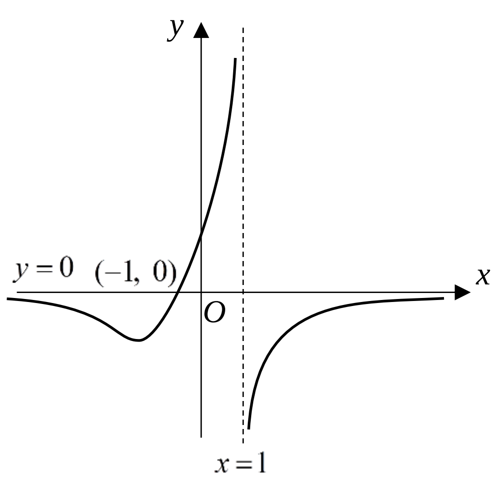
(iii) $y = \mathrm{f}(|x-1|)$ [2]

(b) State the range of values of a such that $\mathrm{f}(|x-1|) = a$ has only positive root(s). [1]

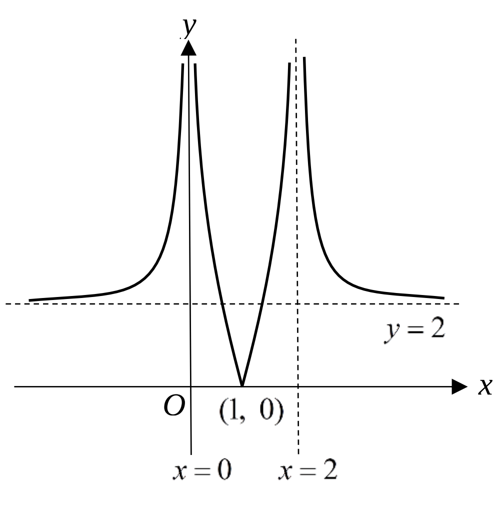
2(a)(i)



(a)(ii)



(a)(iii)



(b)	$0 \le a \le 2$